



Geometry

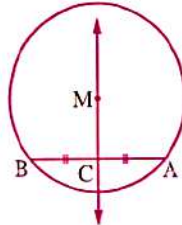
FINAL REVISION

GRADE 9 / PREP 3. SECOND TERM

SIMPLEST MATHS | Mr.Mohamed El-Shourbagy/01093149109

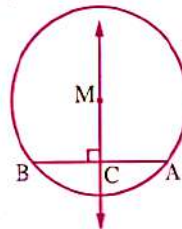
Revision

The straight line passing through the centre of the circle and the midpoint of any chord of it (not passing through the centre) is perpendicular to this chord.



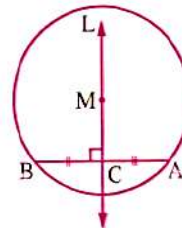
If \overline{AB} is a chord of the circle M and C is the midpoint of \overline{AB} , then : $\overline{MC} \perp \overline{AB}$

The straight line passing through the centre of the circle and perpendicular to any chord of it bisects this chord.



If \overline{AB} is a chord of the circle M and $\overline{MC} \perp \overline{AB}$, where $C \in \overline{AB}$, then : C is the midpoint of \overline{AB}

The perpendicular bisector of any chord of a circle passes through the centre of the circle.

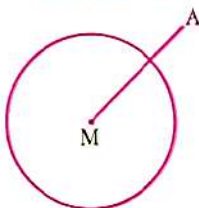


If \overline{AB} is a chord of the circle M, C is the midpoint of \overline{AB} and the straight line $L \perp \overline{AB}$ from the point C, then $M \in$ the straight line L

Position of a point with respect to a given circle

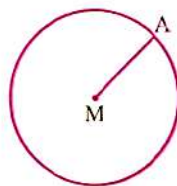
A is outside the circle M

If $MA > r$



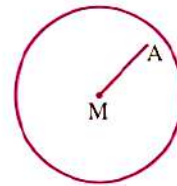
A is on the circle M

If $MA = r$



A is inside the circle M

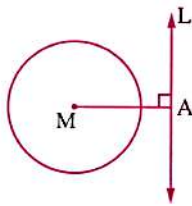
If $MA < r$



Position of a straight line L with respect to a circle M which is at a distance MA from its centre

L lies outside the circle M

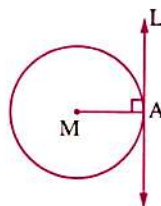
If $MA > r$



- $L \cap$ the circle M = \emptyset
- $L \cap$ the surface of the circle M = \emptyset

L touches the circle M

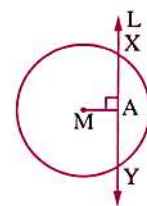
If $MA = r$



- $L \cap$ the circle M = {A}
- $L \cap$ the surface of the circle M = {A}

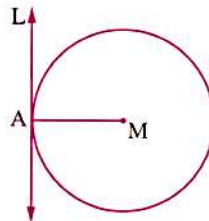
L is a secant to the circle M

If $MA < r$



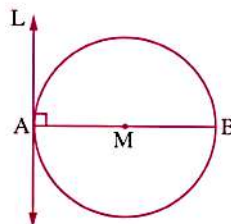
- $L \cap$ the circle M = {X, Y}
- $L \cap$ the surface of the circle M = \overline{XY}
 \overline{XY} is called the chord of intersection

The tangent to a circle is perpendicular to the radius drawn from the point of tangency.



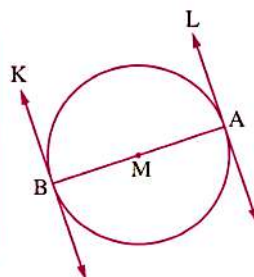
If L is a tangent to the circle M at the point A, then $\overline{MA} \perp L$

The straight line which is perpendicular to the diameter of a circle at one of its endpoints is a tangent to the circle.



If \overline{AB} is a diameter of the circle M, $L \perp \overline{AB}$ at the point A, then L is a tangent to the circle M at the point A

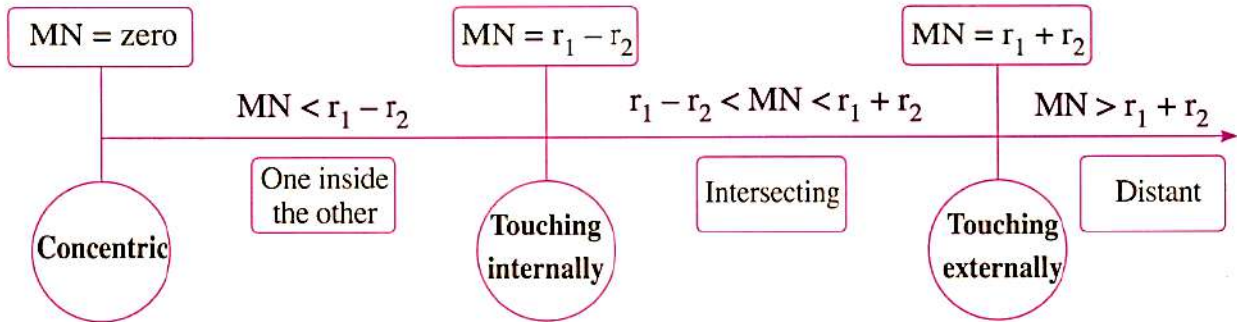
The two tangents which are drawn from the two endpoints of a diameter of a circle are parallel.



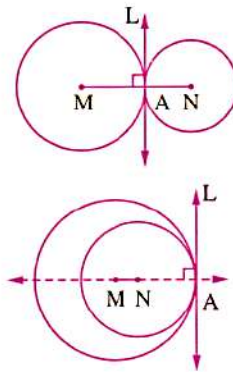
If \overline{AB} is a diameter in the circle M, L and K are tangents to the circle M at A, B, then $L \parallel K$

Position of the circle M with respect to the circle N

To determine the position of the circle M (with radius length r_1) with respect to the circle N (with radius length r_2), find $r_1 - r_2$, $r_1 + r_2$, then use the following diagram to determine the position (where $r_1 > r_2$)

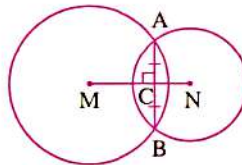


The line of centres of two touching circles passes through the point of tangency and is perpendicular to the common tangent at this point.



If the two circles M and N are touching at A, L is a common tangent to them at A, then $\overline{MN} \perp L$

The line of centres of two intersecting circles is perpendicular to the common chord and bisects it.



If M and N are intersecting circles at A and B, then $\overline{MN} \perp \overline{AB}$, $AC = BC$ (\overline{MN} is the axis of symmetry of \overline{AB})

Remarks on identifying the circle

- It is possible to draw an infinite number of circles passing through a given point.
- There is an infinite number of circles that can be drawn to pass through the two points A and B and all their centres lie on the axis of symmetry of \overline{AB}
- The smallest circle passing through the two points A, B is the circle in which \overline{AB} is a diameter in it and its centre is the midpoint of \overline{AB} and the length of its radius = $\frac{1}{2} AB$
- It is impossible to draw a circle passing through three collinear points.

- There is a unique circle passing through three points as A , B and C which are not collinear and the centre of this circle is the point of intersection of any two axes of symmetry of the axes of the line segments \overline{AB} , \overline{BC} and \overline{AC}

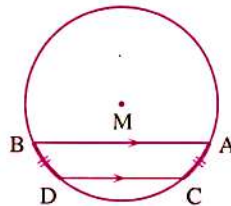
Equality of arcs in measure and length

In the same circle (or in congruent circles) , if the measures of arcs are equal , then the lengths of the arcs are equal and vice versa.



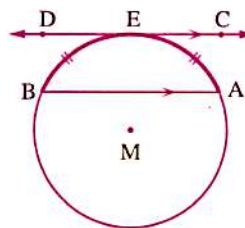
If $m(\widehat{AB}) = m(\widehat{CD})$, then the length of $\widehat{AB} =$ the length of \widehat{CD} and vice versa

If two parallel chords are drawn in a circle , then the measures of the two arcs between them are equal.



If $\overline{AB} \parallel \overline{CD}$, then $m(\widehat{AC}) = m(\widehat{BD})$

If a chord is parallel to a tangent of a circle , then the measures of the two arcs between them are equal.



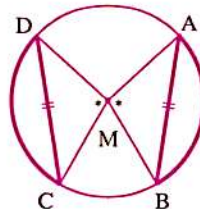
If $\overline{CD} \parallel \overline{AB}$, then $m(\widehat{EA}) = m(\widehat{EB})$

Notice that

$$\frac{\text{The measure of the arc}}{360^\circ} = \frac{\text{The length of the arc}}{2 \pi r}$$

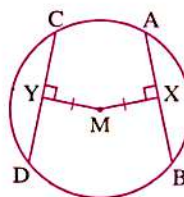
Equality of two chords in length

In the same circle (or in congruent circles) , if the measures of arcs are equal , then their chords are equal in length and vice versa.



If $m(\widehat{AB}) = m(\widehat{CD})$, then $AB = CD$ and vice versa

In the same circle (or in congruent circles) , if chords of a circle are equal in length , then they are equidistant from the centre and vice versa.

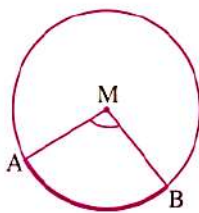


If $AB = CD$, $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{CD}$, then $MX = MY$ and vice versa

Central angle, inscribed angle and angle of tangency and relation between them

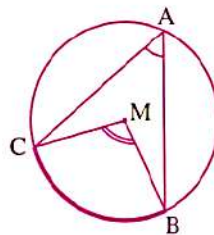
The measure of the central angle

Equals the measure of the subtended arc



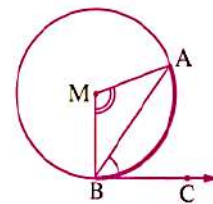
$$m(\angle M) = m(\widehat{AB})$$

Equals twice the measure of the inscribed angle subtended by the same arc



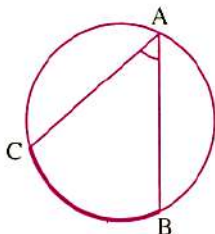
$$m(\angle M) = 2 m(\angle A)$$

Equals twice the measure of the angle of tangency subtended by the same arc



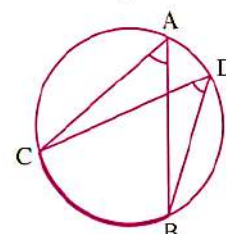
$$m(\angle M) = 2 m(\angle ABC)$$

Equals half the measure of the subtended arc



$$m(\angle A) = \frac{1}{2} m(\widehat{BC})$$

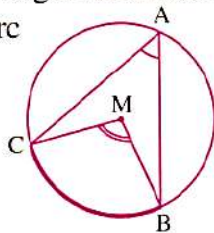
Equals the measure of the inscribed angle subtended by the same arc



$$m(\angle A) = m(\angle D)$$

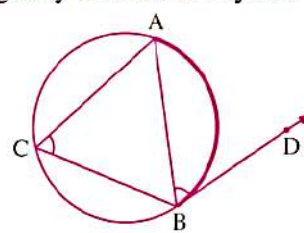
The measure of the inscribed angle

Equals half the measure of the central angle subtended by the same arc



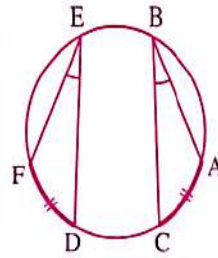
$$m(\angle A) = \frac{1}{2} m(\angle M)$$

Equals the measure of the angle of tangency subtended by the same arc



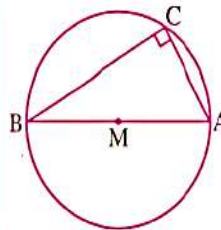
$$m(\angle C) = m(\angle ABD)$$

In the same circle (or in any number of circles), the measures of the inscribed angles subtended by arcs of equal measures are equal and vice versa.



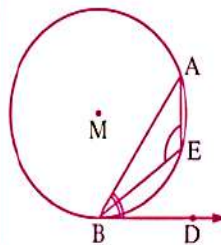
If $m(\widehat{AC}) = m(\widehat{DF})$
 , then $m(\angle B) = m(\angle E)$
 and vice versa

The inscribed angle in a semicircle is a right angle



If \overline{AB} is a diameter, then
 $m(\angle C) = 90^\circ$

The angle of tangency is supplementary to the drawn inscribed angle on the chord of the angle of tangency and in one side of it.

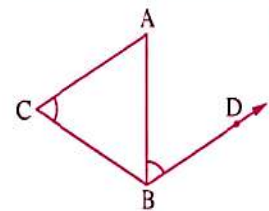


If $\angle AEB$ is inscribed drawn on \overline{AB} , $\angle ABD$ is angle of tangency, then
 $m(\angle ABD) + m(\angle AEB) = 180^\circ$

Notice that

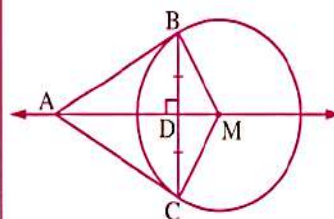
To prove that \overrightarrow{BD} is a tangent to the circumcircle of $\triangle ABC$

Prove that : $m(\angle ABD) = m(\angle ACB)$



Relation between tangents of the circle

The two tangent-segments drawn to a circle from a point outside it are equal in length

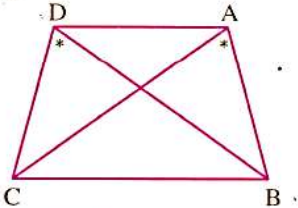


If \overline{AB} and \overline{AC} are tangent segments to the circle M, then

- $AB = AC$
- \overrightarrow{AM} bisects $\angle BAC$ and $\angle BMC$
- $\overrightarrow{AM} \perp \overline{BC}$ and bisects it.

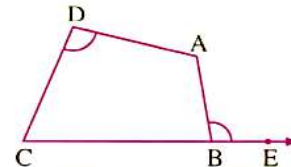
Cyclic quadrilateral

If there are two equal angles in measure and drawn on one of its sides as a base and on one side of this side.



If $m(\angle BAC) = m(\angle BDC)$

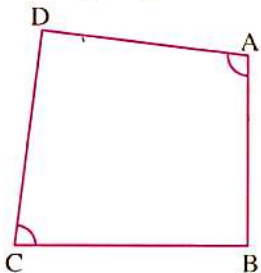
If there is an exterior angle at any of its vertices equal in measure to the measure of the interior angle at the opposite vertex.



If $m(\angle ABE) = m(\angle D)$

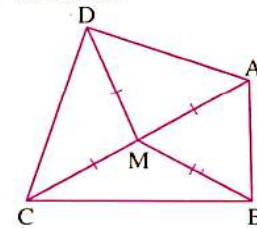
When is the quadrilateral cyclic ?

If there are two opposite supplementary angles.



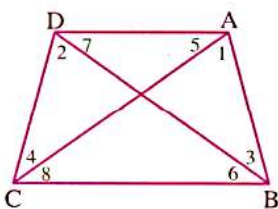
If $m(\angle A) + m(\angle C) = 180^\circ$

If there is a point in the plane of the figure such that it is equidistant from its vertices.

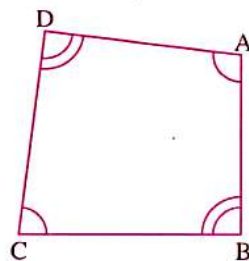


If $MA = MB = MC = MD$

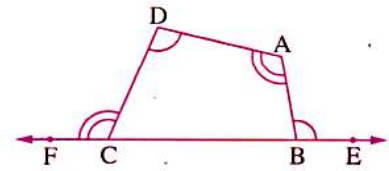
Properties of the cyclic quadrilateral



- $m(\angle 1) = m(\angle 2)$
- $m(\angle 3) = m(\angle 4)$
- $m(\angle 5) = m(\angle 6)$
- $m(\angle 7) = m(\angle 8)$



- $m(\angle A) + m(\angle C) = 180^\circ$
- $m(\angle B) + m(\angle D) = 180^\circ$

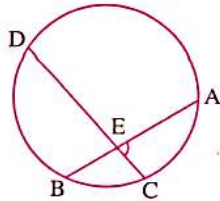


- $m(\angle ABE) = m(\angle D)$
- $m(\angle DCF) = m(\angle A)$

Well known problems

Well known problem (1)

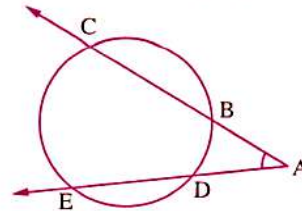
If \overline{AB} and \overline{CD} are two chords in a circle intersecting at the point E, then :



$$m(\angle AEC) = \frac{1}{2} [m(\widehat{AC}) + m(\widehat{BD})]$$

Well known problem (2)

If \overline{CB} and \overline{ED} are two chords in a circle, where $\overline{CB} \cap \overline{ED} = \{A\}$, then :

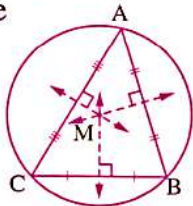


$$m(\angle A) = \frac{1}{2} [m(\widehat{CE}) - m(\widehat{BD})]$$

Circumcircle and inscribed circle of the triangle

The circumcircle of the triangle

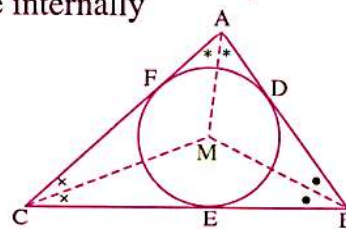
Is the circle that passes through the vertices of the triangle



and its centre is the point of intersection of the perpendicular bisectors of its sides

The inscribed circle of the triangle

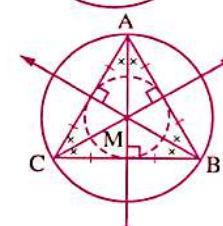
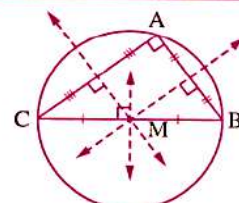
Is the circle that touches all sides of the triangle internally



and its centre is the intersection point of the bisectors of its interior angles

Notice that

- 1 The centre of the circumcircle of the right-angled triangle is the midpoint of its hypotenuse.
- 2 The centre of the circumcircle of the equilateral triangle is the same centre of the inscribed circle to it which is the point of intersection of axes of its sides and the point of intersection of its medians and the point of intersection of the bisectors of its interior angles and also the point of intersection of its altitudes.
- 3 It's possible to draw a circumcircle to a rectangle, a square and an isosceles trapezium while it's impossible to draw a circumcircle to a parallelogram, a rhombus and not isosceles trapezium.



Important theorems and their proofs

Theorem 1

If chords of a circle are equal in length, then they are equidistant from the centre.

Given

$$AB = CD, \overline{MX} \perp \overline{AB} \text{ and } \overline{MY} \perp \overline{CD}$$

R.T.P.

$$MX = MY$$

Construction

Draw \overline{MA} and \overline{MC}

Proof

$$\therefore \overline{MX} \perp \overline{AB}$$

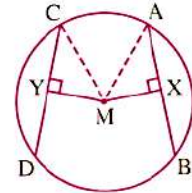
$$\therefore X \text{ is the midpoint of } \overline{AB} \therefore AX = \frac{1}{2} AB$$

$$\therefore \overline{MY} \perp \overline{CD} \therefore Y \text{ is the midpoint of } \overline{CD}$$

$$\therefore CY = \frac{1}{2} CD \therefore AB = CD \text{ (given)} \therefore AX = CY$$

$$\therefore \triangle AXM \text{ and } \triangle CYM, \text{ both have } \begin{cases} AX = CY \text{ (by proof)} \\ MA = MC = r \\ m(\angle AXM) = m(\angle CYM) = 90^\circ \end{cases}$$

$$\therefore \triangle AXM \cong \triangle CYM, \text{ then we get : } MX = MY \quad \text{(Q.E.D.)}$$



Theorem 2

The measure of the inscribed angle is half the measure of the central angle, subtended by the same arc.

Given

In the circle M : $\angle ACB$ is an inscribed angle,
 $\angle AMB$ is a central angle

R.T.P.

$$m(\angle ACB) = \frac{1}{2} m(\angle AMB)$$

Proof

$\therefore \angle AMB$ is an exterior angle of $\triangle AMC$

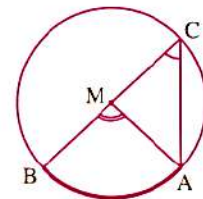
$$\therefore m(\angle AMB) = m(\angle A) + m(\angle C) \quad (1)$$

$\therefore MA = MC$ (two radii lengths)

$$\therefore m(\angle A) = m(\angle C) \quad (2)$$

From (1) and (2) we get : $m(\angle AMB) = 2 m(\angle ACB)$

$$\therefore m(\angle ACB) = \frac{1}{2} m(\angle AMB) \quad \text{(Q.E.D.)}$$



Theorem 3

In the same circle, the measures of all inscribed angles subtended by the same arc are equal.

Given $\angle C$, $\angle D$ and $\angle E$ are inscribed angles subtended by \widehat{AB}

R.T.P. $m(\angle C) = m(\angle D) = m(\angle E)$

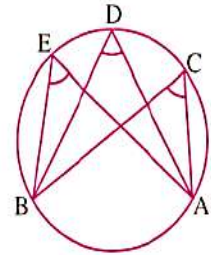
Proof $\therefore m(\angle C) = \frac{1}{2} m(\widehat{AB})$

$, m(\angle D) = \frac{1}{2} m(\widehat{AB})$

$, m(\angle E) = \frac{1}{2} m(\widehat{AB})$

$\therefore m(\angle C) = m(\angle D) = m(\angle E)$

(Q.E.D.)



Theorem 4

In a cyclic quadrilateral, each two opposite angles are supplementary.

Given ABCD is a cyclic quadrilateral

R.T.P. ① $m(\angle A) + m(\angle C) = 180^\circ$

② $m(\angle B) + m(\angle D) = 180^\circ$

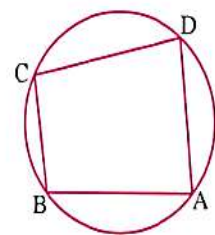
Proof $\therefore m(\angle A) = \frac{1}{2} m(\widehat{BCD})$ and $m(\angle C) = \frac{1}{2} m(\widehat{BAD})$

$\therefore m(\angle A) + m(\angle C) = \frac{1}{2} [m(\widehat{BCD}) + m(\widehat{BAD})]$

$= \frac{1}{2} \text{ the measure of the circle} = \frac{1}{2} \times 360^\circ = 180^\circ$

Similarly : $m(\angle B) + m(\angle D) = 180^\circ$

(Q.E.D.)

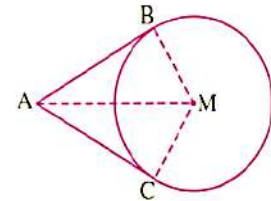


Theorem 5

The two tangent-segments drawn to a circle from a point outside it are equal in length.

Given

A is a point outside the circle M
 , \overline{AB} and \overline{AC} are two tangent-segments
 to the circle at B and C respectively.



R.T.P.

$AB = AC$

Construction

Draw \overline{MB} , \overline{MC} , \overline{MA}

Proof

$\therefore \overline{AB}$ is a tangent to the circle M $\therefore m(\angle ABM) = 90^\circ$

$\therefore \overline{AC}$ is a tangent to the circle M $\therefore m(\angle ACM) = 90^\circ$

\therefore In $\Delta \Delta ABM$, ACM : $\begin{cases} MB = MC \text{ (the lengths of two radii)} \\ \overline{AM} \text{ is a common side.} \\ m(\angle ABM) = m(\angle ACM) = 90^\circ \text{ (proved)} \end{cases}$

$\therefore \Delta ABM \equiv \Delta ACM$ and we deduce that : $AB = AC$ (Q.E.D.)

Theorem 6

The measure of the angle of tangency is equal to the measure of the inscribed angle subtended by the same arc.

Given

$\angle BAC$ is an angle of tangency and $\angle D$ is an inscribed angle.

R.T.P.

$m(\angle BAC) = m(\angle D)$

Proof

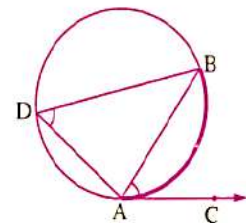
$\therefore \angle BAC$ is an angle of tangency.

$\therefore m(\angle BAC) = \frac{1}{2} m(\widehat{AB})$ (1)

$\therefore \angle D$ is an inscribed angle

$\therefore m(\angle D) = \frac{1}{2} m(\widehat{AB})$ (2)

From (1) and (2) , we deduce that : $m(\angle BAC) = m(\angle D)$ (Q.E.D.)



Some governorates' examinations

1

Cairo Governorate

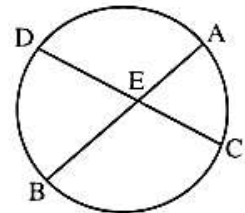
Answer the following questions :

(Calculators are permitted)

1 Choose the correct answer from the given ones :

(1) In the opposite figure :

$m(\widehat{BD}) = 80^\circ$,
 $m(\widehat{AC}) = 60^\circ$, then
 $m(\angle AEC) = \dots\dots\dots$



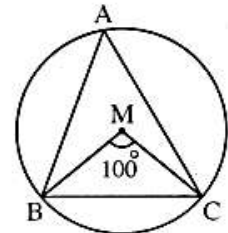
- (a) 20° (b) 30° (c) 70° (d) 140°

(2) The two tangents which are drawn from the two endpoints of a diameter of a circle are

- (a) parallel. (b) intersecting. (c) perpendicular. (d) coincide.

(3) In the opposite figure :

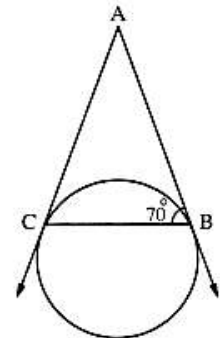
M is a circle , $m(\angle BMC) = 100^\circ$
 , then $m(\angle BAC) = \dots\dots\dots$



- (a) 150° (b) 100°
 (c) 50° (d) 25°

(4) In the opposite figure :

\overline{AB} and \overline{AC} are two tangents to
 the circle at B and C , $m(\angle ABC) = 70^\circ$
 , then $m(\angle A) = \dots\dots\dots$



- (a) 140° (b) 70°
 (c) 40° (d) 35°

(5) Sum of the measures of any two opposite angles in the cyclic quadrilateral equals

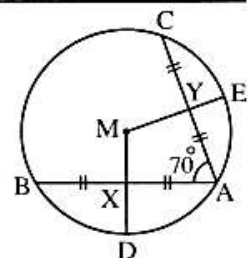
- (a) 90° (b) 180° (c) 270° (d) 360°

(6) Measure of an arc which represents $\frac{1}{3}$ of the measure of the circle equals =

- (a) 60° (b) 90° (c) 120° (d) 180°

2 [a] In the opposite figure :

\overline{AB} and \overline{AC} are two equal chords in length
 in the circle M, X is the midpoint of \overline{AB} and Y
 is the midpoint of \overline{AC} , $m(\angle CAB) = 70^\circ$



- (1) Calculate : $m(\angle DME)$ (2) Prove that : $XD = YE$

[b] In the opposite figure :

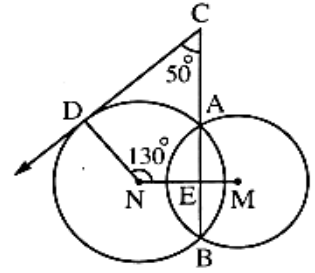
M and N are two circles intersecting at A and B.

and $C \in \overrightarrow{BA}$,

$D \in$ the circle N, $m(\angle MND) = 130^\circ$,

$m(\angle BCD) = 50^\circ$,

Prove that : \overrightarrow{CD} is a tangent to the circle at D

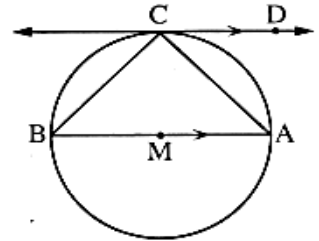


3 [a] In the opposite figure :

\overrightarrow{CD} is a tangent to the circle M at C,

$\overrightarrow{CD} \parallel \overrightarrow{BA}$

Prove that : $m(\angle DCA) = 45^\circ$

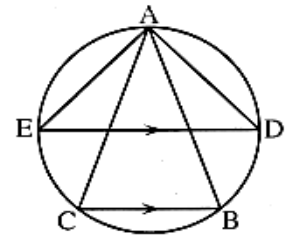


[b] In the opposite figure :

$\overrightarrow{DE} \parallel \overrightarrow{BC}$

Prove that :

$m(\angle DAC) = m(\angle BAE)$

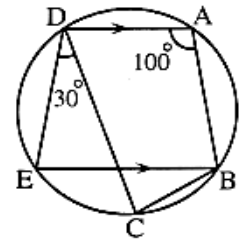


4 [a] In the opposite figure :

$\overrightarrow{AD} \parallel \overrightarrow{BE}$, $m(\angle BAD) = 100^\circ$

and $m(\angle CDE) = 30^\circ$

Find : $m(\angle ADC)$



[b] In the opposite figure :

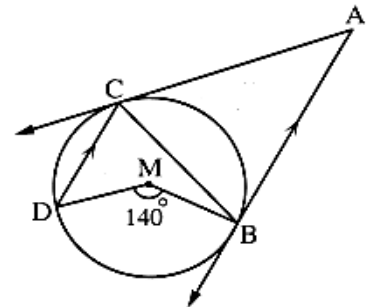
\overrightarrow{AB} and \overrightarrow{AC} are two tangents to the circle

M at B and C

$\overrightarrow{AB} \parallel \overrightarrow{CD}$,

$m(\angle BMD) = 140^\circ$

Find : $m(\angle A)$



5 [a] In the opposite figure :

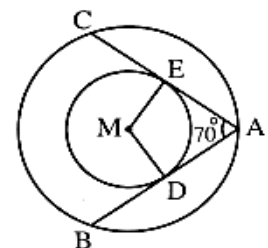
Two concentric circles at M,

\overrightarrow{AB} and \overrightarrow{AC} are two tangent segments to

the smaller circles, $m(\angle A) = 70^\circ$

(1) **Find :** $m(\angle DME)$

(2) **Prove that :** $AB = AC$

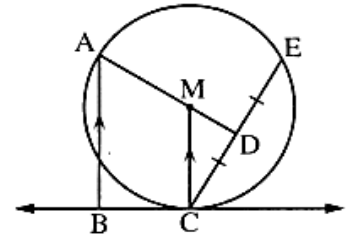


[b] In the opposite figure :

\overline{BC} is a tangent to the circle M at C ,

D is the midpoint of \overline{EC} , $\overline{MC} \parallel \overline{AB}$

Prove that : ABCD is a cyclic quadrilateral.



2

Giza Governorate

Answer the following questions :

(Calculators are permitted)

1 Choose the correct answer from the given ones :

(1) In the opposite figure :

\overline{BC} is a tangent to the circle M at B

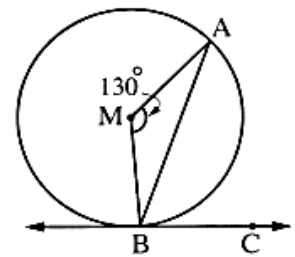
if $m(\angle AMB) = 130^\circ$, then $m(\angle ABC) = \dots\dots\dots$

(a) 280°

(b) 140°

(c) 70°

(d) 65°



(2) The length of the arc which represents $\frac{1}{4}$ of circumference of a circle = $\dots\dots\dots$

(a) $2\pi r$

(b) πr

(c) $\frac{1}{2}\pi r$

(d) $\frac{1}{4}\pi r$

(3) In a cyclic quadrilateral, each two opposite angles are $\dots\dots\dots$

(a) equal.

(b) supplementary.

(c) intersecting.

(d) corresponding.

(4) If surface of circle M \cap surface of circle N = \emptyset , then the two circles are $\dots\dots\dots$

(a) intersecting.

(b) distant.

(c) touching internally.

(d) touching externally.

(5) In the opposite figure :

Circle M, if $m(\angle MBA) = 20^\circ$

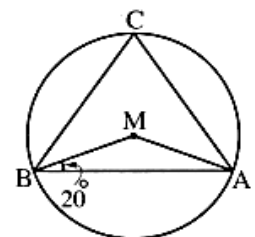
, then $m(\angle C) = \dots\dots\dots$

(a) 120°

(b) 70°

(c) 40°

(d) 30°



(6) In the opposite figure :

If \overline{AD} is a tangent to the circle at A

, $m(\angle DAB) = 105^\circ$

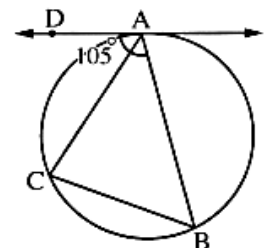
, then $m(\angle ACB) = \dots\dots\dots$

(a) 75°

(b) 60°

(c) 50°

(d) 35°



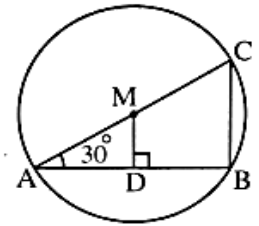
2 [a] In the opposite figure :

A circle of centre M , $\overline{MD} \perp \overline{AB}$,

If $m(\angle A) = 30^\circ$

(1) Prove that : $\overline{MD} \parallel \overline{CB}$

(2) Find : $m(\angle C)$



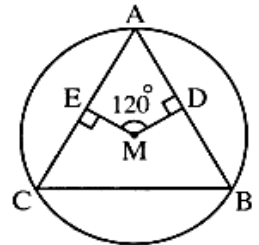
[b] In the opposite figure :

A circle M, $\overline{MD} \perp \overline{AB}$,

$\overline{ME} \perp \overline{AC}$, where $MD = ME$,

$m(\angle DME) = 120^\circ$

Prove that : the triangle ABC is equilateral.



3 [a] In the opposite figure :

If : $AB = AD$, $m(\angle ABD) = 30^\circ$, $m(\angle C) = 60^\circ$

Prove that : ABCD is a cyclic quadrilateral.

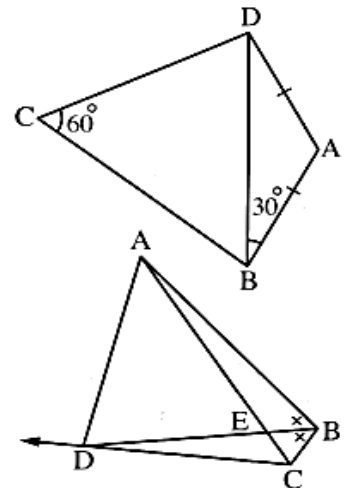
[b] In the opposite figure :

ABCD is a cyclic quadrilateral, \overline{BD} bisects $\angle ABC$,

If $\overline{BD} \cap \overline{AC} = \{E\}$

Prove that : \overline{CD} is a tangent to the circle

passing through the vertices of $\triangle BEC$



4 [a] In the opposite figure :

\overline{BC} is a diameter in the circle M ,

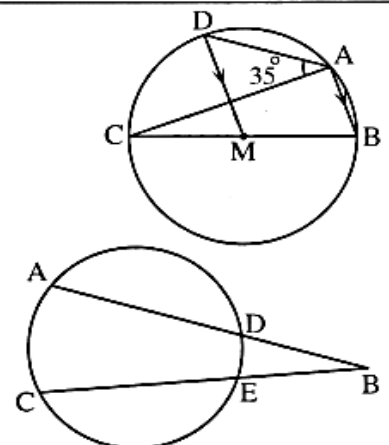
$m(\angle CAD) = 35^\circ$, $\overline{AB} \parallel \overline{DM}$,

Find : $m(\angle ABC)$

[b] In the opposite figure :

If $m(\widehat{AC}) = 120^\circ$, $m(\widehat{DE}) = 50^\circ$

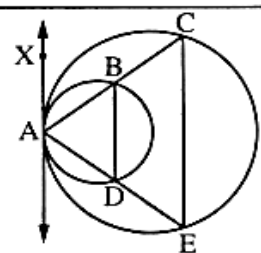
Find : $m(\angle ABC)$



5 [a] In the opposite figure :

If \overline{AX} is a common tangent to the two circles at A.

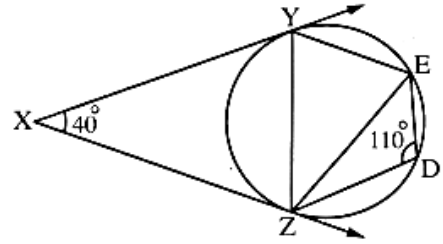
Prove that : $\overline{BD} \parallel \overline{CE}$



[b] In the opposite figure :

\overrightarrow{XY} and \overrightarrow{XZ} are two tangents
to the circle from the point X at Y , Z
, if $m(\angle EDZ) = 110^\circ$, $m(\angle YXZ) = 40^\circ$

Prove that : $m(\widehat{ZDE}) = m(\widehat{ZY})$



3

Alexandria Governorate

Answer the following questions :

(Calculators are permitted)

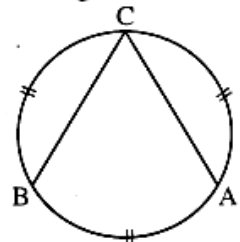
1 Choose the correct answer from the given ones :

- (1) The number of common tangents for the two tangent circles externally is
- (a) 4 (b) 3 (c) 2 (d) infinite number.
- (2) The figure which the circle doesn't passing through its vertices is
- (a) square. (b) rectangle. (c) rhombus. (d) triangle.

(3) In the opposite figure :

$m(\angle C) = \dots\dots\dots$

- (a) 45° (b) 50°
(c) 30° (d) 60°

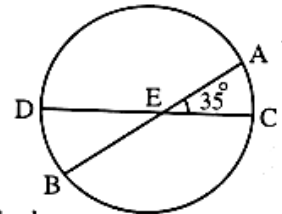


(4) In the opposite figure :

$\angle AEC = 35^\circ$

, then $m(\widehat{AC}) + m(\widehat{DB}) = \dots\dots\dots$

- (a) 17.5° (b) 35°
(c) 70° (d) 140°



(5) The inscribed angle opposite to an arc greater than the semicircle is

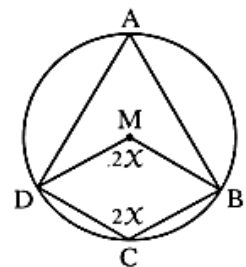
- (a) straight. (b) acute. (c) right. (d) obtuse.

(6) In the opposite figure :

If $m(\angle DMB) = m(\angle DCB) = 2x$

, then $m(\angle A) = \dots\dots\dots$

- (a) 60° (b) 70°
(c) 40° (d) 30°



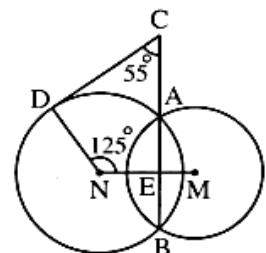
2 [a] In the opposite figure :

M and N are two intersecting circles at A and B

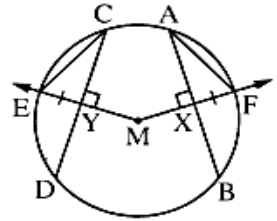
, $C \in \overrightarrow{BA}$, $D \in$ the circle N

, $m(\angle MND) = 125^\circ$ and $m(\angle BCD) = 55^\circ$

Prove that : \overrightarrow{CD} is a tangent to circle N at D

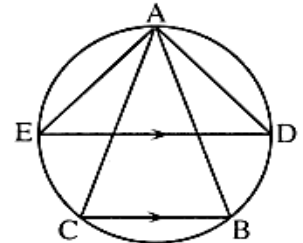


- [b] In the opposite figure : \overline{AB} and \overline{CD} are two chords in the circle M
 , $\overline{MX} \perp \overline{AB}$ and intersects the circle in F
 , $\overline{MY} \perp \overline{CD}$ and intersects the circle at E
 where $FX = EY$



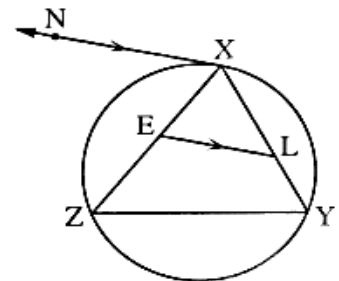
Prove that : (1) $AB = CD$ (2) $AF = CE$

- 3 [a] In the opposite figure :**
 ABC is an inscribed triangle inside a circle
 , $\overline{DE} \parallel \overline{BC}$



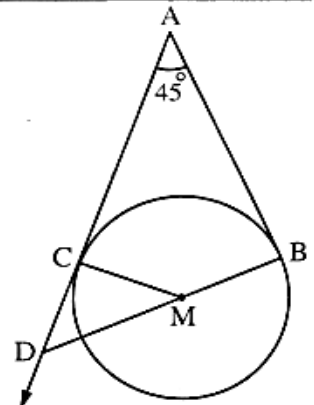
Prove that : $m(\angle DAC) = m(\angle BAE)$

- [b] In the opposite figure :
 XYZ is an inscribed triangle in a circle
 , \overline{LE} paralld tangent \overline{XN}



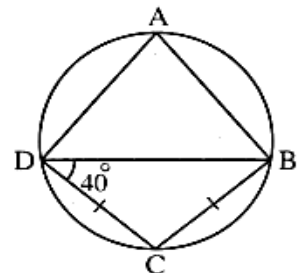
Prove that :
 LYZE is cyclic quadrilateral.

- 4 [a] In the opposite figure :**
 \overline{AB} , \overline{AC} are two tangents
 to circle M at B , C ,
 $m(\angle A) = 45^\circ$

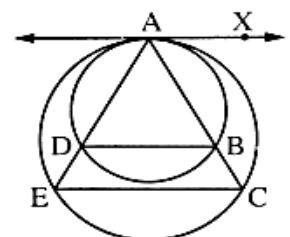


Prove that :
 (1) ABMC is cyclic quadrilateral.
 (2) $AD = AB + MB$

- [b] In the opposite figure :
 ABCD is a quadrilateral
 inscribed in circle ,
 $BC = CD$, $m(\angle BDC) = 40^\circ$
Find : $m(\angle A)$



- 5 [a] In the opposite figure :**
Prove that :
 $\overline{BD} \parallel \overline{CE}$



[b] \overline{AB} and \overline{CD} are two chords in the circle M, \overline{MX} is drawn perpendicular to \overline{AB} to intersect the circle in F and \overline{MY} is drawn perpendicular to \overline{CD} to intersect the circle at E, if $FX = EY$

Prove that :

(1) $AB = CD$

(2) $AF = CE$

3 [a] In the opposite figure :

E is a point outside the circle

Prove that :

$m(\angle DCB) > m(\angle E)$

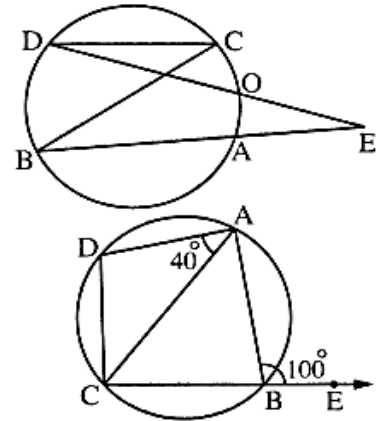
[b] In the opposite figure :

$m(\angle ABE) = 100^\circ$

, $m(\angle CAD) = 40^\circ$

Prove that :

$m(\widehat{CD}) = m(\widehat{AD})$



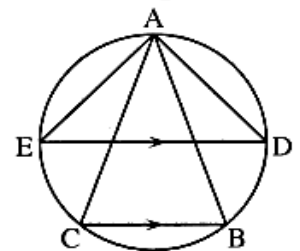
4 [a] Complete :

The straight line passing through the center of the circle and the intersection point of the two tangents are to the chord of tangency of those two tangents.

[b] In the opposite figure :

ABC is an inscribed triangle inside the circle
 $\overline{DE} \parallel \overline{BC}$

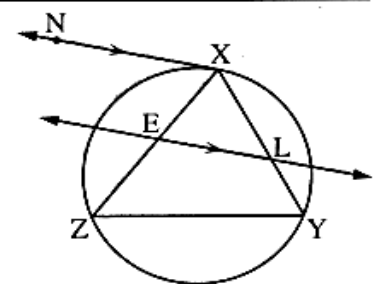
Prove that : $m(\angle DAC) = m(\angle BAE)$



5 In the opposite figure :

XYZ is an inscribed triangle in a circle, if $L \in \overline{XY}$ and \overline{LE} is drawn parallel to the tangent \overline{XN} which touches the circle at X and intersects \overline{XZ} at E

Prove that : LYZE is a cyclic quadrilateral.



5

El-Sharkia Governorate

Answer the following questions :

(Calculators are permitted)

1 Choose the correct answer :

(1) The measure of the circle with radius r is

(a) $2\pi r$

(b) 180°

(c) πr

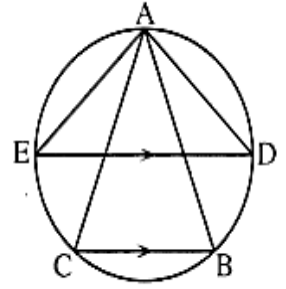
(d) 360°

[b] In the opposite figure :

\overline{ABC} is an inscribed triangle in the circle ,

$\overline{ED} \parallel \overline{BC}$

Prove that : $m(\angle DAC) = m(\angle BAE)$

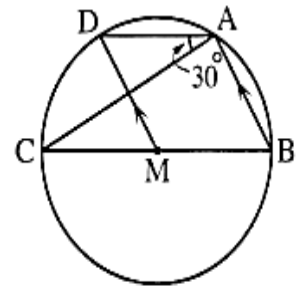


4 [a] In the opposite figure :

\overline{CB} is a diameter of circle M ,

$\overline{AB} \parallel \overline{DM}$, $m(\angle DAC) = 30^\circ$

Find : $m(\angle ACB)$



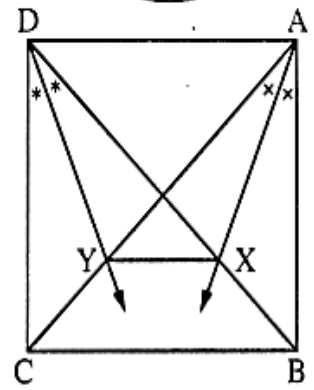
[b] In the opposite figure :

ABCD is a square , \overline{AX} bisects $\angle BAC$

and \overline{DY} bisects $\angle CDB$

(1) Prove that the figure AXYD is cyclic quadrilateral

(2) Find with proof $m(\angle DXY)$



5 In the opposite figure :

\overline{XZ} and \overline{XY} are two tangents at Z and Y ,

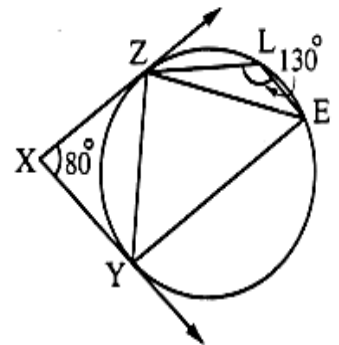
$m(\angle YXZ) = 80^\circ$, $m(\angle ELZ) = 130^\circ$

Prove that :

(1) $ZE = ZY$

(2) $\overline{XZ} \parallel \overline{YE}$

(3) \overline{ZE} is a tangent to the circle passing through the points X , Y and Z



Giza

2

1

- (1) (d) (2) (c) (3) (b)
 (4) (b) (5) (b) (6) (a)

2

[a] $\overline{MD} \perp \overline{AB}$ $\therefore m(\angle ADM) = 90^\circ$
 $\therefore \overline{AC}$ is a diameter in the circle M
 $\therefore m(\angle ABC) = 90^\circ$
 $\therefore m(\angle ADM) = m(\angle ABC) = 90^\circ$
 and they are corresponding angles.
 $\therefore \overline{MD} \parallel \overline{BC}$ (First req.)

In $\triangle ABC$: $\therefore m(\angle A) = 30^\circ$, $m(\angle ABC) = 90^\circ$
 $\therefore m(\angle C) = 180^\circ - (30^\circ + 90^\circ) = 60^\circ$ (Second req.)

[b] $\therefore \overline{MD} \perp \overline{AB}$ \therefore D is the midpoint of \overline{AB}
 $\therefore \overline{ME} \perp \overline{AC}$ \therefore E is the midpoint of \overline{AC}
 $\therefore MD = ME$ $\therefore AB = AC$ (1)

From the quadrilateral ADME
 $m(\angle A) = 360^\circ - (120^\circ + 90^\circ + 90^\circ) = 60^\circ$ (2)

From (1) and (2):

$\therefore \triangle ABC$ is an equilateral triangle. (Q.E.D.)

3

[a] In $\triangle ABD$: $\therefore AB = AD$
 $\therefore m(\angle ABD) = m(\angle ADB) = 30^\circ$
 $\therefore m(\angle A) = 180^\circ - 2 \times 30^\circ = 120^\circ$
 $\therefore m(\angle A) + m(\angle C) = 120^\circ + 60^\circ = 180^\circ$
 $\therefore ABCD$ is a cyclic quadrilateral (Q.E.D.)

[b] $\therefore ABCD$ is a cyclic quadrilateral
 $\therefore m(\angle DCA) = m(\angle DBA)$ (1)
 (drawn on \overline{AD} and on the same side of it)

$\therefore \overline{BD}$ bisects $\angle ABC$
 $\therefore m(\angle DBC) = m(\angle DBA)$ (2)

From (1) & (2): $\therefore m(\angle DBC) = m(\angle DCA)$

$\therefore \overline{CD}$ is a tangent to the circle passing through the vertices of $\triangle BEC$ (Q.E.D.)

4

[a] $\therefore m(\angle CMD) = 2m(\angle CAD)$
 (central and inscribed angles subtended by \widehat{CD})
 $\therefore m(\angle CMD) = 2 \times 35^\circ = 70^\circ$
 $\therefore \overline{AB} \parallel \overline{DM}$, \overline{BM} is a transversal
 $\therefore m(\angle ABC) = m(\angle CMD)$ (corresponding angles)
 $\therefore m(\angle ABC) = 70^\circ$ (The req.)

[b] $\therefore m(\angle ABC) = \frac{1}{2} [m(\widehat{AC}) - m(\widehat{DE})]$
 $\therefore m(\angle ABC) = \frac{1}{2} [120^\circ - 50^\circ]$
 $= \frac{1}{2} \times 70^\circ = 35^\circ$ (The req.)

5

[a] In the small circle
 $\therefore m(\angle XAB)$ (the tangency angle)
 $= m(\angle ADB)$ (the inscribed angle) (1)

In the great circle
 $\therefore m(\angle XAC)$ (the tangency angle)
 $= m(\angle AEC)$ (the inscribed angle) (2)

From (1) and (2):

$\therefore m(\angle ADB) = m(\angle AEC)$ but they are corresponding.
 $\therefore \overline{DB} \parallel \overline{EC}$ (Q.E.D.)

[b] $\therefore \overline{XY}$ and \overline{XZ} are two tangents
 $\therefore XY = XZ$
 \therefore In $\triangle XYZ$: $m(\angle XZY) = m(\angle XYZ)$
 $= \frac{180^\circ - 40^\circ}{2} = 70^\circ$

$\therefore m(\angle XZY)$ (tangency) = $m(\angle YEZ)$ (inscribed)
 $\therefore m(\angle YEZ) = 70^\circ$
 $\therefore DEYZ$ is a cyclic quadrilateral
 $\therefore m(\angle EYZ) + m(\angle D) = 180^\circ$
 $\therefore m(\angle EYZ) = 180^\circ - 110^\circ = 70^\circ$
 $\therefore m(\angle EYZ) = m(\angle YEZ) = 70^\circ$
 \therefore In $\triangle EYZ$: $ZE = ZY$
 $\therefore m(\widehat{ZDE}) = m(\widehat{ZY})$ (Q.E.D.)

Alexandria

3

1

- (1) (b) (2) (c) (3) (d)
 (4) (c) (5) (d) (6) (a)

2

[a] $\therefore \overline{MN}$ is the line of centres, \overline{AB} is the common chord
 $\therefore \overline{AB} \perp \overline{MN} \quad \therefore m(\angle AEN) = 90^\circ$
 \therefore The sum of the measures of the interior angles of the quadrilateral CDNE = 360°
 $\therefore m(\angle CDN) = 360^\circ - (55^\circ + 125^\circ + 90^\circ) = 90^\circ$
 $\therefore \overline{ND} \perp \overline{CD}$
 $\therefore \overline{CD}$ is a tangent to the circle N at D (Q.E.D.)

[b] $\therefore MF = ME$ (lengths of two radii)
 $\therefore XF = YE \quad \therefore MX = MY$
 $\therefore \overline{MX} \perp \overline{AB}, \overline{MY} \perp \overline{CD}$
 $\therefore AB = CD$ (Q.E.D. 1)
 $\therefore \overline{MX} \perp \overline{AB}$
 $\therefore X$ is the midpoint of $\overline{AB} \quad \therefore AX = \frac{1}{2} AB$
 $\therefore \overline{MY} \perp \overline{CD}$
 $\therefore Y$ is the midpoint of $\overline{CD} \quad \therefore CY = \frac{1}{2} CD$
 $\therefore AB = CD \quad \therefore AX = CY$
 $\therefore \triangle AXF, \triangle CYE$
 In them $\begin{cases} AX = CY \\ XF = YE \\ m(\angle AXF) = m(\angle CYE) = 90^\circ \end{cases}$
 $\therefore \triangle AXF \cong \triangle CYE$ then we deduce that $AF = CE$ (Q.E.D. 2)

3

[a] $\therefore \overline{DE} \parallel \overline{BC} \quad \therefore m(\widehat{BD}) = m(\widehat{CE})$
 $\therefore m(\angle DAB) = m(\angle CAE)$
 adding $m(\angle BAC)$ to both sides
 $\therefore m(\angle DAC) = m(\angle BAE)$ (Q.E.D.)
 [b] $\therefore \overline{LE} \parallel \overline{XN}, \overline{XZ}$ is a transversal
 $\therefore m(\angle XEL) = m(\angle NXZ)$ (alternate angles)
 $\therefore m(\angle Y)$ the inscribed = $m(\angle NXZ)$ of tangency
 $\therefore m(\angle Y) = m(\angle XEL)$
 \therefore The figure LYZE is a cyclic quadrilateral. (Q.E.D.)

4

[a] $\therefore \overline{AB}$ touches the circle at B $\therefore \overline{MB} \perp \overline{AB}$
 $\therefore \overline{AC}$ touches the circle at C $\therefore \overline{MC} \perp \overline{AC}$
 $\therefore m(\angle ABM) + m(\angle ACM) = 90^\circ + 90^\circ = 180^\circ$
 \therefore The figure ABMC is a cyclic quadrilateral (Q.E.D. 1)

$\therefore \angle CMD$ is an exterior angle of it
 $\therefore m(\angle CMD) = m(\angle A) = 45^\circ$
 \therefore In $\triangle MCD: m(\angle D) = 180^\circ - (90^\circ + 45^\circ) = 45^\circ$
 $\therefore CD = MC$ (1)
 $\therefore \overline{AC}, \overline{AB}$ are two tangent segments to the circle
 $\therefore AC = AB$ (2)
 Adding (1) and (2) : $\therefore CD + AC = MC + AB$
 $\therefore AD = AB + MC$
 $\therefore MC = MB$ (the lengths of two radii)
 $\therefore AD = AB + MB$ (Q.E.D. 2)

[b] In $\triangle CBD: \therefore CB = CD$
 $\therefore m(\angle CBD) = m(\angle CDB) = 40^\circ$
 $\therefore m(\angle C) = 180^\circ - 2 \times 40^\circ = 100^\circ$
 $\therefore \therefore ABCD$ is a cyclic quadrilateral
 $\therefore m(\angle A) + m(\angle C) = 180^\circ$
 $\therefore m(\angle A) = 180^\circ - 100^\circ = 80^\circ$ (The req.)

5

[a] In the small circle
 $\therefore m(\angle XAB)$ (the tangency angle)
 $= m(\angle ADB)$ (the inscribed angle) (1)
 In the great circle
 $\therefore m(\angle XAC)$ (the tangency angle)
 $= m(\angle AEC)$ (the inscribed angle) (2)
 From (1) and (2) :
 $\therefore m(\angle ADB) = m(\angle AEC)$ but they are corresponding.
 $\therefore \overline{DB} \parallel \overline{EC}$ (Q.E.D.)
 [b] $\therefore AB = CD \quad \therefore m(\widehat{AB}) = m(\widehat{CD})$
 Subtracting $m(\widehat{BD})$ from both sides
 $\therefore m(\widehat{AD}) = m(\widehat{BC}) \quad \therefore m(\angle C) = m(\angle A)$
 $\therefore \triangle ACE$ is isosceles (Q.E.D.)

El-Kalyoubia

4

1

- (1) (c) (2) (c) (3) (a)
 (4) (b) (5) (d) (6) (d)

2

[a] $\therefore \overline{MN}$ is the line of centres, \overline{AB} is the common chord
 $\therefore \overline{AB} \perp \overline{MN} \quad \therefore m(\angle AEN) = 90^\circ$
 \therefore The sum of the measures of the interior angles of the quadrilateral CDNE = 360°
 $\therefore m(\angle CDN) = 360^\circ - (55^\circ + 125^\circ + 90^\circ) = 90^\circ$
 $\therefore \overline{ND} \perp \overline{CD}$
 $\therefore \overline{CD}$ is a tangent to the circle N at D (Q.E.D.)

[b] $\therefore MF = ME$

(lengths of two radii)

$\therefore XF = YE$

$\therefore MX = MY$

$\therefore \overline{MX} \perp \overline{AB}, \overline{MY} \perp \overline{CD}$

$\therefore AB = CD$

(Q.E.D. 1)

$\therefore \overline{MX} \perp \overline{AB}$

$\therefore X$ is the midpoint of $\overline{AB} \quad \therefore AX = \frac{1}{2} AB$

$\therefore \overline{MY} \perp \overline{CD}$

$\therefore Y$ is the midpoint of $\overline{CD} \quad \therefore CY = \frac{1}{2} CD$

$\therefore AB = CD$

$\therefore AX = CY$

$\therefore \triangle AXF, \triangle CYE:$

In them $\begin{cases} AX = CY \\ XF = YE \\ m(\angle AXF) = m(\angle CYE) = 90^\circ \end{cases}$

$\therefore \triangle AXF \cong \triangle CYE$ then we deduce that $AF = CE$
 (Q.E.D. 2)

3

[a] $\therefore m(\angle E) = \frac{1}{2} [m(\widehat{BD}) - m(\widehat{AO})]$

$\therefore m(\angle E) = \frac{1}{2} m(\widehat{BD}) - \frac{1}{2} m(\widehat{AO})$

$\therefore m(\angle DCB) = \frac{1}{2} m(\widehat{BD})$

$\therefore m(\angle E) = m(\angle DCB) - \frac{1}{2} m(\widehat{AO})$

$\therefore m(\angle DCB) = m(\angle E) + \frac{1}{2} m(\widehat{AO})$

$\therefore m(\angle DCB) > m(\angle E)$ (Q.E.D.)

[b] $\therefore \angle ABE$ is an exterior angle of the cyclic quadrilateral ABCD

$\therefore m(\angle D) = m(\angle ABE) = 100^\circ$

In $\triangle ACD: m(\angle ACD) = 180^\circ - (100^\circ + 40^\circ) = 40^\circ$

$\therefore m(\angle ACD) = m(\angle CAD)$

$\therefore CD = AD \quad \therefore m(\widehat{CD}) = m(\widehat{AD})$ (Q.E.D.)

4

[a] an axis of symmetry.

[b] $\therefore \overline{DE} \parallel \overline{BC} \quad \therefore m(\widehat{BD}) = m(\widehat{CE})$

$\therefore m(\angle DAB) = m(\angle CAE)$

adding $m(\angle BAC)$ to both sides

$\therefore m(\angle DAC) = m(\angle BAE)$ (Q.E.D.)

5

$\therefore \overline{LE} \parallel \overline{XN}, \overline{XZ}$ is a transversal

$\therefore m(\angle XEL) = m(\angle NXZ)$ (alternate angles)

$\therefore m(\angle Y)$ the inscribed = $m(\angle NXZ)$ of tangency

$\therefore m(\angle Y) = m(\angle XEL)$

\therefore The figure LYZE is a cyclic quadrilateral (Q.E.D.)

El-Sharkia

5

1

(1) (d)

(2) (b)

(3) (d)

(4) (b)

(5) (c)

(6) (b)

2

[a] $\therefore \overline{CD}$ is a tangent to the circle

$\therefore \overline{MD} \perp \overline{CD} \quad \therefore m(\angle MDC) = 90^\circ$

$\therefore E$ is the midpoint of \overline{AB}

$\therefore \overline{ME} \perp \overline{AB}$

$\therefore m(\angle MEC) = 90^\circ$

$\therefore m(\angle DMF)$

$= 360^\circ - (40^\circ + 90^\circ + 90^\circ)$

$= 360^\circ - 220^\circ = 140^\circ$

$\therefore AE = \frac{1}{2} AB = 8 \text{ cm.}$

$\therefore AM = r = 10 \text{ cm.}$

In $\triangle AEM: \therefore m(\angle AEM) = 90^\circ$

$\therefore (ME)^2 = (AM)^2 - (AE)^2 = 100 - 64 = 36$

$\therefore ME = \sqrt{36} = 6 \text{ cm.}$

$\therefore FE = MF - ME = 10 - 6 = 4 \text{ cm.}$ (Second req.)

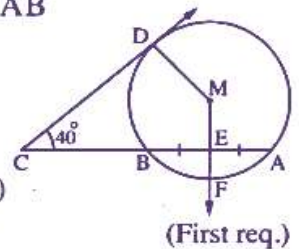
[b] $\therefore \overline{MN}$ is the line of centres

$\therefore \overline{AB}$ is the common chord of the two circles

$\therefore \overline{MN} \perp \overline{AB}$

$\therefore X$ is the midpoint of \overline{AC}

$\therefore \overline{MX} \perp \overline{AC}$



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$\therefore AB = AC \quad \therefore MX = MD$
 $\therefore MY = ME$ (lengths of two radii)
 $\therefore MY - MX = ME - MD$
 $\therefore XY = DE$ (Q.E.D.)

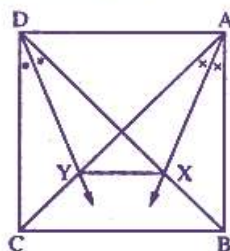
3

[a] $\therefore \overline{AB}$ is a diameter of the circle M
 $\therefore m(\angle ADB) = 90^\circ$
 In $\triangle ABD$: $\therefore m(\angle ABD) = 40^\circ$
 $\therefore m(\angle A) = 180^\circ - (90^\circ + 40^\circ) = 50^\circ$
 $\therefore ABCD$ is a cyclic quadrilateral
 $\therefore m(\angle C) + m(\angle A) = 180^\circ$
 $\therefore m(\angle C) = 180^\circ - 50^\circ = 130^\circ$ (The req.)
[b] $\therefore \overline{DE} \parallel \overline{BC}$
 $\therefore m(\widehat{BD}) = m(\widehat{CE})$
 $\therefore m(\angle DAB) = m(\angle CAE)$
 adding $m(\angle BAC)$ to both sides
 $\therefore m(\angle DAC) = m(\angle BAE)$ (Q.E.D.)

4

[a] $\therefore m(\angle DMC) = 2m(\angle CAD)$
 (central and inscribed angles subtended by \widehat{CD})
 $\therefore m(\angle DMC) = 2 \times 30^\circ = 60^\circ$
 $\therefore \overline{AB} \parallel \overline{DM}$, \overline{BC} is a transversal
 $\therefore m(\angle B) = m(\angle DMC) = 60^\circ$ (corresponding angles)
 $\therefore \overline{BC}$ is a diameter of circle M
 $\therefore m(\angle BAC) = 90^\circ$
 \therefore In $\triangle ABC$: $m(\angle ACB) = 180^\circ - (90^\circ + 60^\circ)$
 $= 30^\circ$ (The req.)

[b] $\therefore ABCD$ is a square, \overline{AC}
 and \overline{BD} are two diagonals
 of the square



$\therefore m(\angle BAC) = m(\angle BDC)$
 $\therefore \frac{1}{2} m(\angle BAC)$
 $= \frac{1}{2} m(\angle BDC)$
 $\therefore m(\angle XAY) = m(\angle XDY)$ but they are drawn
 On \overline{XY} and on one side of it
 \therefore The figure $AXYD$ is a cyclic quadrilateral (Q.E.D. 1)
 $\therefore m(\angle DXY) = m(\angle DAY) = 45^\circ$
 (They are drawn on \overline{DY} and on one side of it)
 (Q.E.D. 2)

5

$\therefore \overline{XY}, \overline{XZ}$ are tangentsegments to the circle at Y and Z
 $\therefore XY = XZ$
 $\therefore m(\angle XYZ) = m(\angle XZY) = \frac{180^\circ - 80^\circ}{2} = 50^\circ$
 $\therefore m(\angle ZEY)$ (the inscribed angle)
 $= m(\angle ZYX)$ (the tangency angle) $= 50^\circ$
 \therefore The figure $LEYZ$ is a cyclic quadrilateral
 $\therefore m(\angle ZYE) = 180^\circ - 130^\circ = 50^\circ$
 $\therefore m(\angle ZEY) = m(\angle ZYE) = 50^\circ$
 $\therefore ZE = ZY$ (Q.E.D. 1)
 $\therefore m(\angle XZY) = m(\angle ZYE) = 50^\circ$
 but they are alternate angles.
 $\therefore \overline{XZ} \parallel \overline{YE}$ (Q.E.D. 2)
 \therefore In $\triangle ZYE$: $m(\angle EZY) = 180^\circ - 2 \times 50^\circ = 80^\circ$
 $\therefore m(\angle EZY) = m(\angle X) = 80^\circ$
 $\therefore \overline{ZE}$ is a tangent to the circle passing through
 The points X, Y and Z (Q.E.D. 3)