Grade 8

(Second Preparatory Year)

Second Semester

Name \ ……………………………………………………………

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Unit 1
Lesson 1

Properties of the ratio

The Ratio

Generally

If a and b are two real numbers, then :

The ratio between a and b is written $a: b$ or $\frac{a}{b}$ and is read a to b where :

a is called the antecedent of the ratio, b is called the consequent of the ratio, a and b are called together the two terms of the ratio.

Properties of the ratio

- The value of the ratio does not change if each of its terms is multiplied or divided by the same non-zero real number.
- The value of the ratio ($\neq 1$) changes if we add or subtract (to or from) each of its two terms a non-zero real number.

The Proportion

It is the equality of two ratios or more.

i.e. If $\frac{a}{b} = \frac{c}{d}$, then the quantities a, b, c and d are proportional.

And vice versa : If a, b, c and d are proportional, then : $\frac{a}{b} = \frac{c}{d}$

- a is called the first proportional. b is called the second proportional.
- c is called the third proportional. d is called the fourth proportional.

a and d are called extremes and b and c are called means.

For example:

The numbers $1, 4, 7$ and 28 are proportional numbers, because $\frac{1}{4} = \frac{7}{28}$

And : 1 is the first proportional, 4 is the second proportional, 7 is the third proportional, 28 is the fourth proportional, 1 and 28 are the extremes of this proportion and are the means.

Properties of proportion

- Property (1)

If $\frac{a}{b} = \frac{c}{d}$, then : $a \times d = b \times c$ (The product of the extremes = the product of the means)

Example 1 :

(1)	The third proportional for the quantities 2,4 and 20 is .... $\therefore$ The quantities $2, 4, x$ and 20 are proportional $\therefore \frac{2}{4} = \frac{x}{20}$ $\therefore 2 \times 20 = 4 \times x$ $\therefore 40 = 4x \qquad \therefore x = 10$
(2)	The fourth proportional for the numbers 4, 12 and 16 is .... $\therefore$ The numbers $4, 12, 16$ and x are proportional $\therefore \frac{4}{12} = \frac{16}{x} \qquad \therefore x = \frac{12 \times 16}{4} = 48$
(3)	If $2, x, 4$ and 6 are proportional, then $x =$ ....

Properties of proportion

- Property (2)

If $a \times d = b \times c$, then $\frac{a}{b} = \frac{c}{d}$

Example 2 : In each of the following, find $\frac{x}{y}$ if:

(1)	$12x = 3y$ solu $\frac{x}{y} = \frac{3}{12} = \frac{1}{4}$.

(2)	$4x - 3y = 0$ **solu** $4x = 3y$ $\frac{x}{y} = \frac{3}{4}$
(3)	If $4x - 3y : 2x + y = \frac{4}{7}$, find in the simplest form the ratio $x : y$ **solu** $7(4x - 3y) = 4(2x + y)$ $\Rightarrow 28x - 21y = 8x + 4y$ $\Rightarrow 28x - 8x = 21y + 4y$ $20x = 25y$ $\frac{x}{y} = \frac{25}{20} = \frac{5}{4}$ $x : y = 5 : 4$
(4)	If $2x^2 - 6y^2 = xy$, find : $x : y$ **solu** $2x^2 - xy - 6y^2 = 0$ $(2x + 3y)(x - 2y) = 0$ $2x + 3y = 0$ $\quad$ $x - 2y = 0$ $2x = -3y$ $\quad$ $x = 2y$ $\frac{x}{y} = -\frac{3}{2}$ $\quad$ $\frac{x}{y} = 2$ $\frac{x}{y} = -\frac{3}{2}$ or , $\frac{x}{y} = 2$ $x : y = -3 : 2$ or $x : y = 2 : 1$
(5)	If $\frac{x+2y}{4x-3y} = \frac{7}{6}$, then prove that : $\frac{x}{y} = \frac{3}{2}$ ....

(6)	If $4a^2 - 9b^2 = 0$, find : a: b ....

Properties of proportion

- Property (3)

If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$

i.e. $\frac{\text{The antecedent of the first ratio}}{\text{The antecedent of the second ratio}} = \frac{\text{The consequent of the first ratio}}{\text{The consequent of the second ratio}}$

For example: If $\frac{a}{4} = \frac{b}{3}$, then $\frac{a}{b} = \frac{4}{3}$ and $\frac{b}{a} = \frac{3}{4}$

Properties of proportion

- Property (4)

If $\frac{a}{b} = \frac{c}{d}$, then $a = c\,k$ and $b = d\,k$ (where k is a constant $\neq 0$)

For example:

If $\frac{a}{b} = \frac{3}{4}$, then : $a = 3\,k$, $b = 4\,k$ (where m is a constant $\neq 0$)

Important remark

If a, b, c and d are proportional quantities and we assume that : $\frac{a}{b} = \frac{c}{d} = k$, then

$a = bk$, $c = dk$

For example:

If $\frac{a}{b} = \frac{c}{d} = 7$, then $a = 7\,b$, $c = 7\,d$

Example 3

(1)	If $a : b = 3 : 5$, find the ratio $20a - 7b : 15a + b$ **solu** $\because \frac{a}{b} = \frac{3}{5}$ $\therefore a = 3k$, $b = 5k$ (where $k \neq 0$) Substituting by a and b in terms of m : $\therefore \frac{20a - 7b}{15a + b} = \frac{60k - 35k}{45k + 5k} = \frac{25k}{50k} = \frac{1}{2}$
(2)	If $\frac{a}{b} = \frac{2}{3}$ and $\frac{x}{y} = \frac{3}{5}$, prove that : $(7aX + 4by), (11ay + bX), 12$ and 14 are proportional quantities. **solu** $\because \frac{a}{b} = \frac{2}{3}$ $\therefore a = 2m, b = 3m$ (where $m \neq 0$) $\because \frac{x}{y} = \frac{3}{5}$ $\therefore x = 3k, y = 5k$ (where $k \neq 0$) [Notice that: We used two different constants m and k] Substituting by a, b, X and y $\therefore \frac{7aX + 4by}{11ay + bX} = \frac{7 \times 2m \times 3k + 4 \times 3m \times 5k}{11 \times 2m \times 5k + 3m \times 3k}$ $= \frac{42mk + 60mk}{110mk + 9mk} = \frac{102mk}{119mk} = \frac{6}{7}$ $\because \frac{12}{14} = \frac{6}{7}$ $\therefore (7aX + 4by), (11ay + bX), 12$ and 14 are proportional quantities.
(3)	If $\frac{x}{y} = \frac{2}{5}$, prove that : $(2x + y), (x + 2y), 12$ and 16 are proportional quantities. ....

(4)	The ratio between two real numbers is $4:7$ If we subtract 16 from each of them , then the ratio between the two obtained numbers is $2:5$ Find the two numbers. **solu** Let the two numbers be a and b $\therefore \frac{a}{b} = \frac{4}{7}$ $\therefore \frac{4\,k - 16}{7\,k - 16} = \frac{2}{5}$ $\therefore 80 - 32 = 20\,k - 14\,k$ $\therefore a = 4 \times 8 = 32, b = 7 \times 8 = 56$ $\therefore a = 4\,k\ , b = 7\,k\ \text{(where } k \neq 0\text{)}$ $\therefore 14\,m - 32 = 20\,m - 80$ $\therefore 48 = 6\,m$ $\therefore m = \frac{48}{6} = 8$ i.e. The two numbers are 32 and 56
(5)	The ratio between two integers is $2:5$ If 2 is subtracted from the first integer and 1 is added to the second , then the ratio becomes $1:4$ Find the two integers. ....

Properties of the ratio (1)

1 : Choose the correct answer from the given ones :

1	If : $24, X, 6$ and 3 are proportional quantities , then $X =$ ___						
(a)	9	(b)	12	(c)	18	(d)	48
2	The fourth proportional for the $2, 6, 9$ is ___						
(a)	12	(b)	18	(c)	27	(d)	54
3	If $\frac{a}{b} = \frac{3}{2}$, then $\frac{a+b}{a-b} =$ ___						
(a)	$\frac{3}{2}$	(b)	$\frac{4}{5}$	(c)	5	(d)	2
4	If : $\frac{a}{b} = \frac{3}{4}$, then $4a - 3b + 5 =$ ___						
(a)	0	(b)	1	(c)	3	(d)	5
5	If : $\frac{a}{b} = \frac{5}{3}$, then $\frac{3a}{5b} =$ ___						
(a)	1	(b)	$\frac{5}{3}$	(c)	3	(d)	5
6	If : $\frac{a}{b} = \frac{c}{d} = \frac{3}{4}$, then $\frac{a+c}{b+d} =$ ___						
(a)	$\frac{3}{4}$	(b)	$\frac{7}{4}$	(c)	$\frac{3}{7}$	(d)	$\frac{9}{16}$
7	If : $\frac{a}{2} = \frac{b}{3}$, then $\frac{b-a}{b+a} =$ ___						
(a)	$\frac{1}{5}$	(b)	$\frac{1}{3}$	(c)	$\frac{2}{5}$	(d)	$\frac{3}{5}$
8	If : $\frac{a}{12} = \frac{b}{5} = \frac{a-2b}{k}$, then $k =$ ___						
(a)	1	(b)	2	(c)	3	(d)	4

9	If : $\frac{a}{5}=\frac{b}{4}=\frac{a+b}{k}$, then $k=$ ___						
(a)	5	(b)	4	(c)	9	(d)	1
10	If : $\frac{X}{Y}=\frac{Z}{1}$ which of the following is right ___						
(a)	$\frac{X}{1}=\frac{Y}{Z}$	(b)	$\frac{X}{Z}=\frac{1}{Y}$	(c)	$\frac{X}{Y}=\frac{1}{Z}$	(d)	$\frac{X}{Z}=\frac{Y}{1}$
11	If : $\frac{X}{2}=\frac{Y}{7}=\frac{X+Y}{K}$, then $K=$ ___						
(a)	4	(b)	10	(c)	9	(d)	14
12	$4X=25Y$, then $\frac{X}{Y}=$ ___						
(a)	$\frac{4}{25}$	(b)	$\frac{2}{5}$	(c)	$\frac{5}{2}$	(d)	$\frac{25}{4}$
13	If : A, X, B and $2X$ are proportional, then : $\frac{A}{B}=$ ___						
(a)	2: 1	(b)	1: 2	(c)	1: 3	(d)	1: 4
14	If: $\frac{3a}{5b}=\frac{1}{2}$, then : $\frac{a}{b}=$						
(a)	$\frac{6}{5}$	(b)	$\frac{5}{6}$	(c)	$\frac{2}{3}$	(d)	$\frac{3}{2}$
15	If : $\frac{a+b}{5}=\frac{a-b}{3}$, then : $\frac{a}{b}=$						
(a)	$\frac{1}{2}$	(b)	2	(c)	4	(d)	$\frac{1}{4}$
16	If : $2a=3b$, then $\frac{5b}{a}=$						
(a)	$\frac{5}{3}$	(b)	$\frac{5}{2}$	(c)	$\frac{15}{2}$	(d)	$\frac{10}{3}$
17	If $3x=5y$, then $\frac{5y}{3x}=$						
(a)	1	(b)	2	(c)	$\frac{3}{5}$	(d)	$\frac{5}{3}$
18	If $\frac{x}{2}=\frac{y}{7}=\frac{2x+y}{a}$, then a =						
(a)	9	(b)	11	(c)	16	(d)	5

(2) Find the value of x in each of the following, if:

(1)	$(2x-3):(x-5)=1:4$ …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… ……………………………………………………………………………
(2)	$(x-5):(5x+3)=2:3$ …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… ……………………………………………………………………………
(3)	$(x^2-8):(2x^2+1)=1:3$ …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… ……………………………………………………………………………

(3)

(1)	If $\frac{x-2y}{x+3y}=\frac{1}{3}$, find : $\frac{y}{x}$ …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… ……………………………………………………………………………

(2)	If $\frac{2x+3}{2x-3} = \frac{2y+5}{2y-5}$, prove that: $\frac{x}{y} = \frac{3}{5}$ ....
(3)	If $x^2 - 4y^2 = 3xy$, find : $x : y$ ....
(4)	If $3x^2 - 10xy + 7y^2 = 0, x \neq y$, find the ratio : $x : y$ ....
(5)	If $\frac{x}{y} = \frac{2}{3}$, find the value of the ratio: $\frac{3x+2y}{6y-x}$ ....
(6)	If $4a = 3b$, then find the value of : $\frac{4a+b}{2a-b}$ ....

(7)	If $\frac{a}{b}=\frac{1}{3},\frac{c}{d}=\frac{7}{2}$, find the ratio : $\frac{2ac+bd}{bc-3ad}$ …………………………………………………… …………………………………………………… …………………………………………………… ……………………………………………………
(8)	If $7x-3y: x+y=3:1$, find the ratio : $12x+9y: 11x-3y$ …………………………………………………… …………………………………………………… …………………………………………………… ……………………………………………………
(9)	If $\frac{21x+a}{7x+b}=\frac{a}{b}$, where $x\neq 0$, then find the value of : $\frac{a+2b}{2a}$ …………………………………………………… …………………………………………………… …………………………………………………… ……………………………………………………
(10)	Find the number that if it is added to each of the numbers $3, 5, 8$ and 12 , they become proportional. …………………………………………………… …………………………………………………… …………………………………………………… ……………………………………………………
(11)	Find the number which is subtracted from each of the following numbers to be proportional $16, 21, 14$ and 18 …………………………………………………… …………………………………………………… …………………………………………………… ……………………………………………………

(4) Prove that: a,b,c and d are proportional quantities if:

(1)	$\frac{a+b}{b}=\frac{c+d}{d}$ ....
(2)	$\frac{a}{b-a}=\frac{c}{d-c}$ ....
(3)	$\frac{a-b}{a+b}=\frac{c-d}{c+d}$ ....
(4)	$\frac{a^2-2c^2}{b^2-2d^2}=\frac{a^2}{b^2}$ where a, b, c and d are positive quantities. ....

(5)	
(1)	Find the number which if it is added to the two terms of the ratio $7:11$, it will be $2:3$ ....
(2)	Find the number that if we subtract thrice of it from each of the two terms of the ratio $\frac{49}{69}$, the ratio becomes $\frac{2}{3}$ ....
(3)	Find the number which if its square is added to each of the two terms of the ratio $7:11$, it becomes $4:5$ ....

(4)	Find the positive number which if we add its square to each of the two terms of the ratio $5:11$, it becomes $3:5$ ....
(5)	Two integers, the ratio between them is $3:7$ and if we subtracted 5 from each term , the ratio between them becomes $1:3$, find the two numbers. ....
(6)	Two integers, the ratio between them is $2:3$, if you add to the first 7 and subtract from the second 12 , the ratio between them becomes $5:3$ Find the two numbers. ....
(7)	Two positive real numbers, the ratio between them is 4:7 and the square of the small number exceeds 5 times the great number by 39 , find the two numbers. ....

Unit 1
Lesson 1

Continue the properties of proportion

learn

- If a, b, c and d are proportional quantities and we assume that : $\frac{a}{b} = \frac{c}{d} = k$,

  then a = bk , c = dk

Generally

- If a, b, c, d, e, f, ... are proportional quantities and we assume that:

  $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \cdots = k$, then

  a = bk ,

  c = dk

  e = fk , ...

(1) If a,b,c and d are proportional quantities, prove that:

(1)	$\frac{2a+3c}{7a-5c} = \frac{2b+3d}{7b-5d}$ solu Let $\frac{a}{b} = \frac{c}{d} = k$ $\therefore$ a = bk , c = dk L.H.S. $= \frac{2bk+3dk}{7bk-5dk} = \frac{k(2b+3d)}{k(7b-5d)} = \frac{2b+3d}{7b-5d} =$ R.H.S.
(2)	$\frac{a+c}{b+d} = \frac{a^2+c^2}{ab+cd}$ solu Let $\frac{a}{b} = \frac{c}{d} = k$ $\therefore$ $a = bk$, $C = dk$ $\because \frac{a+c}{b+d} = \frac{bk+dk}{b+d} = \frac{k(b+d)}{(b+d)} = k$ (1) , $\because \frac{a^2+c^2}{ab+cd} = \frac{(bk)^2+(dk)^2}{bk\times b+dk\times d} = \frac{b^2k^2+d^2k^2}{b^2k+d^2k} = \frac{k^2(b^2+d^2)}{k(b^2+d^2)} = k$ (2) From (1) and (2) we deduce that: $\frac{a+c}{b+d} = \frac{a^2+c^2}{ab+cd}$

(3)	If a, b, c, d, e and f are positive proportional quantities, prove that: $\sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}} = \frac{a}{b}$ solu $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$ $\therefore$ a = bk , c = dk , e = fk $\therefore \sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}} = \sqrt{\frac{(bk)^2+(dk)^2+(fk)^2}{b^2+d^2+f^2}} = \sqrt{\frac{b^2k^2+d^2k^2+f^2k^2}{b^2+d^2+f^2}}$ $= \sqrt{\frac{k^2(b^2+d^2+f^2)}{(b^2+d^2+f^2)}} = \sqrt{k^2} = k$ $\because \frac{a}{b} = \frac{bk}{b} = k$ $\therefore \sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}} = \frac{a}{b}$
(4)	If $\frac{a}{b} = \frac{c}{d}$, prove that : $\frac{5a-2c}{5b-2d} = \frac{4a+3c}{4b+3d}$ ....

Property(5)

If we consider the proportion : $\frac{9}{15} = \frac{6}{10} = \frac{3}{5}$

- If we add the antecedents and consequents of the 1st and the 2nd ratios, we get the ratio $\frac{9+6}{15+10} = \frac{15}{25} = \frac{3}{5}$ which is one of given ratios.

- Also if we add the antecedents and consequents of the 2nd and the 3rd ratios, we get the ratio $\frac{6+3}{10+5} = \frac{9}{15} = \frac{3}{5}$ = one of the given ratios.

- If we add the antecedents and consequents of the three given ratios, we get the ratio $\frac{9+6+3}{15+10+5} = \frac{18}{30} = \frac{3}{5}$ = one of the given ratios.

- i.e. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \cdots$ and $k_1, k_2, k_3, \dots$ are non-zero real numbers , then $\frac{k_1a+k_2c+k_3e+\cdots}{k_1b+k_2d+k_3f+\cdots}$ one of the given ratios

Example 2 :

If $\frac{a}{4} = \frac{b}{5} = \frac{c}{3}$, find : $\frac{a-b+c}{a+b-c}$

Solution

Multiplying the two terms of the 2nd ratio by (-1) , then add the antecedents and the consequents of the three ratios :

$\therefore \frac{a-b+c}{4-5+3} = \frac{a-b+c}{2}$ = one of the given ratios.

(1) Multiplying the two terms of the 3rd ratio by (-1) , then add the antecedents and the consequents of the three ratios :

$\therefore \frac{a+b-c}{4+5-3} = \frac{a+b-c}{6}$ = one of the given ratios.

From (1) and (2) : $\therefore \frac{a-b+c}{2} = \frac{a+b-c}{6}$

$\therefore \frac{a-b+c}{a+b-c} = \frac{2}{6} = \frac{1}{3}$

(2)	If $\frac{a+b}{11}=\frac{b+c}{9}=\frac{c+a}{4}$, prove that : $\frac{a+b+c}{5a+4b+3c}=\frac{6}{25}$ **Solution** Adding the antecedents and consequents of the three ratios. $\therefore \frac{\text{The sum of antecedents}}{\text{The sum of consequents}}=$ one of the given ratios. $\therefore \frac{2a+2b+2c}{24}=$ one of the given ratios. $\therefore \frac{a+b+c}{12}=$ one of the given ratios. Multiplying the two terms of the 1st ratio by 3 and the 3rd by 2 and adding the antecedents and consequents of the three ratios $\therefore \frac{\text{The sum of antecedents}}{\text{The sum of consequents}}=$ one of the given ratios. $\therefore \frac{3a+3b+b+c+2c+2a}{33+9+8}=$ one of the given ratios. $\therefore \frac{5a+4b+3c}{50}=$ one of the given ratios. From (1) and (2) : $\therefore \frac{a+b+c}{12}=\frac{5a+4b+3c}{50}$ $\therefore \frac{a+b+c}{5a+4b+3c}=\frac{12}{50}=\frac{6}{25}$
(3)	If $\frac{a+4b}{x+2y}=\frac{4b+7c}{2y+5z}=\frac{7c+a}{5z+x}$, prove that : $\frac{a}{2b}=\frac{x}{y}$ **Solution** Multiplying the two terms of the 2nd ratio by (-1), then add the antecedents and the consequents of the three ratios: $\therefore \frac{a+4b-4b-7c+7c+a}{x+2y-2y-5z+5z+x}=\frac{2a}{2x}=\frac{a}{x}=$ one of the given ratios. (1) Multiplying the two terms of the 3rd ratio by (-1), then add the antecedents and the consequents of the three ratios: $\therefore \frac{a+4b+4b+7c-7c-a}{x+2y+2y+5z-5z-x}=\frac{8b}{4y}=\frac{2b}{y}=$ one of the given ratios. (2) From (1) and (2) : $\therefore \frac{a}{x}=\frac{2b}{y}$ $\therefore \frac{a}{2b}=\frac{x}{y}$

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(4)	If $\frac{x}{a-2b}=\frac{y}{b-2c}=\frac{z}{c-2a}$, prove that: $\frac{x+2y-z}{3a-5c}=\frac{y+2z}{b-4a}$ …………………………………………………………………………………………… …………………………………………………………………………………………… …………………………………………………………………………………………… …………………………………………………………………………………………… …………………………………………………………………………………………… …………………………………………………………………………………………… …………………………………………………………………………………………… …………………………………………………………………………………………… …………………………………………………………………………………………… ……………………………………………………………………………………………
(5)	If $\frac{a}{b}=\frac{2}{3},\frac{a}{c}=\frac{3}{5}$ and $a+b+c=75$, find the value of each of : a, b and c …………………………………………………………………………………………… …………………………………………………………………………………………… …………………………………………………………………………………………… …………………………………………………………………………………………… …………………………………………………………………………………………… …………………………………………………………………………………………… ……………………………………………………………………………………………

Continue the properties of proportion (1)

	(1) If a,b,c and d are proportional quantities, prove that:
(1)	$\frac{3a+c}{5a-2c}=\frac{3\,b+d}{5\,b-2\,d}$ ....
(2)	$\frac{3a-2c}{5a+3c}=\frac{3\,b-2\,d}{5\,b+3\,d}$ ....
(3)	$\frac{a^2+c^2}{ab+cd}=\frac{a}{b}$ ....
(4)	$\frac{a^2+c^2}{b^2+d^2}=\frac{ac}{bc}$ ....

(5)	$\frac{ac}{bd} = \left(\frac{a-c}{b-d}\right)^2$ ……………………………………………………………………………………… ……………………………………………………………………………………… ……………………………………………………………………………………… ………………………………………………………………………………………
(6)	$\left(\frac{a+b}{c+d}\right)^2 = \frac{2a^2-3b^2}{2c^2-3d^2}$ ……………………………………………………………………………………… ……………………………………………………………………………………… ……………………………………………………………………………………… ……………………………………………………………………………………… ………………………………………………………………………………………
(7)	$\frac{a^2-2ac+c^2}{ac} = \frac{b^2-2bd+d^2}{bd}$ ……………………………………………………………………………………… ……………………………………………………………………………………… ……………………………………………………………………………………… ……………………………………………………………………………………… ……………………………………………………………………………………… ………………………………………………………………………………………

(2) If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, prove that :	
(1)	$\frac{a+5c}{b+5d} = \frac{c-3e}{d-3f}$ ……………………………………………………………………………………… ……………………………………………………………………………………… ……………………………………………………………………………………… ………………………………………………………………………………………

(2)	$\frac{2a+7c-4e}{2b+7d-4f}=\frac{a-8e}{b-8f}$ …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… ……………………………………………………………………………………
(3)	$\frac{2a^4b^2+3a^2e^2-5e^4f}{2b^6+3b^2f^2-5f^5}=\frac{a^4}{b^4}$ …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… ……………………………………………………………………………………
(4)	$\sqrt{\frac{5a^2-7ce}{5b^2-7df}}=\frac{2a+c}{2b+d}$ …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… ……………………………………………………………………………………

(3)	
(1)	If $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$, prove that: (1) $\frac{2y-z}{3x-2y+z} = \frac{1}{2}$ …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… (2) $\sqrt{3x^2 + 3y^2 + z^2} = 2x + y$ …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… ……………………………………………………………………………………
(2)	If $x = \frac{y}{2} = \frac{z}{3}$, then prove that : $\frac{x+y-2z}{x-3z} = \frac{3}{8}$ …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… ……………………………………………………………………………………
(3)	If $\frac{a}{2} = \frac{b}{3} = \frac{c}{4}$, prove that : $2a - 5b + 3c$ = one of the given ratios. …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… ……………………………………………………………………………………

(4)	If $\frac{a}{2}=\frac{b}{3}=\frac{c}{4}=\frac{2a-b+5c}{3x}$, then find the value of : x …………………………………………………… …………………………………………………… …………………………………………………… …………………………………………………… ……………………………………………………
(5)	If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{2}{3}$, and $5a-3c+e=18$ Find the value of : $5b-3d+f$ …………………………………………………… …………………………………………………… …………………………………………………… …………………………………………………… …………………………………………………… ……………………………………………………
(6)	If $\frac{x+y}{19}=\frac{y+z}{7}$, prove that : $\frac{x+2y+z}{13}=\frac{x-z}{6}$ …………………………………………………… …………………………………………………… …………………………………………………… …………………………………………………… ……………………………………………………
(7)	If $\frac{y}{x-z}=\frac{x}{y}=\frac{x+y}{z}$, prove that each ratio is equal to 2(unless $x+y=0$), then find X: y: z …………………………………………………… …………………………………………………… …………………………………………………… …………………………………………………… ……………………………………………………

(8)	If $\frac{x}{a-b+c}=\frac{y}{b-c+a}=\frac{z}{c-a+b}$, prove that: $\frac{x+y}{a}=\frac{y+z}{b}$
(9)	If $\frac{\chi}{2a+b}=\frac{y}{2b-c}=\frac{z}{2c-a}$, then prove that: $\frac{2\chi+y}{4a+4b-c}=\frac{2\chi+2y+z}{3a+6b}$
(10)	If $\frac{a}{2x-y}=\frac{b}{2y-x}$, prove that : $\frac{2a+b}{a+2b}=\frac{x}{y}$
(11)	If $\frac{\text{a}}{2X+\text{y}}=\frac{\text{b}}{3\text{y}-X}=\frac{\text{c}}{4X+5\text{y}}$, prove that : $\frac{\text{a}+2\text{ b}}{4\text{ b}+\text{c}}=\frac{7}{17}$

(12)	If $\frac{a}{2}=\frac{b}{7}=\frac{c}{3}$, find the value of : $\frac{a+2b}{b-c}$ ……………………………………………………………………………… ……………………………………………………………………………… ……………………………………………………………………………… ……………………………………………………………………………… ………………………………………………………………………………
(13)	If $\frac{a+b}{4}=\frac{b+c}{5}=\frac{c+a}{7}$, prove that: $\frac{a+b+c}{8}=\frac{a}{3}$ ……………………………………………………………………………… ……………………………………………………………………………… ……………………………………………………………………………… ……………………………………………………………………………… ………………………………………………………………………………
(14)	If $\frac{x+y}{3}=\frac{y+z}{8}=\frac{z+x}{6}$, prove that : $\frac{x+y+z}{2x+3y+3z}=\frac{17}{50}$ ……………………………………………………………………………… ……………………………………………………………………………… ……………………………………………………………………………… ……………………………………………………………………………… ………………………………………………………………………………
(15)	If $\frac{x+y}{5}=\frac{y+z}{8}=\frac{z+x}{7}$, prove that : $\frac{x}{2}=\frac{y}{3}=\frac{z}{5}$ ……………………………………………………………………………… ……………………………………………………………………………… ……………………………………………………………………………… ……………………………………………………………………………… ………………………………………………………………………………

(16)	If $\frac{x+y}{25}=\frac{x-y}{11}=\frac{x+y-z}{8}$, prove that : $x: y: z=18: 7: 17$ ....
(17)	If $\frac{a+3b}{x+6y}=\frac{3b+5c}{6y+10z}=\frac{5c+a}{10z+x}$, prove that : $\frac{a}{b}=\frac{x}{2y}$ and find $a: b: c$ ....
(18)	If $\frac{x}{7}=\frac{y}{3}$, prove that : $(2x-3y), (x+2y), 10$ and 26 are proportional. ....

.

Unit 1
Lesson 2

Continued Proportion

learn

Definition :The quantities a, b and c are said to be in continued proportion if $\frac{a}{b}=\frac{b}{c}$ In this proportion, a is called the first proportion c is called the third proportion and b is called the middle proportion (proportional mean).

For Example :-

The numbers 4,6 and 9 form a continued proportion because : $\frac{4}{6}=\frac{6}{9}$ or because :

$$(6)^2 = 4 \times 9$$

where 6 is the middle proportion, 4 is the first proportion and 9 is the third proportion.

Notice That

- **If a, b and c are in continued proportion, then $b^2 = ac$ i.e . $b = \pm\sqrt{ac}$ and the two quantities a and c should be either both positive or both negative.**
- **For any two positive numbers or any two negative numbers x and y , there are two middle proportions ($\sqrt{xy}$ and $-\sqrt{xy}$)**

Example 1 :

(1)	The middle proportional between 5 and 20 is …………… The middle proportional $= \pm\sqrt{5 \times 20} = \pm\sqrt{100} = \pm 10$
(2)	The middle proportional between 3 and $\frac{1}{3}$ is …………… The middle proportional $= \pm\sqrt{3 \times \frac{1}{3}} = \pm\sqrt{1} = \pm 1$
(3)	The middle proportional between $3x^3$ and $27x$ is …………… The middle proportional $= \pm\sqrt{3x^3 \times 27x} = \pm\sqrt{81x^4} = \pm 9x^2$

(4)	The first proportional of 12 and 18 is .... solu Let the first proportional be a $\therefore \frac{a}{12} = \frac{12}{18}$ $\therefore a = \frac{12 \times 12}{18} = 8$
(5)	The third proportional of -6 and 12 is .... solu Let the third proportional be c $\therefore \frac{-6}{12} = \frac{12}{c}$ $\therefore c = \frac{12 \times 12}{-6} = -24$
(6)	Find the middle proportional between 32 and 18 ....
(7)	Find the first proportional of 8 and 16 ....

.

Remark

If a, b and c are in continued proportion and we assume that : $\frac{a}{b} = \frac{b}{c} = k$

, then

$\therefore b = ck$

$\because \frac{a}{b} = k$

$\therefore a = bk$

Substituting for b from (1) : $\therefore a = (ck)k$

$\therefore a = ck^2$

i.e. If $\frac{a}{b} = \frac{b}{c} = m$, then $\begin{cases} b = ck \\ a = ck^2 \end{cases}$

General Definition :-

The quantities a, b, c, d, ... are in continued proportion if : $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \cdots$

Example 2 :

(1) If a, b and c are in continued proportion, prove that: $\frac{4a^2-3b^2}{4b^2-3c^2} = \frac{a}{c}$

solu

$$\frac{a}{b} = \frac{b}{c} = k \quad \therefore \quad b = ck \quad , \quad a = c\,k^2$$

$$\therefore \frac{4a^2 - 3b^2}{4b^2 - 3c^2} = \frac{4(ck^2)^2 - 3(ck)^2}{4(ck)^2 - 3c^2} = \frac{4c^2k^4 - 3c^2k^2}{4c^2k^2 - 3c^2} = \frac{c^2k^2(4k^2-3)}{c^2(4k^2-3)} = k^2$$

$$\because \frac{a}{c} = \frac{ck^2}{c} = k^2$$

From (1) and (2), we deduce that: $\frac{4a^2-3b^2}{4b^2-3c^2} = \frac{a}{c}$

.

(2)	If b is the middle proportional between a and c , prove that : $\frac{a-b}{a}=\frac{a-c}{a+b}$ **Solution** $\because$ b is the middle proportional between a and c $\therefore$ a , b and c are in continued proportion $\frac{a}{b}=\frac{b}{c}=k \qquad \therefore b=ck \quad , a=ck^2$ $\because \frac{a-b}{a}=\frac{ck^2-ck}{ck^2}=\frac{ck(k-1)}{ck^2}=\frac{k-1}{k}$ # (1) $\frac{a-c}{a+b}=\frac{ck^2-c}{ck^2+ck}=\frac{c(k^2-1)}{ck(k+1)}=\frac{c(k-1)(k+1)}{ck(k+1)}=\frac{k-1}{k}$ #(2) From (1) and (2), we deduce that : $$\frac{a-b}{a}=\frac{a-c}{a+b}$$
(3)	If b is the middle proportional between a and c , prove that : $ab-c^2=(b-c)(a+b+c)$ **Solution** $\because ab-c^2=ck^2\times ck-c^2=c^2k^3-c^2=c^2(k^3-1)$ #(1) $(b-c)(a+b+c)=(ck-c)(ck^2+ck+c)=c(k-1)\times c(k^2+k+1)$ $=c^2(k-1)(k^2+k+1)=c^2(k^3-1)$ #(2) From (1) and (2), we deduce that: $ab-c^2=(b-c)(a+b+c)$

(4)	If a, b and c are in continued proportion, prove that: $\frac{3c^2-4b^2}{3b^2-4a^2}=\frac{c^2}{b^2}$ ………………………………………………………………………… ………………………………………………………………………… ………………………………………………………………………… ………………………………………………………………………… ………………………………………………………………………… ………………………………………………………………………… …………………………………………………………………………

Remark

If a, b, c and d are in continued proportion and we assume that : $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}=k$, then :

$\frac{c}{d}=k$ $\therefore c=dk$ (1)

$\frac{b}{c}=k$ $\therefore b=ck$

Substituting for c from (1) :

$\therefore b=(dk)k$

$\therefore b=dk^2$ (2)

$\frac{a}{b}=k$

$\therefore a=bk$

Substituting for b from (2) :

$\therefore a=(dk^2)k$

$\therefore a=dk^3$

i.e. If $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}=k$, then $c=dk$, $b=dk^2$ and $a=dk^3$

Example 3 :

(5) If a, b, c and d are in continued proportion

, prove that: $\frac{a+d}{b-c+d} = \frac{a-c}{b-c}$

Solution

$\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = k$ $\qquad \therefore c = dk \quad , \quad b = dk^2 \quad , a = dk^3$

$$\therefore \frac{a+d}{b-c+d} = \frac{dk^3+d}{dk^2-dk+d} = \frac{d(k^3+1)}{d(k^2-k+1)}$$

$$= \frac{(k+1)(k^2-k+1)}{k^2-k+1} = k+1 \qquad (1)$$

$$\frac{a-c}{b-c} = \frac{dk^3-dk}{dk^2-dk} = \frac{dk(k^2-1)}{dk(k-1)} = \frac{(k-1)(k+1)}{(k-1)} = k+1 \qquad (2)$$

$$= \frac{(k+1)(k^2-k+1)}{k^2-k+1} = k+1$$

$$\frac{a-c}{b-c} = \frac{dk^3-dk}{dk^2-dk} = \frac{dk(k^2-1)}{dk(k-1)} = \frac{(k-1)(k+1)}{(k-1)} = k+1$$ (2)

From (1) and (2), we deduce that: $\frac{a+d}{b-c+d} = \frac{a-c}{b-c}$

(6) If a, b, c and d are in continued proportion, prove that : $\frac{a+2b}{b+2c} = \frac{c+a}{d+b}$

....

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....

(7)	If the quantities a , 2 b, 3c and 4 d are in continued proportion, prove that: $(2b-3c)$ is the middle proportional between $(a-2b)$ and $(3c-4d)$ **Solution** Let $\frac{a}{2b}=\frac{2b}{3c}=\frac{3c}{4d}=k$ $\therefore 3c=4dk$, $2b=4dk^2$, $a=4dk^3$ Proving that : $(2b-3c)$ is the middle proportional between (a-2b) and (3c-4d) means proving that : $(2b-3c)^2=(a-2b)(3c-4d)$ $\because (2b-3c)^2=(4dk^2-4dk)^2=(4dk(k-1))^2=16d^2k^2(k-1)^2$ #(1) $\because (a-2b)(3c-4d)=(4d^3-4d^2)(4dk-4d)$ $=4dk^2(k-1)\times 4d(k-1)=16d^2k^2(k-1)^2$ #(2) From (1) and (2), we deduce that : $(2b-3c)^2=(a-2b)(3c-4d)$ $\therefore (2b-3c)$ is the middle proportional between $(a-2b)$ and $(3c-4d)$

Continued Proportion (3)

(1) If a,b,c and d are in continued proportion, prove that:

(1)	$\frac{a-2b}{b-2c} = \frac{3b+4c}{3c+4d}$ …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… ……………………………………………………………………………………
(2)	$\frac{3a+5c}{3b+5d} = \frac{a-4c}{b-4d}$ …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… ……………………………………………………………………………………
(3)	$\frac{3a-5c}{a-b+c} = \frac{3b-5d}{b-c+d}$ …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… ……………………………………………………………………………………

(4)	$\frac{a-d}{a+b+c} = \frac{a-2b+c}{a-b}$ ....
(5)	$\frac{c^2-d^2}{a-c} = \frac{bd}{a}$ ....
(6)	$\frac{a^2-3c^2}{b^2-3d^2} = \frac{b}{d}$ ....

(7)	$\frac{ab - cd}{b^2 - c^2} = \frac{a + c}{b}$ ....
(8)	$\frac{a + 5b}{b + 5c} = \sqrt{\frac{b}{d}}$ ....
(9)	$\sqrt[3]{\frac{5a^3 - 3c^3}{5b^3 - 3d^3}} = \frac{a+c}{b+d}$ ....

Ex (2)	
(1)	If a, 3, 9 and b are in continued proportion, find the value of each of a and b …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… ……………………………………………………………………………………
(2)	If 3, ℓ, 12 and m are in continued proportion, find the value of each of ℓ and m …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… ……………………………………………………………………………………
(3)	Find the number that if we subtract it from each of the numbers 3, 7, 19, then they become in continued proportion. …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… ……………………………………………………………………………………

(4)	If b is the middle proportional between a and c and a = 4c = 4 , then find the value of : $a^2 + b^2 + c^2$ ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… …………………………………………………………………………………………
(5)	If $y^2 = xz$, prove that: $\frac{x(x-y)}{y(y-z)} = \frac{y^2}{z^2}$ ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… …………………………………………………………………………………………
(6)	If $\frac{a^2+b^2}{b^2} = \frac{b^2+c^2}{c^2}$, prove that b is the middle proportional between a and c where a c is a positive quantity. ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… …………………………………………………………………………………………

Unit 2
Lesson 1

Cartesian product

Learn

The ordered pair

(a, b) is called an ordered pair

- **a is called the first projection**
- **b is called the second projection**

Remarks

- **If $a \neq b$, then $(a, b) \neq (b, a)$**

For example: $(5,3) \neq (3,5)$

- **The ordered pair is not a set.i.e. $(a, b) \neq \{a, b\}$**
- **(a, a) is an ordered pair, while in the sets, we don't write {a, a}, but we write {a} without repeating the element a**
- **There is an empty set of elements and denoted by the symbol ∅, but there is not an empty ordered pair.**

The equality of two ordered pairs

If $(a, b) = (\mathcal{X}, y)$, then $a = \mathcal{X}, b = y$

For example:

- **If $(a, b) = (3, -4)$, then $a = 3$, $b = -4$**
- **If $(\chi, 2) = (-5, y)$, then $x = -5, y = 2$**

(1) Complete the following

(1)	If $(3,8) = (3, \sqrt{y})$, then $\sqrt[3]{y} = \ldots$ solu $\therefore \sqrt{y} = 8$ $\therefore y = 8^2 = 64$ $\therefore \sqrt[3]{y} = \sqrt[3]{64} = 4$	(1)	If $(2^{x-1}, -3) = (1, y)$, then $2x - y$ solu $\therefore y = -3$ $2^{x-1} = 1$, $x - 1 = 0$ $\therefore x = 1$ $\therefore 2x - y = 2 \times 1 - (-3)$ $= 2 + 3 = 5$

.

(2)	If $(32, x+y) = (y^5, 2)$, then $x = \cdots$ **solu** $\therefore y^5 = 32 = 2^5$ $\therefore y = 2$ $x + y = 2$ $y = 2$ $\therefore x + 2 = 2 \quad \therefore \quad x = 0$	(2)	If $(x^2 - 1, 4) = (48, 2y)$, $xy = \cdots$ solu $\therefore x^2 - 1 = 48$ $\therefore x^2 = 49$ $\therefore x = \pm\sqrt{49} = \pm 7$, $2y = 4 \quad \therefore y = \frac{4}{2} = 2$ $\therefore xy = \pm 7 \times 2 = \pm 14$

(2) Find the values of x and y in each of the following :	
(1)	$(x+1, y^2) = (3, 9)$ ....
(2)	$(x^2 - 1, 8) = (48, \sqrt[3]{y})$ ....
(3)	$(x^3 - 5, 8) = (3, 3y - 7)$ ....
(4)	$(x^2 - 2, 2y) = (y, \sqrt[3]{64})$ ....

The Cartesian product of two finite sets

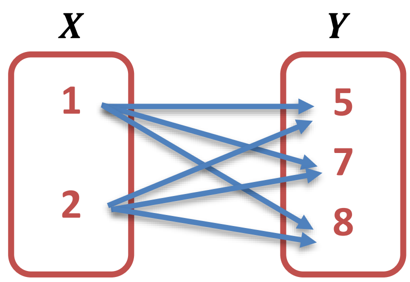
For any two finite and non empty sets X and Y , we get :
The Cartesian product of the set X by the set Y and it is denoted by $X \times Y$ is the set of all ordered pairs whose first projection of each of them belongs to X and the second projection of each of them belongs to Y i.e. $X \times Y = \{(a, b): a \in X, b \in Y\}$

For example : If $X = \{1, 2\}, Y = \{5, 7, 8\}$, then :

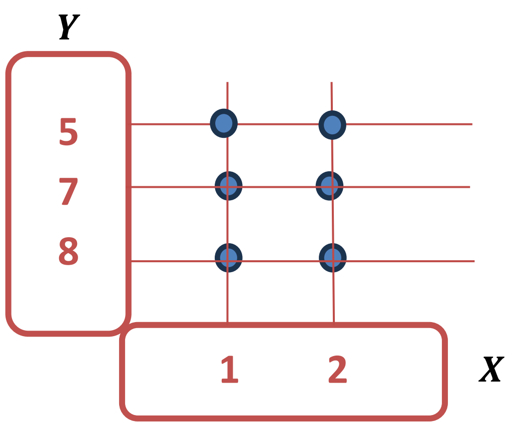
$$X \times Y = \{1, 2\} \times \{5, 7, 8\} = \{(1, 5), (1, 7), (1, 8), (2, 5), (2, 7), (2, 8)\}$$

- We can represent $X \times Y$ by two ways as follows :

1^{st} way : The arrow diagram



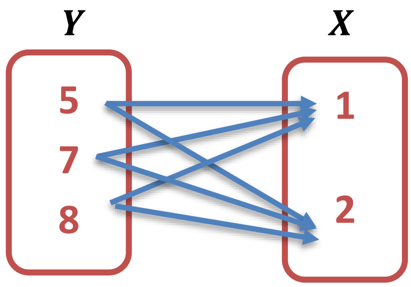
2^{nd} way : The graphical (Cartesian) diagram



Remarks

- For any two finite and non empty sets X and Y, then $X \times Y \neq Y \times X$ where $X \neq Y$
- For any set X , then $X \times \emptyset = \emptyset \times X = \emptyset$ where $\emptyset$ is the empty set.
- If $(a, b) \in X \times Y$, then $a \in X, b \in Y$

Example 3 :

(1)	If $X = \{1, 2\}, Y = \{5, 7, 8\}$, then : $Y \times X = \{5, 7, 8\} \times \{1, 2\}$ $= \{(5, 1), (5, 2), (7, 1), (7, 2), (8, 1), (8, 2)\}$ • Similarly, we can represent $Y \times X$ by two ways as follows : The arrow diagram 
(2)	If $X = \{1, 2\}, Y = \{3, 4, 5\}$, 1- find : $X \times Y$ 2- represent $Y \times X$ by two ways The arrow diagram and The Cartesian diagram ....

The Cartesian product of a set by itself

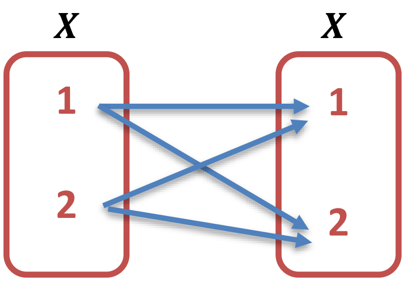
The Cartesian product of a set by itself

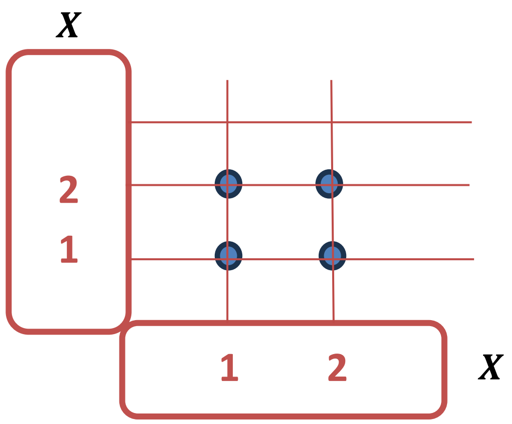
The Cartesian product of the set X by itself and we denote it by $X \times X$ or by X^2 (it is read X two) is the set of all ordered pairs whose first projections and second projections belong both to X
i.e. $X \times X = \{(a, b): a \in X, b \in X\}$
For example: If $X = \{1, 2\}$, then :

$$X \times X = \{1, 2\} \times \{1, 2\} = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$$

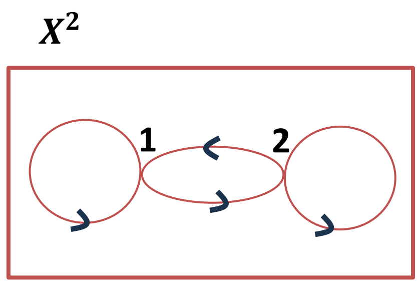
- **We can represent X × X by two ways as follows :**

The arrow diagram **or** **The Cartesian diagram**





Notice that : The figure ◯ is called a loop to show that the arrow goes from the point and returns to the same point.



Example 4 :

(1)

If $X = \{2, 3, 4\}$ and $Y = \{a, b\}$, find each of :

(1) $X \times Y$ (2) $Y \times X$ (3) $X \times X$ (4) Y^2

Solution

(1) $X \times Y = \{(2, a), (2, b), (3, a), (3, b), (4, a), (4, b)\}$
(2) $Y \times X = \{(a, 2), (a, 3), (a, 4), (b, 2), (b, 3), (b, 4)\}$
(3) $X \times X = \{(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4), (4, 2), (4, 3), (4, 4)\}$
(4) $Y^2 = \{(a, a), (a, b), (b, a), (b, b)\}$

(2)

If $X = \{3, 4, 5\}$ and $Y = \{5, 6\}$, find each of the following:
(1) $Y \times X$ and represent it by an arrow diagram
(2) X^2 and represent it by a Cartesian diagram

....
....
....
....
....

Remarks

The number of the elements of the Cartesian product

If we denote the number of elements of the set X by $n(X)$ and the number of elements of the set Y by $n(Y)$, then the number of elements of the Cartesian product $X \times Y$ is denoted by $n(X \times Y)$, and :

- $n(X \times Y) = n(Y \times X) = n(X) \times n(Y)$
- $n(X \times X) = n(X) \times n(X) = [n(X)]^2$
- $n(X \times \emptyset) = n(X) \times n(\emptyset) = 0$ [Because $n(\emptyset) = 0$]

Notice that:

If X, Y are two finite and non empty sets, $X \neq Y$, then $X \times Y \neq Y \times X$,

But $n(X \times Y) = n(Y \times X)$

Example 5 :

(1)	If $X = \{2, -1, 0\}$ and $Y = \{5, -7\}$, then $n(X) = 3, n(Y) = 2$, then : • $n(X \times Y) = 3 \times 2 = 6$ • $n(Y \times X) = 2 \times 3 = 6$ • $n(X^2) = 3^2 = 9$ • $n(Y^2) = 2^2 = 4$
(2)	If $X = \{0, 2\}, n(Y) = 5$, then $n(X \times Y) = \cdots$ $\because n(X) = 2, n(Y) = 5$ $\therefore n(X \times Y) = 2 \times 5 = 10$
(3)	If $n(Y) = 4$, $n(X \times Y) = 8$, then $n(X) = \cdots$ $n(X) = \frac{n(X \times Y)}{n(Y)} = \frac{8}{4} = 2$
(4)	If $n(X^2) = 9, n(Y^2) = 16$, then $n(Y \times X) = \cdots$ ……………………………………………………………………………….. ………………………………………………………………………………..
(5)	If $X = \{a, b\}, Y = \{3, 5, 7\}, Z = \{5, 7, 9\}$, represent the sets X, Y and Z by Venn diagram, then find : (1) $X \times (Y \cup Z)$, $Y \cup Z = \{3, 5, 7, 9\}$ $X \times (Y \cup Z) = \{a, b\} \times \{3, 5, 7, 9\}$ $= \{(a, 3), (a, 5), (a, 7), (a, 9), (b, 3), (b, 5), (b, 7), (b, 9)\}$ (2) $(X \times Y) \cup (X \times Z)$ $X \times Y = \{a, b\} \times \{3, 5, 7\} = \{(a, 3), (a, 5), (a, 7), (b, 3), (b, 5), (b, 7)\}$ $X \times Z = \{a, b\} \times \{5, 7, 9\} = \{(a, 5), (a, 7), (a, 9), (b, 5), (b, 7), (b, 9)\}$ $(X \times Y) \cup (X \times Z) = \{(a, 3), (a, 5), (a, 7), (a, 9), (b, 3), (b, 5), (b, 7), (b, 9)\}$ (3) $X \times (Y \cap Z) = \{a, b\} \times \{5, 7\} = \{(a, 5), (a, 7), (b, 5), (b, 7)\}$ (4) $X \times (Z - Y) = \{a, b\} \times \{9\} = \{(a, 9), (b, 9)\}$

(6)	If $X = \{1, 2, 3, 4\}, Y = \{3, 4, 5\}$, represent X and Y by Venn diagram, then find : $(1)(X \cap Y) \times Y$ $(2)(X - Y) \times Y$ (3) $(Y - X) \times X$ ....
(7)	If $X = \{3, 4\}, Y = \{4, 5\}$ and $Z = \{6, 5\}$, then find : $(1)X \times (Y \cap Z)$ (2) $(X - Y) \times Z$ (3) $(X - Y) \times (Y - Z)$ ....

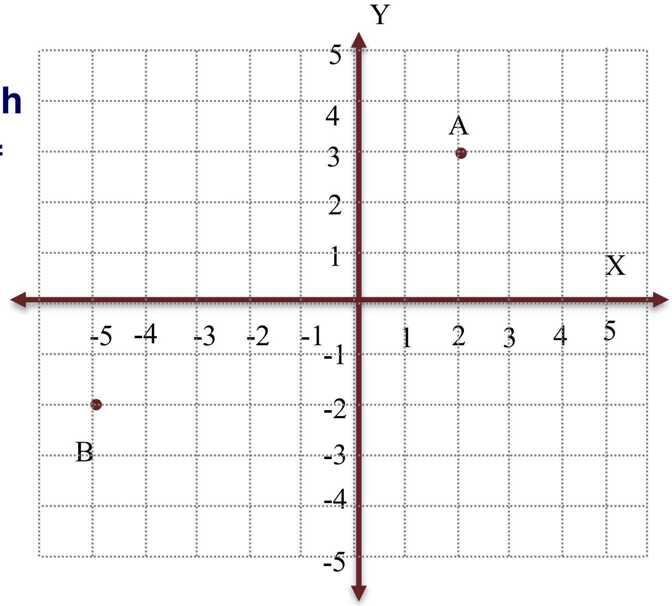
The Cartesian product of two infinite sets is also an infinite set.

- If X is an infinite set, then $X \times X$ is also an infinite set. Examples of this include:
- $N \times N = \{(x, y): x \in N, Y \in N\}$
- $Z \times Z = \{(x, y): x \in Z, Y \in Z\}$
- $Q \times Q = \{(x, y): x \in Q, Y \in Q\}$
- $R \times R = \{(x, y): x \in R, Y \in R\}$

Example 6:

(1)

The figure shown represents a small part of the orthogonal square grid, the Cartesian product $Z \times Z$, and each intersection point represents one of the ordered pairs in $Z \times Z$, where:



(A) $(2,3) \in Z \times Z$

(B) $(-5,-2) \in Z \times Z$

(2)

To which of the Cartesian products $R \times R$, $Q \times Q$, $Z \times Z$, $N \times N$ does each of the following ordered pairs belong? (Write all possible Cartesian products)

(1) $(\sqrt{4} \text{ ، } 5)$

Solution

$\because (\sqrt{4} \text{ ، } 5) = (2 \text{ ، } 5)$

$\therefore (\sqrt{4} \text{ ، } 5) \in N \times N$ ،$(\sqrt{4} \text{ ، } 5) \in Z \times Z$ ،$(\sqrt{4} \text{ ، } 5) \in Q \times Q$ ،$(\sqrt{4} \text{ ، } 5) \in R \times R$

(2) $(\sqrt[3]{-8} \text{ ، } 3)$

Solution

$\because (\sqrt[3]{-8} \text{ ، } 3) = (-2 \text{ ، } 3)$

$\therefore (\sqrt[3]{-8} \text{ ، } 3) \in Z \times Z$ ،$(\sqrt[3]{-8} \text{ ، } 3) \in Q \times Q$ ، $(\sqrt[3]{-8} \text{ ، } 3) \in Q \times Q$

(3) $\left(2\frac{1}{2} \text{ ، } \frac{1}{3}\right)$

Solution

$\because \left(2\frac{1}{2} \text{ ، } \frac{1}{3}\right) = \left(\frac{5}{2} \text{ ، } \frac{1}{3}\right)$ $\therefore \left(2\frac{1}{2} \text{ ، } \frac{1}{3}\right) \in Q \times Q$ ،$\left(2\frac{1}{2} \text{ ، } \frac{1}{3}\right) \in R \times R$

(4) $(\sqrt{5} \text{ ، } 1)$

Solution

$\therefore (\sqrt{5} \text{ ، } 1) \in R \times R$

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(3)

To which of the Cartesian products $R \times R$, $Q \times Q$, $Z \times Z$, $N \times N$ does each of the following ordered pairs belong? (Write all possible Cartesian products)

1. $(-\sqrt{9} ، 2)$

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2. $(\sqrt[3]{27} ، 5)$

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3. $\left(3\frac{1}{5} ، \frac{2}{5}\right)$

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4. $(5 ، \sqrt{7})$

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Cartesian product (1)

1 : Choose the correct answer from the given ones :								
1	If : $(x^3, y+3) = (1, \sqrt{4})$, then : $x - y =$ ___							
(a)	3	(b)	2	(c)	1	(d)	0
2	If : $(a+1, 5) = (-2, b-1)$, then : $2a + b =$ ___							
(a)	-12	(b)	zero	(c)	2	(d)	12
3	If : $X = \{3\}$, then : $X^2 =$ ___							
(a)	9	(b)	$(3,3)$	(c)	$\{9\}$	(d)	$\{(3,3)\}$
4	If : $X = \{5\}, Y = \emptyset$, then $n(X \times Y) =$ ___							
(a)	1	(b)	2	(c)	5	(d)	zero
5	If : $n(X) = 3, Y = \{4, 5\}$, then $n(X \times Y) =$ ___							
(a)	2	(b)	3	(c)	5	(d)	6
6	If : $X = \{5, 6, 7\}$, then $n(X^2) =$ ___							
(a)	3	(b)	6	(c)	9	(d)	12
7	If $n(X) = 3, n(X \times Y) = 12$, then $n(Y) =$ ___							
(a)	4	(b)	9	(c)	15	(d)	36
8	If : $n(X) = 5, n(X \times Y) = 15$, then $n(Y) =$ __							
(a)	1	(b)	5	(c)	3	(d)	15
9	If : $X \times Y = \{(1,3), (1,4)\}$, then $n(X) =$ ___							
(a)	1	(b)	2	(c)	3	(d)	4
10	If : $(3,5) \in \{3,6\} \times \{x, 8\}$ then $x =$ ___							
(a)	8	(b)	6	(c)	5	(d)	3
11	If : $(X - Y) \times Y = \{(1,2), (1,3)\}$ and $n(X \times Y) = 6$, then $X =$ ___							
(a)	$\{1\}$	(b)	$\{1,2\}$	(c)	$\{1,3,6\}$	(d)	$\{1,3,2\}$
12	The cartesian product $\{2\} \times \mathbb{R}$ represent graphically by a straight line passing through the two points $(2,0)$ and ___							
(a)	$(0,2)$	(b)	$(2,5)$	(c)	$(5,2)$	(d)	$(-2,2)$

13	If : $(\|x\|, 4) = (3, y^2)$ and the point (x, y) lies in the second quadrant, 18 then $x + y =$ __						
(a)	7	(b)	1	(c)	-1	(d)	-7
19	If $(9, 4) \in \{9, 7\} \times \{a, 5\}$, then $a =$ ……………….						
(a)	9	(b)	4	(c)	7	(d)	5

Example 2 :

(1) Find : a, b if $(a - 7, 26) = (-2, b^3 - 1)$

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(2) If $(x - 1, 11) = (8, y + 3)$, then find : $\sqrt{x + 2y}$

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(3) If $(x^2, 27) = (1, y^3)$ and the point (x, y) lies in the second quadrant, find the value of : $\sqrt{y - x}$

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(4) If $(2x, 4) = (8, y + 1)$, then find the value of : $\sqrt{x^2 + y^2}$

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(5)	If $X = \{2, 5\}, Y = \{1, 3, 7\}$, then find : (1) $X \times Y$ (2) $n(Y^2)$ …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… ……………………………………………………………………………
(6)	If $X = \{3, 7\}, Y = \{1, 2, 5\}$ Find : $X \times Y$, $n(Y^2)$ …………………………………………………………………………… …………………………………………………………………………… ……………………………………………………………………………
(7)	If $X \times Y = \{(1, 1), (1, 3), (1, 5)\}$ Find : (1) X , Y (2) $Y \times X$ …………………………………………………………………………… …………………………………………………………………………… ……………………………………………………………………………
(8)	If $X = \{1, 3, 5\}, Y = \{4, 5\}$ Find : $(X \cap Y) \times (X \cup Y)$ …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… ……………………………………………………………………………
(9)	If $X \times Y = \{(2, 3), (2, 2), (2, 4)\}$ Find each of the following : (1) X , Y (2) $X \times (X \cap Y)$ …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… ……………………………………………………………………………

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Example 3 :

(1) To which of the Cartesian products $R \times R$, $Q \times Q$, $Z \times Z$, $N \times N$ does each of the following ordered pairs belong? (Write all possible Cartesian products)

(1) $\left(\sqrt[3]{-125} ، 7\right)$

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(2) $\left(\sqrt{9} ، 4\right)$

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(3) $\left(2 ، \sqrt{3}\right)$

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(4) $\left(\frac{1}{2} ، 1\frac{1}{3}\right)$

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Unit 2 Lesson 2 Relation and Function

The relation

The relation from set X to set Y is a connection that connects some or all the elements of set X with some or all the elements of set Y and it is denoted by " R "

- The relation R from X to Y is a set of ordered pairs whose first projection belongs to X and its second projection belongs to Y and the first projection is connected with the second projection by this relation.

If $(a, b) \in R$ where $a \in X, b \in Y$
So, we express this as " a Rb "

- The relation R from set X to set Y is a subset of the Cartesian product $X \times Y$ i.e. $R \subset X \times Y$
- The relation can be expressed by an arrow diagram or a Cartesian diagram (graphical).

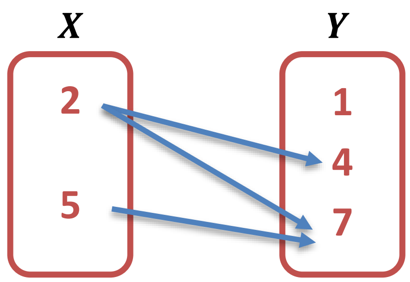
Example 1 :

(1) If $X = \{2, 5\}, Y = \{1, 4, 7\}$ and R is a relation from X to Y where " aRb " means " $a < b$ " for every $a \in X, b \in Y$, state the relation R and represent it by an arrow diagram and by a Cartesian diagram.

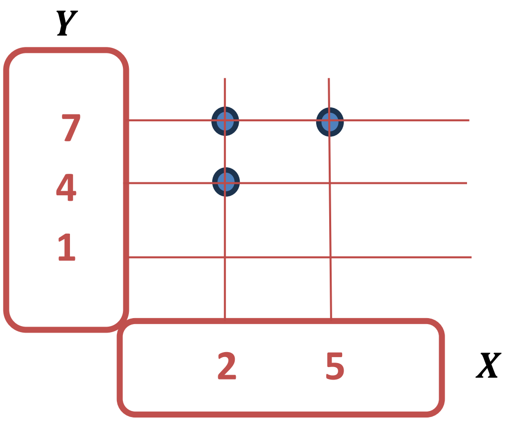
Solution

$\therefore$ The relation $R = \{(2, 4), (2, 7), (5, 7)\}$
The following figures represent the arrow diagram and the Cartesian diagram of this relation :

The arrow diagram



The Cartesian diagram



(2)	If $X = \{1, 2, 3\}, Y = \{3, 4, 5, 6\}$ and R is a relation from X to Y where " aRb " means " $a + b = 6$ " for every $a \in X$ and $b \in Y$, state the relation R and represent it by an arrow diagram. ....
(3)	If $X = \{0, 1, 2, 3\}, Y = \{0, 1, 2, 3, 4, 5, 6\}$ and R is a relation from X to Y where " a R b " means " $a = \frac{1}{2}$ b " for each $a \in X, b \in Y$, write R and represent it by an arrow diagram and a Cartesian diagram. ....

Codomain

is called "the range of the function" and it is a subset from the codomain of the function.

Generally

A relation from X to Y is said to be a function if one of the following cases is satisfied :

- In the relation, each element of the set X appears only once as a first projection in one of the ordered pairs of the relation.
- In the arrow diagram which represents the relation, each element of X has one and only one arrow going out of it to one element of Y
- In the Cartesian diagram which represents the relation, each vertical line has one and only one point lying on it of the points which represent the relation.

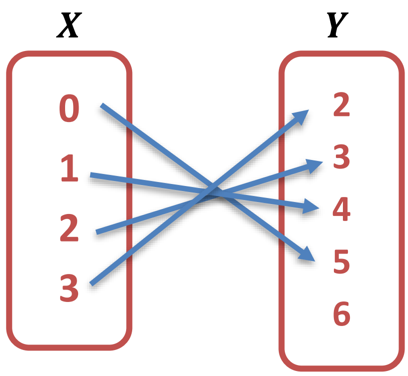
Example 2 :

(1) If $X = \{0, 1, 2, 3\}$, $Y = \{2, 3, 4, 5, 6\}$ and R is a relation from X to Y where " aRb " means " $a + b = 5$ " for each $a \in X, b \in Y$, write the relation R and represent it by an arrow diagram.
Mention giving reasons if R is a function from X to Y or not.
And if it is a function, find its range.

Solution

· $R = \{(0, 5), (1, 4), (2, 3), (3, 2)\}$

- R represents a function from X to Y because each element of X is connected with only one element of Y The range of the function $= \{5, 4, 3, 2\}$



(2) If $X = \{3, 2, 1, \text{zero}, \frac{1}{2}, \frac{1}{3}\}$ and R is a relation on X where " aRb " means " a is the multiplicative inverse of b " for each $a \in X, b \in X$, write R and represent it by an arrow diagram and mention giving reasons if R represents a function or not.

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(3) If $X = \{1, 2, 3\}$, $Y = \{1, 4, 6, 9\}$ and R is a relation from X to Y where " aRb " means " $a = \sqrt{b}$ " for each $a \in X, b \in Y$, write the relation R and represent it by an arrow diagram. Mention giving reasons if R is a function from X to Y or not, and if it is a function, mention its range.

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The symbolic representation of the function

The symbolic representation of the function

- The function is usually denoted by one of the following letters. f or m or q or . . and the function f from the set X to the set Y is written mathematically as :
  $f: X \longrightarrow Y$ and is read as f is a function from X to Y

Or m: X ⟶ Y and is read as m is a function from X to Y and so on . . .

- If the ordered pair (x, y) belongs to the function, then the element y is called the image of the element $\mathcal{X}$ by the function f and we express it by one of the following two forms :
  $f: X \longmapsto y$ it is read as f maps X to y

Or $f: f(\mathcal{X}) = \text{y}$ it is read as f is a function where $f(\mathcal{X}) = \text{y}$

For example : If f: X ⟶ Y where $f: x \longmapsto x^2$, then $f: 3 \longmapsto 9$

, also can be written in the form : $f(\mathcal{X}) = \mathcal{X}^2$, hence $f(3) = 9$

Remark

The mathematical form $f(\mathcal{X}) = \mathcal{X}^2$ is called the rule of the function f , and it is used to find the image of any element of the domain by the function f

Remember

If f is a function from the set X to the set Y i. e. $f: X \longrightarrow Y$, then :

- X is called the domain of the function f
- Y is called the codomain of the function f
- The set of images of the elements of the set X by the function f is called the range of the function f which is a subset of the codomain Y

Example 3 :

(1)	If $f: f(x) = x^2 - 2x + 5$ (1) Find : $f(1), f(0), f(-2), f\left(\frac{1}{2}\right)$ and $f(\sqrt{5})$ **Solution** $f(1) = (1)^2 - 2 \times (1) + 5 = 1 - 2 + 5 = 4$ $, f(0) = (0)^2 - 2 \times (0) + 5 = 0 - 0 + 5 = 5$ $, f(-2) = (-2)^2 - 2 \times (-2) + 5 = 4 + 4 + 5 = 13$ $, f\left(\frac{1}{2}\right) = (\frac{1}{2})^2 - 2 \times (\frac{1}{2}) + 5 = \frac{1}{4} - 1 + 5 = 4\frac{1}{4}$ $f(\sqrt{5}) = (\sqrt{5})^2 - 2 \times (\sqrt{5}) + 5 = 5 - 2\sqrt{5} + 5 = 10 - 2\sqrt{5}$
(2)	If $f(x) = 2x + b$ and $g(x) = x^2 + b$ and if $f(2) + g(-4) = 30$, then find : $f(-2) - g(2)$ **Solution** $\because f(2) = 2 \times 2 + b = 4 + b \quad , g(-4) = (-4)^2 + b = 16 + b$ $\because f(2) + g(-4) = 30$ $\therefore 4 + b + 16 + b = 30$ $\therefore 20 + 2b = 30$ $\therefore 2b = 30 - 20 = 10$ $\therefore b = 10 \div 2 = 5$ $\therefore f(X) = 2X + 5 \quad , \quad g(X) = X^2 + 5$ $\therefore f(-2) = 2 \times (-2) + 5 = 1 \quad , \quad g(2) = 2^2 + 5 = 9$ $\therefore f(-2) - g(2) = 1 - 9 = -8$
(3)	If $f(x) = x^2 - 3x$ and $g(x) = x - 3$, find $f(\sqrt{2}) + 3g(\sqrt{2})$. Prove that $f(3) = g(3) = 0$. ....

(4) If $f(x) = x^2 - 2x - 5$, prove that $f(1 + \sqrt{6}) = f(1 - \sqrt{6}) = 0$.

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(5) If the set of the function f is $\{(1,3), (2,5), (3,7), (4,9), (5,11)\}$, write:

- The domain of f.

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- The range of f.

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- The rule of the function f.

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(6) The function $f: \mathbb{R} \to \mathbb{R}$ where $f(x) = ax^2 + bx + 5$, and $a = 0$, b is a real number not equal to zero.

- Find the degree of the function f.

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- If $f(3) = 11$, find the value of b.

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The Relation and Function (2)

Example 1 :

(1)	If : $X = \{1, 3, 4\}, Y = \{1, 2, 3, 4, 5\}$ and R is a relation from X to Y where " aRb " means " $a + b = 6$ " for all $a \in X, b \in Y$, Write R and represent it by an arrow diagram. Is R a function ? and why ? ....
(2)	If : $X = \{1, 2, 5\}$ and R is a relation on X where aRb means " $a + 2b$ = an odd number" for each $a \in X$ and $b \in X$. Write R and represent it by an arrow diagram. Is R a function? ....
(3)	If : $X = \{-1, 0, 1, 2, 3\}, Y = \{0, 1, 4, 6, 9\}$ and R is a relation from X to Y where " aRb " means " $a^2 = b$ " for each of $a \in X, b \in Y$ Write R and represent it by a cartesian diagram. ....

(4)	If $X = \{1, 2, 3\}, Y = \{1, 3, 6, 9, 12\}$ and R is a relation from X to Y , where " aRb " means " $a = \frac{1}{3}b$ " for each $a \in X, b \in Y$ Write R and show that R is a function, then write its range. ....
(5)	If the function $f = \{(0, 4), (1, 3), (2, 2), (3, 1)\}$ (1) Write each of domain and range of the function f (2) Write the rule of the function f ....
(6)	If : $f: X \longrightarrow Y, X = \{-1, 2, 3\}, Y = \{2, 3, 5, 7\} R = \{(-1, 3), (3, 5), (2, 7)\}$ Find : (1) The domain of the function f (2) The codomain of the function f (3) The range of the function f ....
(7)	If : $X = \{1, 3, 4, 5\}, Y = \{1, 2, 3, 4, 5, 6\}$ and R is a relation from X to Y where " a R b " means " $a + b = 7$ " for all $a \in X, b \in Y$ Write R and represent it by an arrow diagram, show that R is a function ? Write its range. ....

(8)	If $X = \{3, 4, 5\}, Y = \{1, 5, 4, 6\}$ and R is a relation from X to Y where " a R b " means " $a + b = 9$ " for each $a \in X$ and $b \in Y$, write R and represent it by an arrow diagram and show that R is a function, write its range. ....
(9)	If $X = \{0, 1, 4, 7\}, Y = \{1, 3, 5, 6\}$ and R is a relation from X to Y where " a R b" means " $a + b < 8$ " for each $a \in X, b \in Y$. Write R and represent it by an arrow diagram and also by a Cartesian diagram. Is R a function ? and why ? ....
(10)	If : $X = \left\{\frac{1}{3}, \frac{1}{2}, 1, 2, 3\right\}$ and R is a relation on X where " aRb " means " $ab = 1$ " for every $a \in X, b \in X$, represent R by an arrow diagram, show that R is a function and write its range. ....
(11)	If : $X = \{2, 3, 4\}, Y = \{y: y \in \mathbb{N}, 2 \le y < 9\}$ where $\mathbb{N}$ is a set of the natural numbers, and R is a relation from X to Y where " aRb " means : " $a = \frac{1}{2}b$ " for all $a \in X$ and $b \in Y$, write R and represent it by an arrow diagram. Show that R is a function from X to Y and write its range. ....

(12)	If : $X = \{1, 2, 4\}$ and R is a relation on X where " aRb" means " a is twice of b " for each $a \in X, b \in X$ (1) Write R and represent it by an arrow diagram. (2) Is R a function? and why? ....
(13)	If : $X = \{0, 1, 2, 3\}, Y = \{-3, -2, -1, 0\}$ and R is a relation from X to Y where " aRb " means that the number " a is the additive inverse to the number b " for every $a \in X$ and $b \in Y$ (1) Find the relation R (2) Represent the relation R by an arrow diagram. (3) Is R a function? Why? ....

2 : Choose the correct answer from the given ones :

1	If : $f(X) = X + 3$, then $f(-2) = \dots$						
(a)	9	(b)	-3	(c)	1	(d)	5
2	If : $f(X) = 5\chi - 7$, then $f(3) = \cdots$						
(a)	2	(b)	3	(c)	8	(d)	15
3	If : $f(X) = 5\chi - 3$, then $f(0) =$						
(a)	5	(b)	2	(c)	3	(d)	-3
4	If : $f(X) = 7\chi - \frac{1}{2}$, then $f\left(\frac{1}{2}\right) = \dots$						
(a)	7	(b)	$\frac{1}{2}$	(c)	7	(d)	$\frac{7}{2}$
5	If : $f(X) = X^2 - \sqrt{2}X$, then : $f(\sqrt{2}) = \dots.$						
(a)	4	(b)	2	(c)	6	(d)	0
6	If : $f(X) = X^2 - X + 3$, then : $f(3) =$						
(a)	3	(b)	6	(c)	9	(d)	12

Example 2 :

(1)	If : $f(X) = 2x^2 - 5x + 2$, then prove that : $f(2) = f\left(\frac{1}{2}\right)$ ………………………………………… ………………………………………… ………………………………………… ………………………………………… …………………………………………
(2)	If : $f(x) = 2x - 1$, then prove that : $f(2) - 3f(1) =$ zero ………………………………………… ………………………………………… ………………………………………… ………………………………………… ………………………………………… …………………………………………

(3)	If : $f(x) = x^2 - x + 3$, then find : $f(-2), f(0), f(1)$
(4)	If f is a function on X where $X = \{3, 4, 5, 6\}$ and $f(3) = 3, f(4) = 5, f(5) = 5$, $f(6) = 5$ (1) Represent f by an arrow diagram. (2) Write the set of f and mention its range.
(5)	If the function $f = \{(0, 4), (1, 3), (2, 2), (3, 1)\}$ (1) Write each of domain and range of the function f (2) Write the rule of the function f

Unit 2
Lesson 3
The linear function

The linear function

First The linear function

-Definition

The function $f: \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = aX + b, a \in \mathbb{R} - \{0\}, b \in \mathbb{R}$
is called a linear function (it is a polynomial function of the first degree).

Examples of linear functions :

- $f: \mathbb{R} \longrightarrow \mathbb{R}, f(X) = X - 1$, $f: \mathbb{R} \longrightarrow \mathbb{R}, f(X) = 2\chi + 1$

Notice that:

In each of the shown functions , the index of X is 1 , therefore each of them is a function of the first degree.

The graphical representation of the linear function

- The linear function $f: \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = aX + b$, $a \in \mathbb{R} - \{0\}, b \in \mathbb{R}$ is represented graphically by a straight line intersecting :
- The y -axis at the point $(0, b)$ The X-axis at the point $\left(\frac{-b}{a}, 0\right)$
- To represent a linear function, it is enough to find two ordered pairs belonging to the function.
- You can find a third ordered pair to check that the three points are on the same straight line.

EX (1)

(1) Graph the following linear functions : $f: f(X) = 2X - 3$

$\therefore f(-1) = 2(-1) - 3 = -5$

$\therefore (-1, -5) \in f$

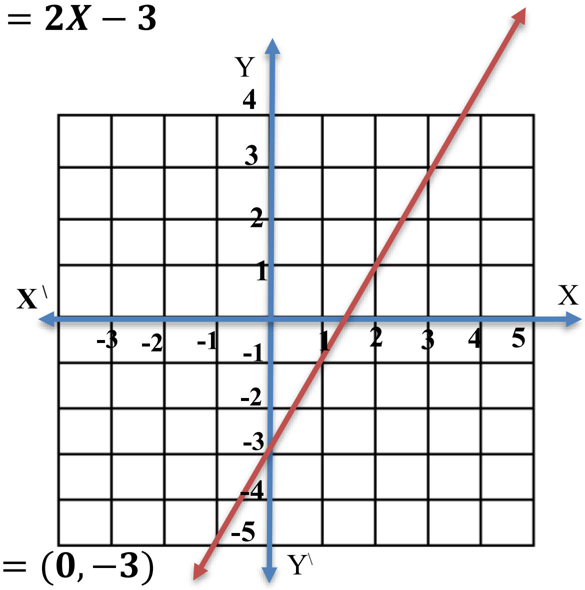
, $f(1) = 2 \times 1 - 3 = -1$

$\therefore (1, -1) \in f$

and $f(2) = 2 \times 2 - 3 = 1$

$\therefore (2, 1) \in f$

x	-1	1	2
$y = f(x)$	-5	-1	1

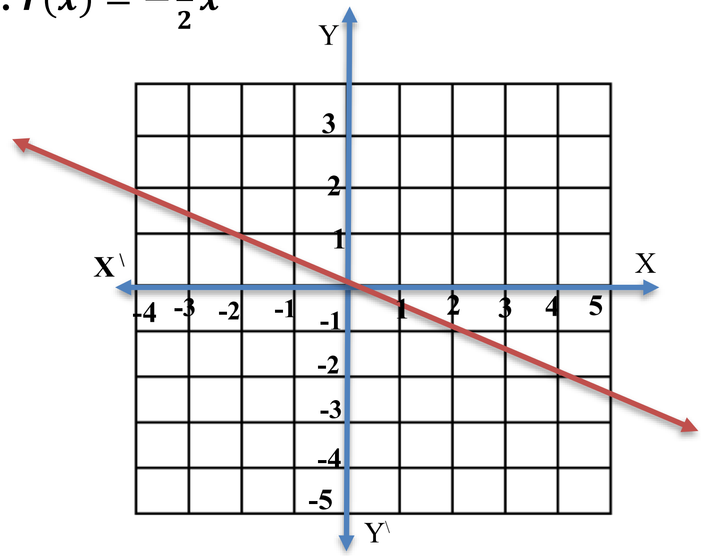


The point of intersection with $y-$axis $= (0, b) = (0, -3)$

The point of intersection with $x-$axis $= \left(-\frac{b}{a}, 0\right) = \left(\frac{3}{2}, 0\right)$

(2) Graph the following linear functions : $r(x) = -\frac{1}{2}x$

x	0	2	-4
$y = r(x)$	0	-1	2



From the opposite graph notice that,

the straight line L passes through the origin point $O(0, 0)$

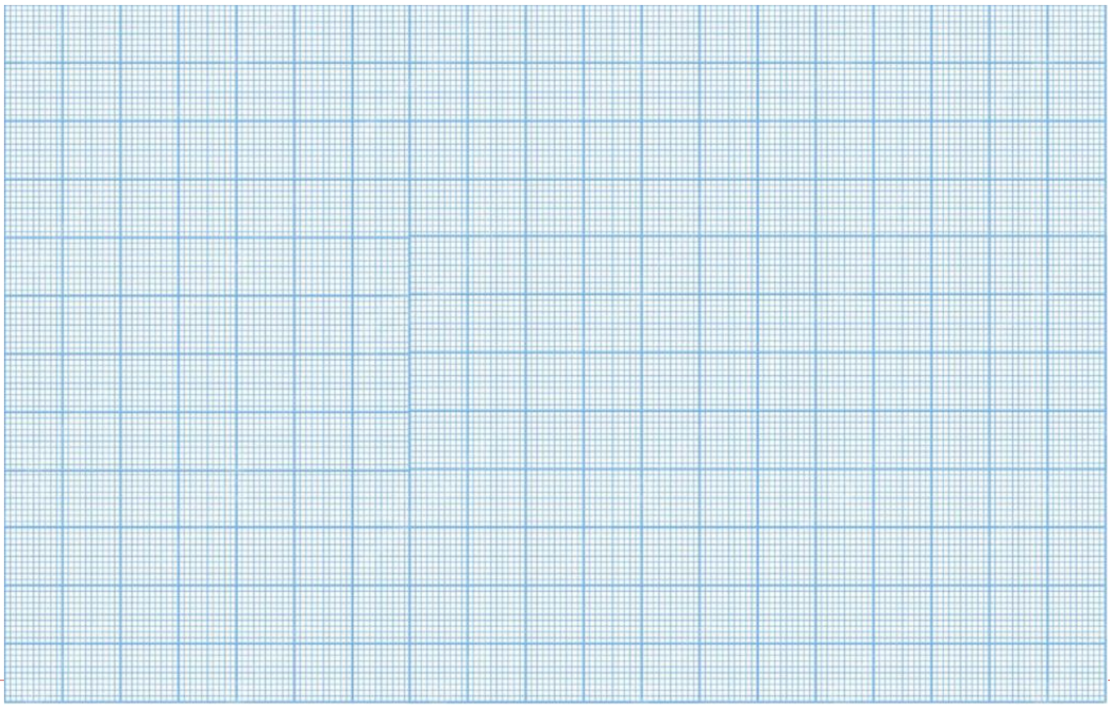
Learn

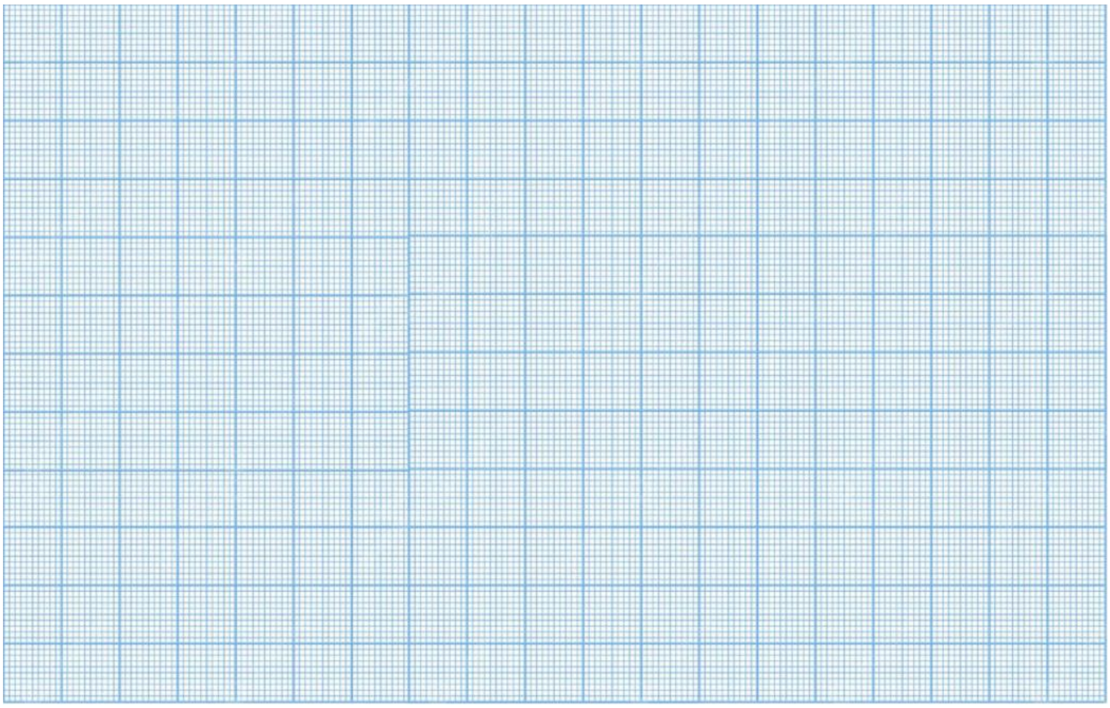
Generally

- The function $f: \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = aX$, $a \in \mathbb{R}^*$ is represented graphically by a straight line passing through the origin point $(0, 0)$

Represent graphically each of the following linear functions :

(1) $f: f(x) = 3x - 3$



(2)	$f: f(X) = 2X$ 

EX (2)

(1)	**If the point (a, −a) lies on the straight line representing the function $f: f(X) = X - 6$, find the value of a** $\because$ (a, −a) lies on the straight line representing the function f $\therefore$ $(a, -a)$ satisfies the function $\therefore a - 6 = -a$ $\therefore 2a = 6 \qquad \therefore a = 3$
(2)	**If the straight line representing the function $f: \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = aX + b$ intersects the y -axis at $(0, 3)$ and $f(2) = 7$, find the value of each of a, b** ……………………………………………………………………………… ……………………………………………………………………………… ……………………………………………………………………………… ……………………………………………………………………………… ………………………………………………………………………………

Second The constant function

Definition

The function $f:\mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = b, b \in \mathbb{R}$ is called a constant function.

For example:

$f: f(X) = 5$ is a constant function where

$f(1) = 5$, $f(0) = 5$, $f(-2) = 5$.

The graphical representation of the constant function

The constant function $f: f(\mathcal{X}) = b$ (where $b \in \mathbb{R}$) is represented by a straight line parallel to $\mathcal{X}$-axis and passing through the point $(0, b)$ and this line is :

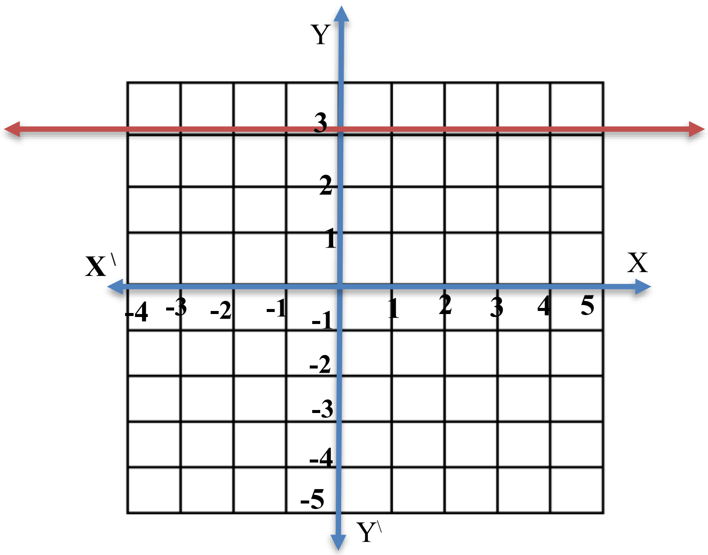
- above X-axis if $b > 0$
  below $\mathcal{X}$-axis if $b < 0$
- coincident with X-axis if $b = 0$

(1) The straight line is above x-axis and passes through $(0, 2)$
(2) The straight line is below x-axis and passes through $(0, -3)$
(3) The straight line is coincident with $\mathcal{X}$-axis and passes through $(0, 0)$

EX (3)

(1) raph the following linear functions : $f(x) = 3$

x	1	2	3
$y = f(x)$	3	3	3



(2) If the straight line which represents the function $f:\mathbb{R}\longrightarrow\mathbb{R}, f(X) = aX + b$ intersects the X-axis at the point $(3, 0)$ and intersects the y -axis at the point $(0, -3)$, then find the values of the two constants a and b and find the value of $f(1)$

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(3) If the cost of producing $x$books is given by the relation: $f(x) = 10 + 5x$ where $x$is the number of books and $f(x)$is the total cost in pounds. Construct a table showing some values of $x$and their corresponding total costs, then represent them graphically. Then find the number of books that can be produced with a total cost of 90 pounds.

Solution:

$f(x) = 10 + 5x$

$f(0) = 10$, $f(2) = 20$, $f(4) = 30$

x	0	2	4
f(x)	10	20	30

If the total cost is 90 pounds:

$10 + 5x = 90$

$5x = 80$, $x = \frac{80}{5} = 16$

Therefore, the number of books that can be produced with a total cost of 90 pounds is 16 books .

The linear function (3)

(1) Represent the following functions graphically, where x∈R:

(1)	$f : f(\mathcal{X}) = -5$
(2)	$f : f(\mathcal{X}) = -2\mathcal{X}$

(3)	$f : f(\mathcal{X}) = 5 - \frac{1}{2}\chi$
(4)	$f : f(\mathcal{X}) = 2 - \mathcal{X}$

(5)	If the straight line which represents the function $f: \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = 6X - a$ intersects the y -axis at the point $(b, 2)$, find the value of each of a, b …………………………………………………………………… …………………………………………………………………… …………………………………………………………………… ……………………………………………………………………
(6)	If the function $f: f(X) = 3X - 6$ is represented by a straight line passing through the point (a, 2a), find the value of a, then find the intersection point of the straight line with the y -axis. …………………………………………………………………… …………………………………………………………………… …………………………………………………………………… ……………………………………………………………………
(7)	If $f: \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = 2X + a$ and $f(3) = 9$, find : (1) The value of a (2) The coordinates of the intersection point of the straight line representing the function with the X-axis. …………………………………………………………………… …………………………………………………………………… …………………………………………………………………… …………………………………………………………………… …………………………………………………………………… ……………………………………………………………………

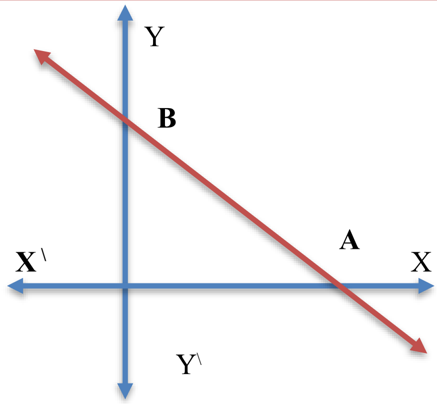
(8) If the straight line representing the function $f: \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = aX + b$ cuts a positive part of the y-axis of length 3 units and passes through the point $(1, 5)$, find the value of each of : a , b

(9) The opposite figure represents the function f where $f(\chi) = 4 - 2X$

Find :

(1) The coordinates of A, B

(2) The area of $\triangle$ AOB



(10) A car moves at a constant speed from a point 50 km away from Cairo.
If the distance d(in kilometers) of the car from Cairo after time t(in hours) is given by:

$$d(t) = 100t + 50$$

Graph the function d, then find the time required for the car to be 550 km away from Cairo.

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(11)	The price of a photocopying machine after t years is given in pounds by the function f, where: $$f(t) = 12000 - 750t$$ Find the price of the machine after 4 years of purchase, and determine after how many years the price becomes 3000 pounds. ....
(12)	If the cost of renting a football field (in pounds) is given by the function f, where: $$f(t) = 120t + 80$$ and t is the number of hours of rental, find the cost of renting the field for two hours, and after how many hours the cost will be 680 pounds. ....

Unit 2 Lesson 4
First-Degree Equations and Inequalities in One Variable

learn

"The general form of a first-degree equation in one variable is:

$$ax + b = 0$$

where a and b are real numbers, and $a \neq 0$.

Example 1: Find in $\mathbb{R}$ the S.S. of each of the following equations

(1)	$3x + 2 = 1$ Solution $\therefore 3x = 1 - 2$ $\therefore 3x = -1$ $\therefore x = -1 \times \frac{1}{3}$ $\therefore x = -\frac{1}{3}$ $\therefore$ The S.S $= \left\{-\frac{1}{3}\right\}$	(1)	$\sqrt{3}x - 1 = 2$ Solution $\therefore \sqrt{3}x = 2 + 1$ $\therefore \sqrt{3}x = 3$ $\therefore x = \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ $\therefore x = \frac{3\sqrt{3}}{3}$ $\therefore x = \sqrt{3}$ $\therefore$ The S.S $= \{\sqrt{3}\}$
(2)	$7x - \sqrt{7} = 6\sqrt{7}$ Solution $\therefore 7x = 6\sqrt{7} + \sqrt{7}$ $\therefore 7x = 7\sqrt{7}$ $\therefore x = \frac{7\sqrt{7}}{7} = \sqrt{7}$ $\therefore$ The S.S.$= \{\sqrt{7}\}$	(2)	$x - \sqrt{5} = 1$ Solution $\therefore x = 1 + \sqrt{5}$ $\therefore$ The S.S $= \{1 + \sqrt{5}\}$
(3)	$2x + 5 = 4$	(3)	$x - \sqrt{3} = 2$ ....

Second Solving inequalities of the first degree in one unknown in $\mathbb{R}$

- Each of the inequalities :
- $2x < 5$
- $-3x + 2 \leq 1$
- $-5 + x > 2x - 1 \geq 3 + x$
- is called an inequality of the first degree in one unknown denoted by x

Second Solving inequalities of the first degree in one unknown in $\mathbb{R}$

The methods of solving these inequalities in $\mathbb{R}$ depend on the properties of the inequality relation which will be summarized in the following:

- Let a, b and c be three real numbers and assuming that $a < b$, then :

  $a + c < b + c$ whether c is positive or negative (the addition property)

  $ac < bc$ if c is positive (the property of multiplying by a positive real number)

  $ac > bc$ if c is negative (the property of multiplying by a negative real number)

  i.e. When we multiply (or divide) the two sides of an inequality by a negative number, we should change the symbol of the inequality.

Example 2 :Find in $\mathbb{R}$ the S.S. of each of the following inequalities, then represent the solution on the number line :

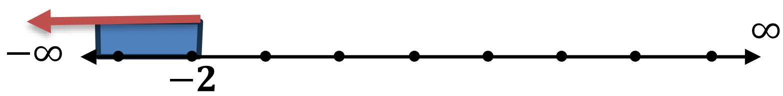
(1) $2x + 6 < 2$

Solution

$\therefore 2x < 2 - 6$

$\therefore 2x < -4$

$\therefore \frac{2x}{2} < \frac{-4}{2}$

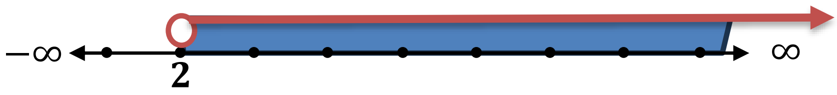
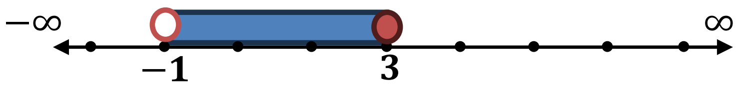
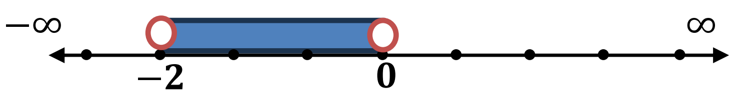


$\therefore x < -2$

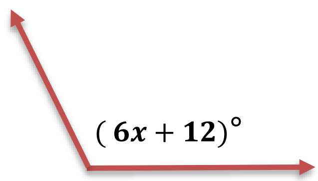
$\therefore$ The S.S. is all the real numbers which are less than -2

The S.S. $=]-\infty, -2[$

.

(2)	$5-4x \leq -3$ **Solution** $\therefore -4x \leq -3-5$ $\therefore -4x \leq -8$ (dividing both sides by -4) $\therefore \quad x \geq 2$ (Notice the change in the symbol of the inequality because we divided by a negative number)  $\therefore$ The S.S. $= [2, \infty[$
(3)	$-3 < 2x-1 \leq 5$ **Solution** $\because -3+1 < 2x-1+1 \leq 5+1$ $\therefore \quad -2 < 2x \leq 6$ (dividing all sides by 2) $\therefore \quad -1 < x \leq 3$  $\therefore$ The S.S. =]- 1,3]
(4)	$3 < 3-5x < 13$ **Solution** $\because 3-3 < 3-5x-3 < 13-3$ $\therefore \quad 0 < -5x < 10$ (dividing all sides by -5) $\therefore \quad 0 > x > -2$ (Notice the change in the symbols of the inequality because we divided by a negative number).  $\therefore$ The S.S. $=]-2, 0[$

(5)	$-16 < 5x + 4 \leq 9$ ....
(6)	$x - 2 \geq 3x - 5$ $\because x \geq 3x - 5 + 2$ $\therefore x \geq 3x - 3$ $\therefore x - 3x \geq -3$ $\therefore -2x \geq -3$ (dividing all sides by -2) $\therefore x \leq \frac{3}{2}$ (Notice the change in the symbol of the inequality) $\therefore$ The S.S. $=]-\infty, \frac{3}{2}]$
(7)	$x - 1 < 3x - 3 \leq x + 5$ Solution $\therefore x + 2 - x < 3x - x \leq x + 8 - x$ (adding $-x$ to all sides) $\therefore 2 < 2x \leq 8$ (dividing all sides by 2) $\therefore 1 < x \leq 4$ $\therefore$ The S.S. $=]1, 4]$

(8)	$2x+1>4x-3>2x-11$ ....
(9)	$\frac{3x-4}{6}<x+1<\frac{x+3}{2}$ ....
(10)	In the figure opposite, if ∠A is obtuse, what are the possible values of x ?  ....

First-Degree Equations and Inequalities in One Variable (4)

(1) Find the solution set for each of the following equations in R .	
(1)	$x + 5 = 0$ ....
(2)	$5x + 6 = 1$ ....
(3)	$\lvert x + 2\sqrt{3} \rvert = 3$ ....
(4)	$4x - 1 = \lvert -2 \rvert$ ....
(5)	$2 - \sqrt{6}x = \lvert -8 \rvert$ ....

(2) Find the solution set for each of the following inequalities in R in the form of an interval, then represent the solution on the number line :

(1)	$2x > 6$ ……………………………………………………………… ……………………………………………………………… ……………………………………………………………… ……………………………………………………………… ………………………………………………………………
(2)	$-7x \geq -14$ ……………………………………………………………… ……………………………………………………………… ……………………………………………………………… ……………………………………………………………… ……………………………………………………………… ………………………………………………………………
(3)	$5 - x > 3$ ……………………………………………………………… ……………………………………………………………… ……………………………………………………………… ………………………………………………………………
(4)	$\frac{1}{2}x + 1 \leq 2$ ……………………………………………………………… ……………………………………………………………… ……………………………………………………………… ………………………………………………………………
(5)	$3 - 2x \leq 7$ ……………………………………………………………… ……………………………………………………………… ……………………………………………………………… ……………………………………………………………… ………………………………………………………………

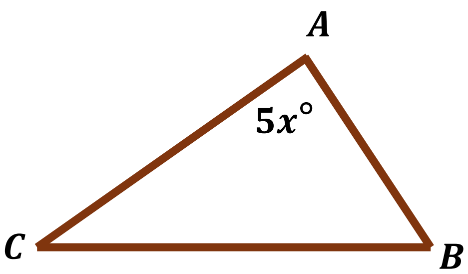
(3) Find the solution set for each of the following inequalities in R in the form of an interval, then represent the solution on the number line :

(1)	$3 < x + 2 \le 6$ …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… ……………………………………………………………………………………
(2)	$-5 < x + 3 < 9$ …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… ……………………………………………………………………………………
(3)	$\sqrt[3]{-8} \le x + 1 \le \sqrt{9}$ …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… ……………………………………………………………………………………
(4)	$-8 \le 3x + 1 \le 4$ …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… ……………………………………………………………………………………
(5)	$\lvert -3\rvert < 2x - 1 < 5$ …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… ……………………………………………………………………………………

(4) Find the solution set for each of the following inequalities in R in the form of an interval, then represent the solution on the number line :

(1)	$3x < 2x + 4$ …………………………………………………………………………………………… …………………………………………………………………………………………… …………………………………………………………………………………………… ……………………………………………………………………………………………
(2)	$x - 1 \leq 3 - x$ …………………………………………………………………………………………… …………………………………………………………………………………………… …………………………………………………………………………………………… …………………………………………………………………………………………… …………………………………………………………………………………………… ……………………………………………………………………………………………
(3)	$1 - x \geq 2x - 3$ …………………………………………………………………………………………… …………………………………………………………………………………………… …………………………………………………………………………………………… …………………………………………………………………………………………… ……………………………………………………………………………………………
(4)	$7x - 12 \geq 5x - 8$ …………………………………………………………………………………………… …………………………………………………………………………………………… …………………………………………………………………………………………… ……………………………………………………………………………………………
(5)	$5x - 3 < 2x + 9$ …………………………………………………………………………………………… …………………………………………………………………………………………… …………………………………………………………………………………………… …………………………………………………………………………………………… ……………………………………………………………………………………………

(5) Find the solution set for each of the following inequalities in R in the form of an interval, then represent the solution on the number line :

(1)	$-x < x < 4 - x$ ....
(2)	$x - 1 < 3x - 1 \leq x + 1$ ....
(3)	$2 + 2x \leq 3x + 3 < 5 + 2x$ ....
(4)	$\frac{3x - 4}{6} < x + 1 < \frac{x + 3}{2}$ ....
(5)	In the figure opposite, if ∠A is obtuse, what are the possible values of x .... 

Unit 3 Lesson 1 — The Axioms of inequality

Learn

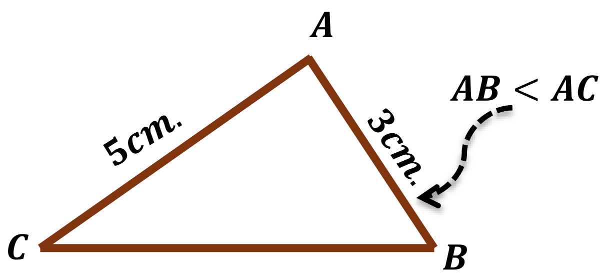
The concept of inequality

- Through our study of the sets of numbers, we had shown the relation of inequality that is used for comparing two different numbers, we expressed that by using one of the two signs

  $>$ that is read ⟶ «is greater than » or $<$ that is read ⟶ « is smaller than »
- Since the lengths of line segments and measures of angles are numbers, then we can use the relation of inequality to compare between the lengths of two line segments or between the measures of two angles.

For example:

In $\triangle ABC$: If AC = 5 cm. and AB = 3 cm.,



then we deduce that :

The length of $\overline{AC}$ is greater than the length of $\overline{AB}$, then we write $AC > AB$

or the length of $\overline{AB}$ is smaller than the length of $\overline{AC}$, then we write $AB < AC$

The concept of inequality

For any four numbers a, b, c, d:

(1) $a > b$, $a + c > b + c$

(2) $a > b$, $a - c > b - c$

(3) $a > b$, $c > 0$, $ac > bc$

(4) $a > b$, $b > c$, $a > c$

(5) $a > b$, $c > d$, $a + c > b + d$

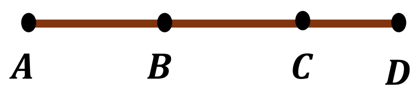
Example 1 :

(1)

In the opposite figure :

If B and C belong to $\overline{AD}$ such that $AB > CD$

, prove that : AC > BD



Solution

$\because$ AB > CD   adding BC to both sides

$\therefore$ AB + BC > CD + BC

$\therefore$ AC > BD

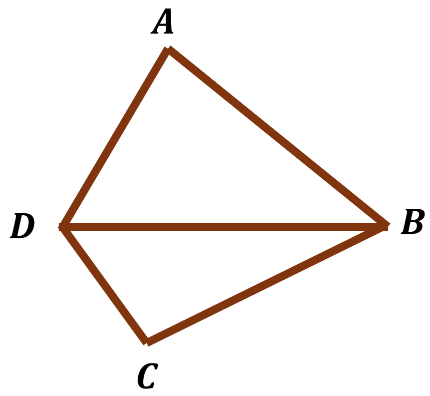
(2)

In the opposite figure :

If m(∠ADB) > m(∠ABD) and

m(∠CBD) < m(∠CDB)

, prove that : m(∠ADC) > m(∠ABC)



Solution

$\because$ m(∠CBD) < m(∠CDB)

$\therefore$ m(∠CDB) > m(∠CBD)   (1)

$\because$ m(∠ADB) > m(∠ABD)   (2)

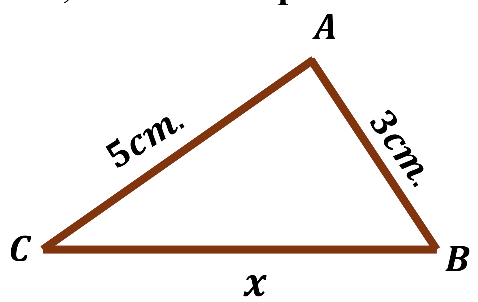
Adding (1) and (2) :

$\therefore$ m(∠CDB) + m(∠ADB) > m(∠CBD) + m(∠ABD)

$\therefore$ m(∠ADC) > m(∠ABC)

(3)

In the figure opposite, find the set of possible values of x, where x is a positive real number



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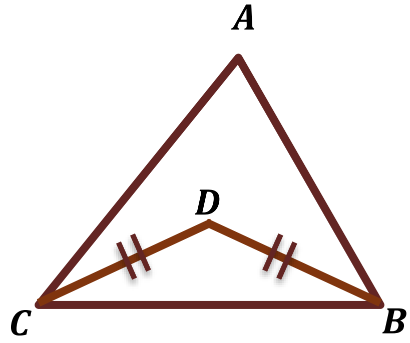
(4)

In the opposite figure :

If $m(\angle ABC) > m(\angle ACB)$ and $BD = DC$

, prove that : $m(\angle ABD) > m(\angle ACD)$



Solution

$\because DB = DC$

$\therefore m(\angle DBC) = m(\angle DCB)$

$\because m(\angle ABC) > m(\angle ACB)$

Subtracting (1) from (2) :

$\therefore m(\angle ABC) - m(\angle DBC) > m(\angle ACB) - m(\angle DCB)$

$\therefore m(\angle ABD) > m(\angle ACD)$

Remember that

The measure of any exterior angle of a triangle is greater than the measure of any interior angle of the triangle except its adjacent angle.

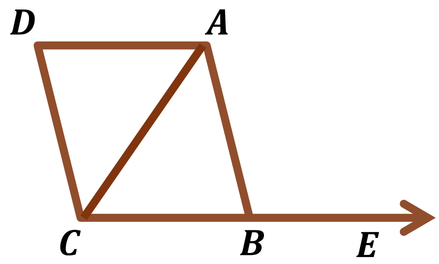
(5)

In the opposite figure :

ABCD is a parallelogram and $E \in \overrightarrow{CB}$

Prove that:

$m(\angle ABE) > m(\angle ACD)$



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The Axioms of inequality (1)

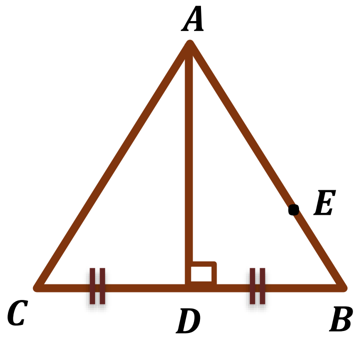
EX (1)

(1)

In the opposite figure :

$E \in \overline{AB}$, $\overline{AD} \perp \overline{BC}$ and D is the midpoint of $\overline{BC}$

Prove that : $AC > AE$



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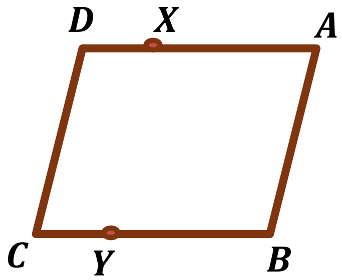
(2)

In the opposite figure :

ABCD is a parallelogram, $X \in \overline{AD}$ and $Y \in \overline{BC}$

such that $DX < BY$ .Prove that: $AX + AB > CY + CD$



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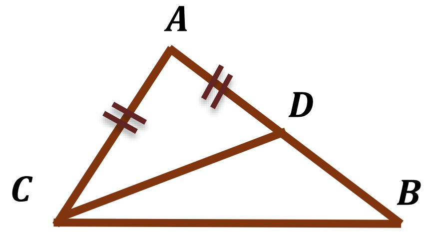
(3)

In the opposite figure : $D \in \overline{AB}$ where $AD = AC$

Prove that : $m(\angle ACB) > m(\angle B)$



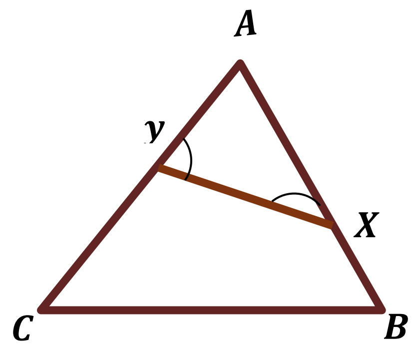
(4)

In the opposite figure :

ABC is a triangle in which : $AC > AB, X \in \overline{AB}$

and $Y \in \overline{AC}$ where $m(\angle AXY) = m(\angle AYX)$

Prove that: $YC > XB$



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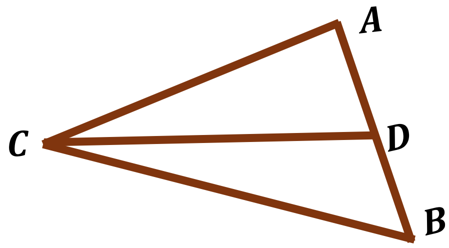
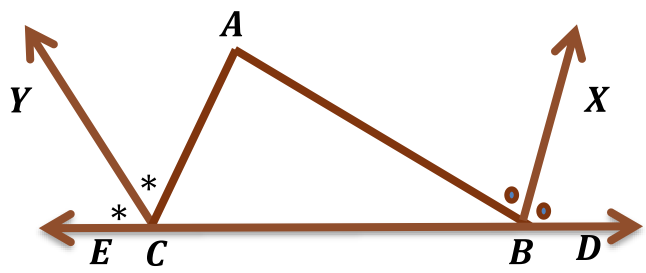
(5)	In the opposite figure : ABC is a triangle in which : $\overline{AB} \equiv \overline{AC}$ and $D \in \overline{AB}$ Prove that : $m(\angle ADC) > m(\angle ACB)$  …………………………………………………… …………………………………………………… …………………………………………………… …………………………………………………… …………………………………………………… …………………………………………………… …………………………………………………… …………………………………………………… …………………………………………………… …………………………………………………… ……………………………………………………
(6)	10 In the opposite figure : $B \in \overrightarrow{DE}$, $C \in \overrightarrow{DE}$ such that $m(\angle ACB) > m(\angle ABC)$, $\overrightarrow{BX}$ bisects $\angle ABD$ and $\overrightarrow{CY}$ bisects $\angle ACE$ Prove that : $m(\angle ABX) > m(\angle ACY)$  …………………………………………………… …………………………………………………… …………………………………………………… …………………………………………………… …………………………………………………… …………………………………………………… …………………………………………………… …………………………………………………… …………………………………………………… ……………………………………………………

Unit 3
Lesson 2

Inequality in Triangle

Theorem

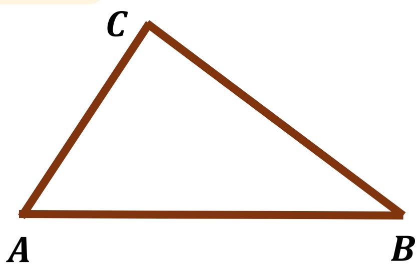
- In a triangle, if two sides have unequal lengths, then the longer is opposite to the angle of the greater measure.

Remark

i.e. In $\triangle$ ABC :

If AB > BC > AC ,

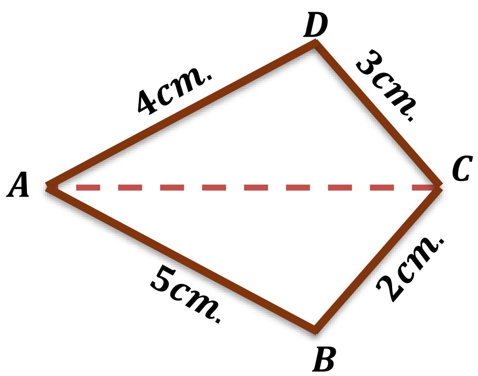
m(∠C) > m(∠A) > m(∠B) ,



Example 1 :

ABCD is a quadrilateral in which AB = 5 cm. , BC = 2 cm. , CD = 3 cm. and DA = 4 cm.

Prove that : m(∠DCB) > m(∠DAB)



Solution

Construction Draw $\overline{AC}$

$\triangle$ ACD : ∵ AD = 4 cm. and CD = 3 cm.

(1) ∴ AD > CD

∴ m(∠ACD) > m(∠CAD) #(1)

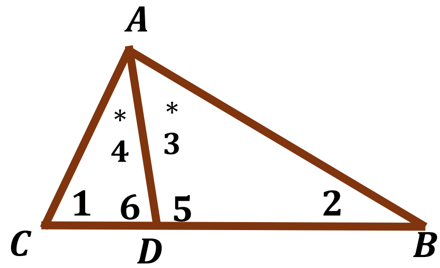
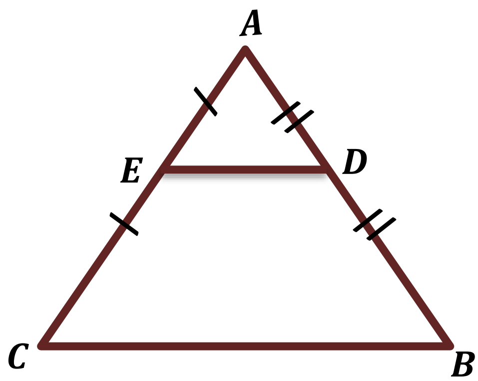
In $\triangle$ ABC: ∵ AB = 5 cm. and CB = 2 cm.

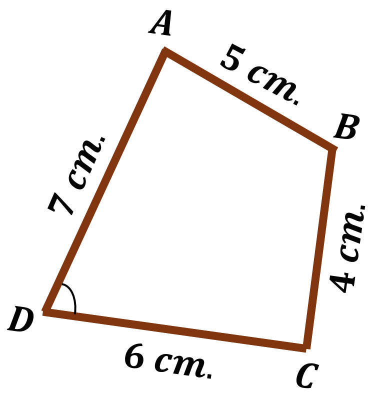
∴ AB > CB

∴ m(∠ACB) > m(∠CAB) #(2)

Adding (1) and (2) : ∴ m(∠ACD) + m(∠ACB) > m(∠CAD) + m(∠CAB)

∴ m(∠DCB) > m(∠DAB) (Q.E.D.)

(2)	ABC is a triangle in which AB > AC and ∠BAC is bisected by $\overrightarrow{AD}$ which intersects $\overline{BC}$ at D Prove that : △ ABD is an obtuse-angled triangle.  Solution △ ABC : ∵ AB > AC ∴ $m(\angle 1) > m(\angle 2)$ ∵ $\overrightarrow{AD}$ bisects ∠BAC ∴ $m(\angle 3) = m(\angle 4)$ ∴ $m(\angle 1) + m(\angle 4) > m(\angle 2) + m(\angle 3)$ but $m(\angle 1) + m(\angle 4) = m(\angle 5)^{(*)}$ because ∠5 is an exterior angle of △ ADC ∴ $m(\angle 5) > m(\angle 2) + m(\angle 3)^{(**)}$ ∴△ ABD is an obtuse-angled triangle.
(3)	In the opposite figure : ABC is a triangle in which AB > AC , D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively. Prove that: $m(\angle AED) > m(\angle ADE)$  ....

(4)	In the opposite figure : ABCD is a quadrilateral in which : AB = 5 cm. , BC = 4 cm. , CD = 6 cm. and DA = 7 cm. Prove that : m(∠ABC) > m(∠ADC)  ………………………………………………………………………………
(5)	ABCD is a parallelogram in which : AC > BD Prove that: ∠D is an obtuse angle. ………………………………………………………………………………

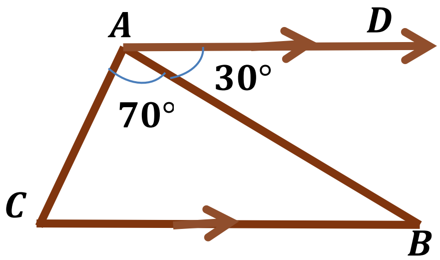
Theorem

In a triangle, if two angles are unequal in measure, then the greater angle in measure is opposite to a side greater in length than that opposite to the other angle.

Example 1 :

In the opposite figure : ABC is a triangle in which $m(\angle BAC) = 70^\circ$, $\overrightarrow{AD} // \overrightarrow{BC}$ and $m(\angle DAB) = 30^\circ$

Prove that: $AB > AC$



Solution

(6) $\because \overrightarrow{AD} // \overrightarrow{BC}$ and $\overleftrightarrow{AB}$ is a transversal to them.

$\therefore m(\angle B) = m(\angle DAB) = 30^\circ$ (alternate angles)

$\therefore$ In $\triangle ABC$: $m(\angle C) = 180^\circ - (30^\circ + 70^\circ) = 80^\circ$

$\therefore m(\angle C) > m(\angle B)$

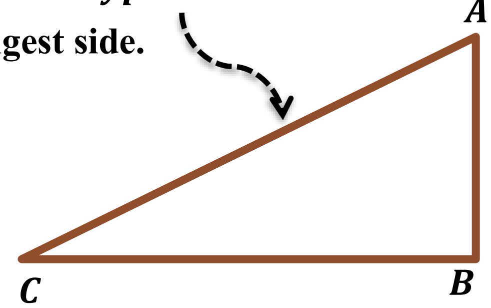
$\therefore AB > AC$ (Q.E.D.)

Corollaries

Corollary (1)

In the right-angled triangle, the hypotenuse is the longest side.

In the opposite figure :



If $\triangle ABC$ is right-angled at B , then $m(\angle B) > m(\angle A)$, $m(\angle B) > m(\angle C)$ because $\angle B$ is a right angle and each of $\angle A$ and $\angle C$ is acute, so we find that:

$AC > BC$ and $AC > AB$ (according to the previous theorem).

Notice that:

In the obtuse-angled triangle, the side opposite to the obtuse angle is the longest side in the triangle.

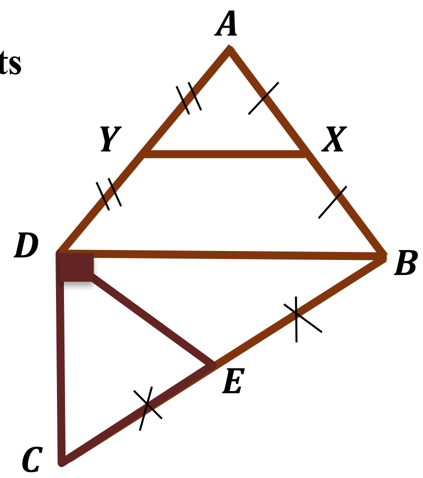
Example 1 :

(1)

In the opposite figure :

ABCD is a quadrilateral X, Y and E are the midpoints of $\overline{AB}$, $\overline{AD}$ and $\overline{BC}$ respectively and $m(\angle BDC) = 90^\circ$

Prove that: $DE > XY$



Solution

$\triangle ABD$: $\because$ X is the midpoint of $\overline{AB}$ and

Y is the midpoint of $\overline{AD}$

$\therefore XY = \frac{1}{2}BD$ #(1)

In $\triangle DBC$: $\because m(\angle BDC) = 90^\circ$ and E is the midpoint of $\overline{BC}$

$DE = \frac{1}{2}BC$ #(2)

$\because \overline{BC}$ is the hypotenuse of $\triangle BDC$

$\therefore BC > BD$

$\therefore \frac{1}{2}BC > \frac{1}{2}BD$ #(3)

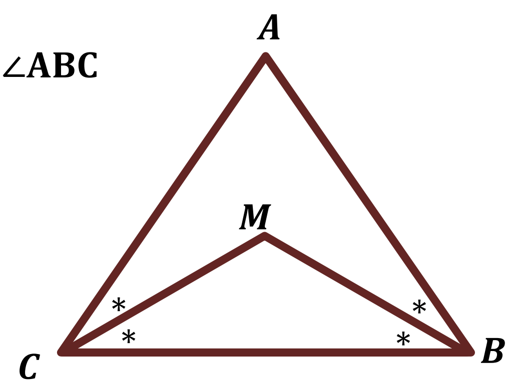
From (1), (2) and (3): $\therefore DE > XY$

(2)

In the opposite figure :

ABC is a triangle in which $AC > AB$, $\overrightarrow{BM}$ bisects $\angle ABC$

and $\overrightarrow{CM}$ bisects $\angle ACB$ Prove that: $MC > MB$



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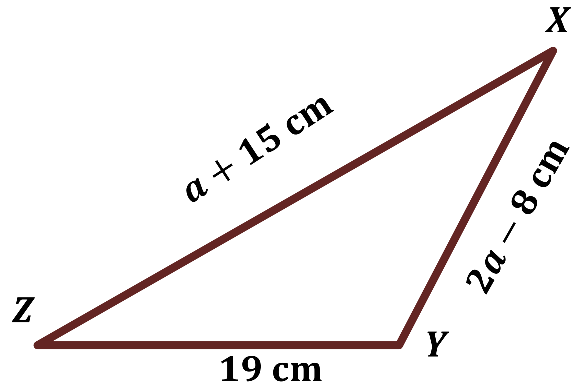
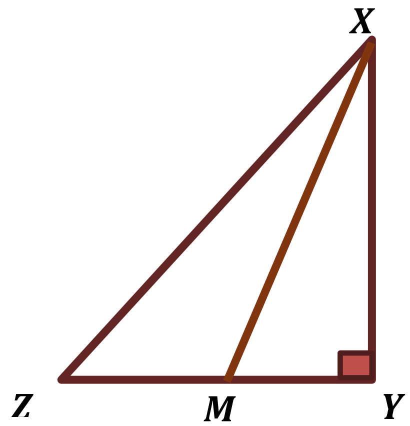
(3)	In the figure opposite, find the set of values of $a$that make $m(\angle Z) < m(\angle Y)$ solu $\because \quad XZ + XY \quad > ZY$ $\therefore a + 15 + 2a - 8 > 19$ $\therefore \quad 3a + 7 \quad > 19$ $\therefore \quad 3a \quad > 19 - 7$ $\therefore \quad 3a \quad > 12$ $\therefore \quad a \quad > 4$ $\because m(\angle Z) < m(\angle Y)$ $\therefore \quad XY < XZ$ $\therefore 2a - 8 < a + 15$ $\therefore 2a - a < 15 + 8$ $\therefore \quad a \quad < 23$ $\therefore 4 < a < 23$ $\therefore a \in]4, 23[$ 
(4)	In the opposite figure : XYZ is a right-angled triangle at Y and M ∈ $\overline{YZ}$ Prove that: XZ > XM  …………………………………………………… …………………………………………………… …………………………………………………… …………………………………………………… …………………………………………………… …………………………………………………………………………………………… …………………………………………………………………………………………… …………………………………………………………………………………………… …………………………………………………………………………………………… …………………………………………………………………………………………… …………………………………………………………………………………………… ……………………………………………………………………………………………

Inequality in Triangle (2)

EX (1)

(1) Arrange the measures of the angles of $\triangle$ ABC in each of the following cases ascendingly :

(1) If AB = 12 cm. BC = 15 cm. and AC = 10 cm.

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(2) If AB = 5.7 cm., BC = 8.5 cm. and AC = 6 cm.

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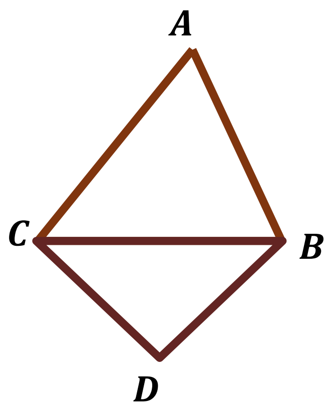
(2) In the opposite figure :

AC > AB and DB = DC

Prove that: $m(\angle ABD) > m(\angle ACD)$



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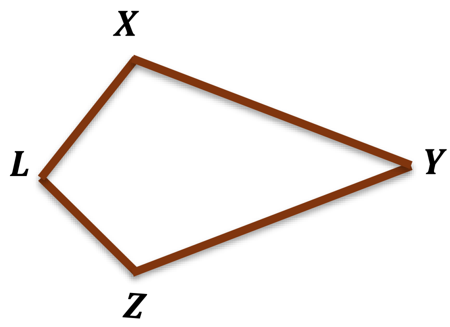
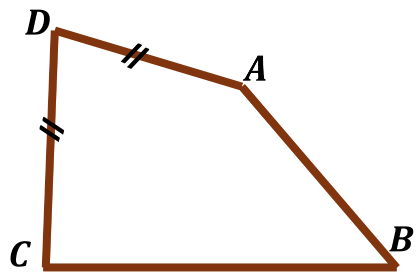
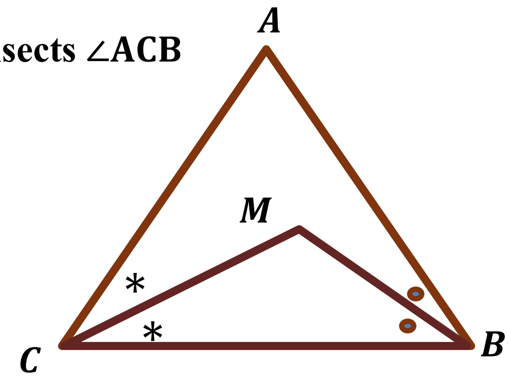
(3)	In the opposite figure : XY > XL and YZ > ZL. Prove that : m(∠XLZ) > m(∠XYZ)  …………………………………………………
(4)	In the opposite figure : ABCD is a quadrilateral in which : AD = DC and BC > AB. Prove that : m(∠A) > m(∠C)  …………………………………………………
(5)	ABCD is a quadrilateral in which : $\overline{AB}$ is the longest side, $\overline{CD}$ is the shortest one. Prove that : m(∠BCD) > m(∠BAD) …………………………………………………

(6) In the opposite figure :

ABC is a triangle, $\overrightarrow{BM}$ bisects $\angle ABC$ and $\overrightarrow{CM}$ bisects $\angle ACB$

If $MC > MB$

, prove that : $m(\angle ABC) > m(\angle ACB)$



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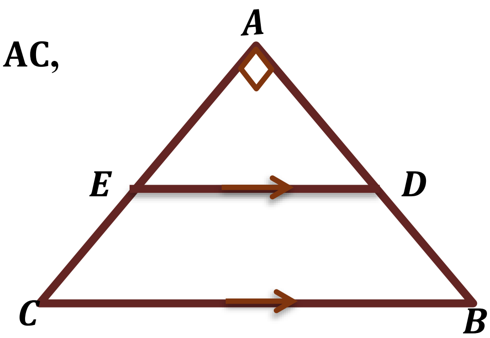
(7) In the opposite figure :

ABC is a triangle in which : $m(\angle A) = 90°$, $AB > AC$,

$D \in \overline{AB}$, $E \in \overline{AC}$ and $\overline{DE} // \overline{BC}$

Prove that : $m(\angle AED) > 45°$



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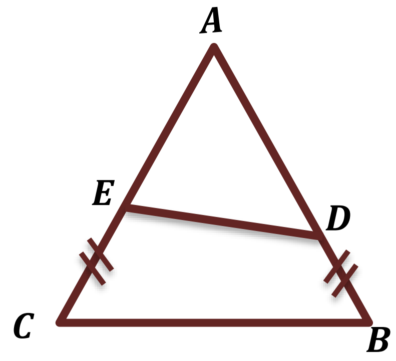
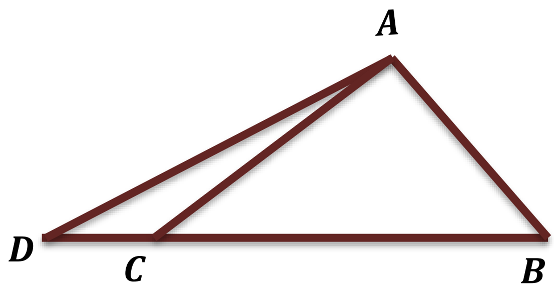
(8)	In the opposite figure : ABC is a triangle in which : $AB > AC, D \in \overline{AB}$ and $E \in \overline{AC}$ where $BD = CE$ Prove that : $m(\angle AED) > m(\angle ADE)$  …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… ……………………………………………………………………………………
(9)	In the opposite figure : $C \in \overline{BD}$ such that $AC > AB$ Prove that : $m(\angle ABD) > m(\angle D)$  …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… ……………………………………………………………………………………

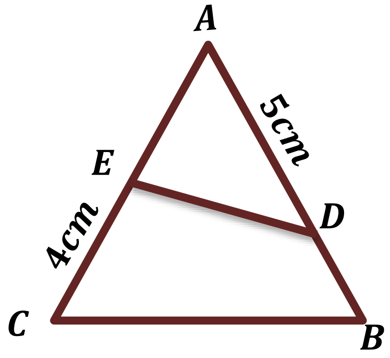
(10) In the opposite figure :

ABC is an equilateral triangle

whose side length = 7 cm. , D ∈ $\overline{AB}$ such that

AD = 5 cm. and E ∈ $\overline{AC}$ such that CE = 4 cm.

Prove that : $m(\angle AED) > 60^\circ$



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(11) In the figure opposite, find the set of values of $a$that satisfy $m(\angle Z) < m(\angle Y)$

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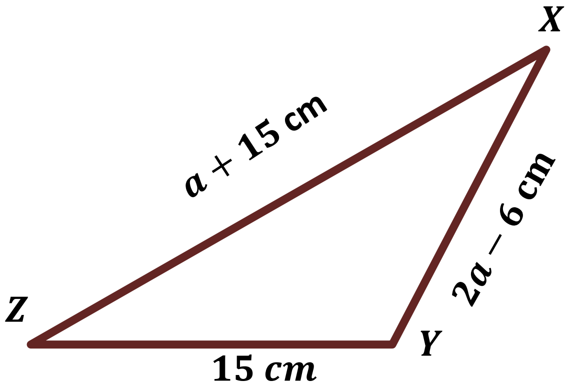

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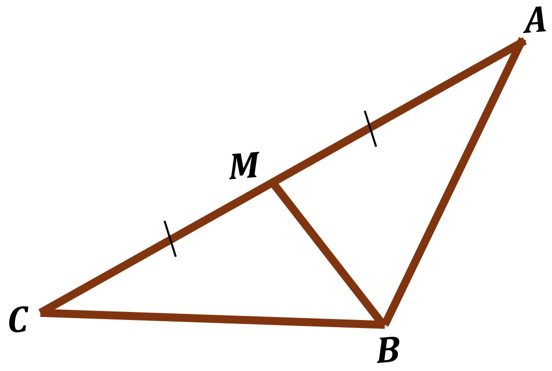
(12)

In the opposite figure :

$\overline{BM}$ is a median in the triangle ABC and BM < AM

Prove that: ∠ABC is an obtuse angle.



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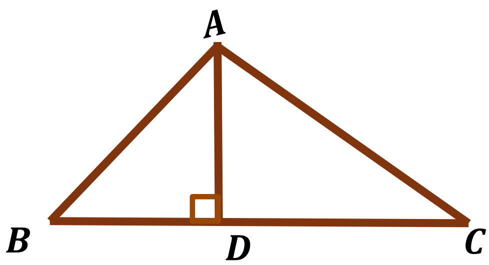
(13)

In the opposite figure :

ABC is a triangle in which : AC > AB, $\overline{AD} \perp \overline{BC}$ and intersects it at D

Prove that : m(∠BAD) < m(∠CAD)



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EX (2)

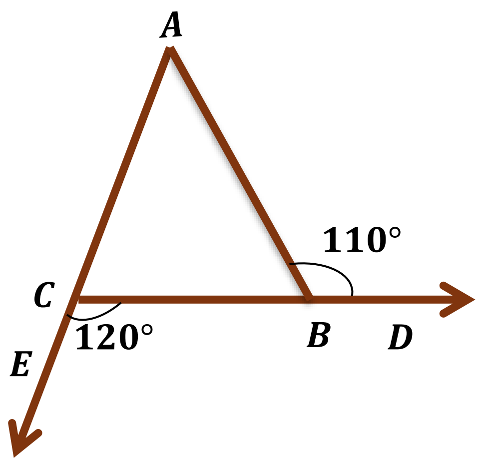
(1) In the opposite figure:

ABC is a triangle, $D \in \overrightarrow{CB}$,

$E \in \overrightarrow{AC}$, $m(\angle ABD) = 110^\circ$

and $m(\angle BCE) = 120^\circ$

Prove that: $AB > BC$



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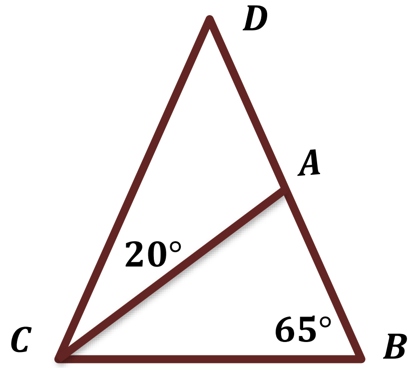
(2) In the opposite figure :

$AB = AC$, $m(\angle ABC) = 65^\circ$

, $m(\angle ACD) = 20^\circ$, $A \in \overline{BD}$

Prove that: $AB > AD$



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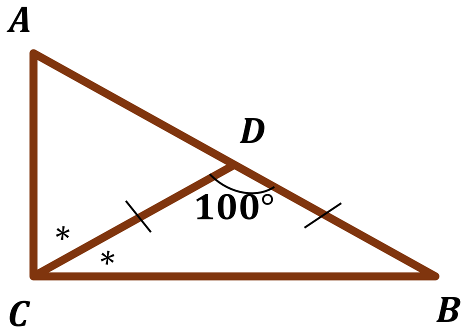
(3)

In the opposite figure : ABC is a triangle, $\overrightarrow{CD}$ bisects $\angle C$ and intersects $\overline{AB}$ at point D , $m(\angle BDC) = 100^\circ$ and $DB = DC$

Prove that: $AC > DB$



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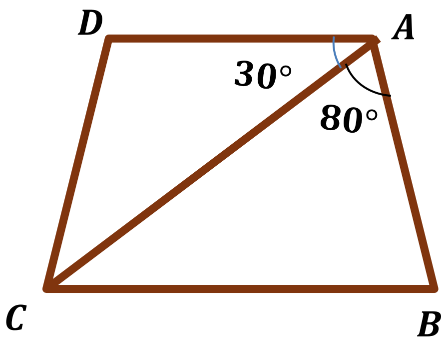
(4)

In the opposite figure :

$\overline{AD} // \overline{BC}$, $m(\angle BAC) = 80^\circ$ and $m(\angle DAC) = 30^\circ$

Prove that: $BC > AB$



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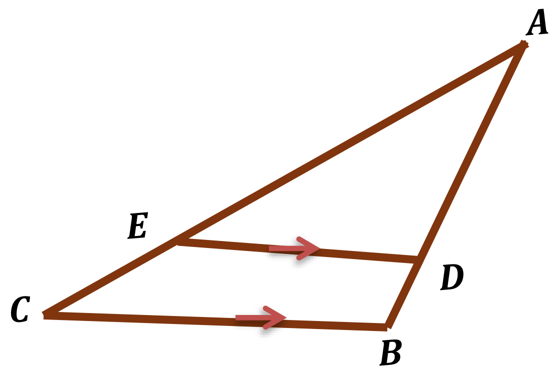
(5)

In the opposite figure :

ABC is an obtuse-angled triangle at B

, $\overline{DE}//\overline{BC}$

Prove that: AE > AD



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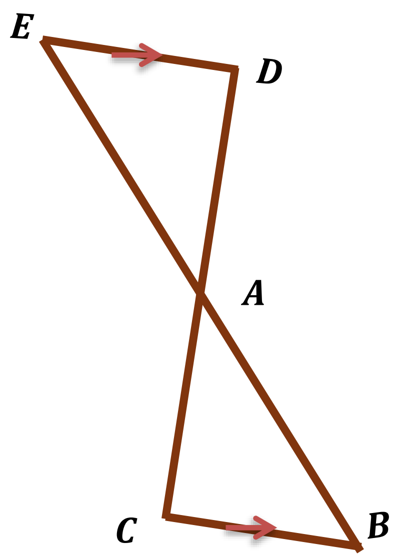
(6)

In the opposite figure :

AB > AC, $\overline{DE}//\overline{BC}$ and

$\overline{DC} \cap \overline{BE} = \{A\}$

Prove that: AE > AD



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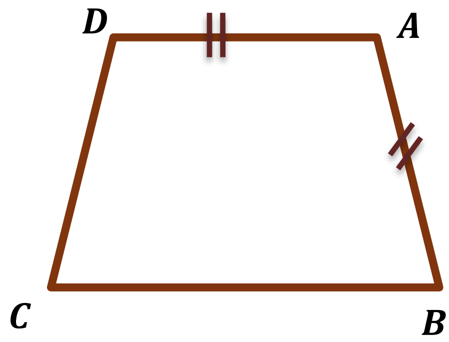
(7)

In the opposite figure :

ABCD is a quadrilateral , AB = AD

and m(∠D) > m(∠B)

Prove that: BC > CD



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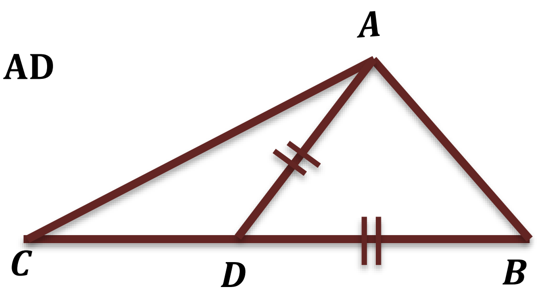
(8)

In the opposite figure :

ABC is a triangle and D ∈ $\overline{BC}$ where BD = AD

Prove that: BC > AC



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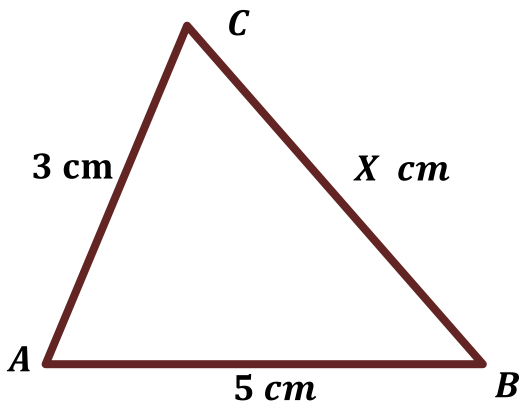
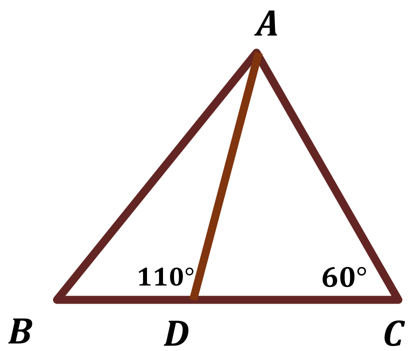
(9) In the figure opposite, find the set of values of X that make $m(\angle B) < m(\angle A)$



(10) In the opposite figure :

$m(\angle ADB) = 110^\circ$ and $m(\angle C) = 60^\circ$

Prove that: $AB + AC > 2AD$



Unit 3 Lesson 2

Inequality in two Triangles

theorem

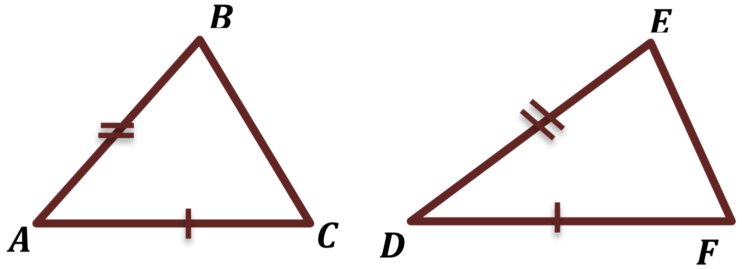
- If two sides of a triangle are equal in length to two sides of another triangle, and the measures of the included angles between these sides are different, then the larger angle corresponds to the longer opposite side.

For example,

if $m(\angle A) > m(\angle D)$, $AC = DF$,

and $AB = DE$,

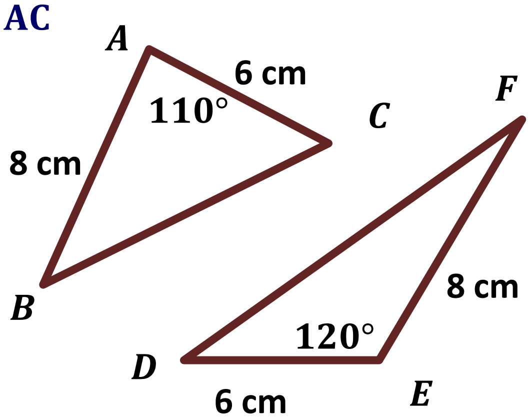
then $BC > EF$.



Example 1:

(7) Compare the length of $\overline{AD}$ and the length of $\overline{AC}$

Solution



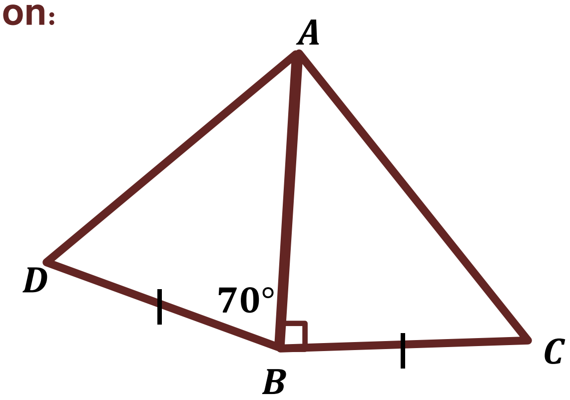
$\because$ EF = AB , DE = AC

$\because$ m(∠E) > m(∠A)

$\therefore$ FD > BC

(8) Compare the length of $\overline{FD}$ and the length of $\overline{BC}$

Solution:



$\because$ BD = BC , AB Common side

$\because$ m(∠ABD) < m(∠ABC)

$\therefore$ AD < AC

(9) In the figure opposite: Compare the length of $\overline{ZY}$ and the length of $\overline{MZ}$

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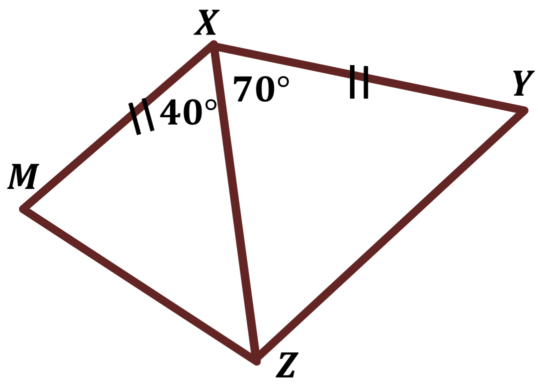


Note

- The length of the side opposite the angle included between two sides in a triangle increases as the measure of that angle increases.

Example 2 :

(1)

In the figure opposite:

Compare the length of $\overline{FD}$ and the length of $\overline{AC}$

Solution:

The sum of the interior angles of triangle FGD is $180°$

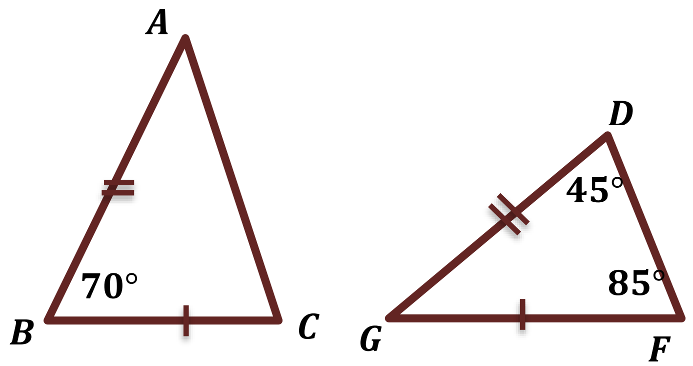
$\therefore$ m $(\angle G) = 180° - (85° + 45°)$

$= 180° - 130° = 50°$

$\because$ DG = AB , FG = BC

m$(\angle G) <$ m$(\angle B)$

FD $<$ AC



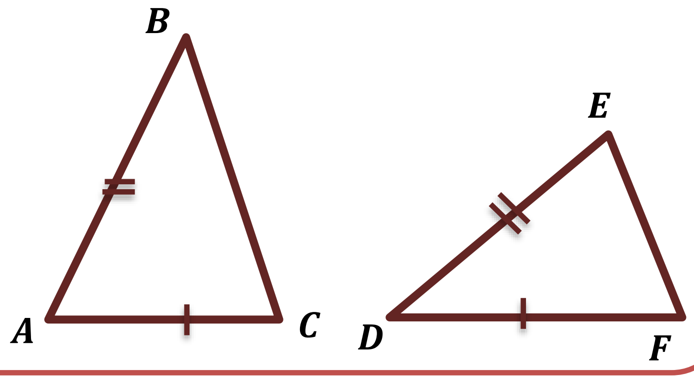
The converse of the theorem

- If two sides of a triangle are equal in length to two sides of another triangle, and the third side differs in length in each triangle, then the larger side corresponds to the larger angle.

For example, in the figure opposite:

If $BC > EF$, $AC = DF$, and $AB = DE$,

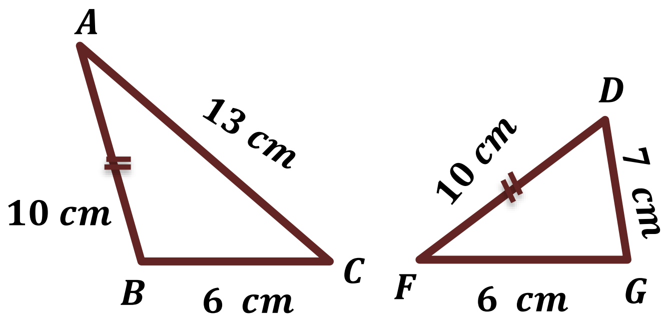
then $m(\angle A) > m(\angle D)$



Example 3 :

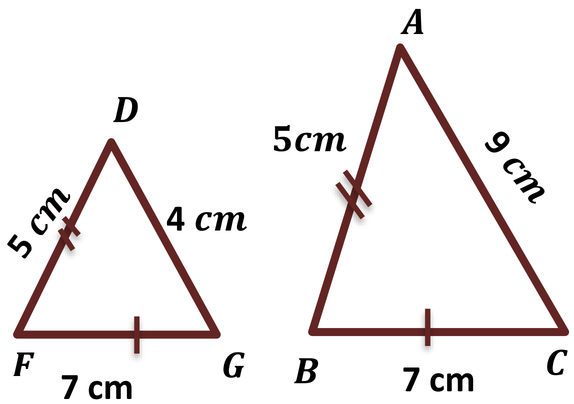
(5) Compare: $m(\angle B)$ and $m(\angle F)$

Solution:



$FD = BA = 10$,
$FG = BC = 6$
$DG < AC$
$\therefore m(\angle F) < m(\angle B)$

(6) In the figure opposite: Compare $m(\angle F)$ and $m(\angle B)$



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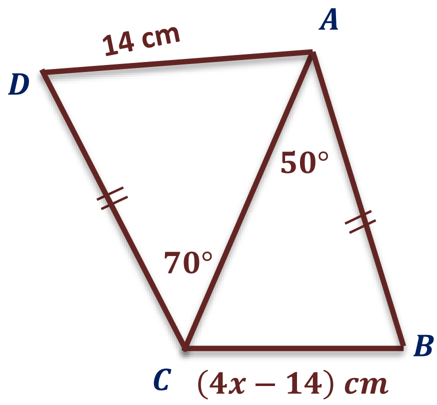
Example (5):
Find the set of possible values of X in each of the following (where x is a real number)

(1)



Solution:

$\because BC = CD, \quad AC$ (Common side)

$\because m(\angle CAB) < m(DCA)$

$\therefore CB < DA$

$\therefore 4x - 14 < 14$

$\therefore 4x < 14 + 14$

$\therefore 4x < 28$

$\therefore \frac{4x}{4} < \frac{28}{4}$

$x < 7$

$\because CB > 0$

$4x - 14 > 0$

$\therefore 4X > 14$

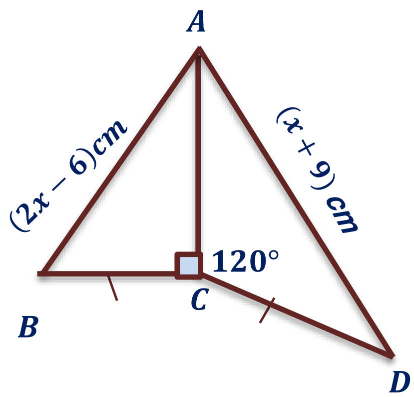
$\therefore \frac{4x}{4} > \frac{14}{4}$

$x > 3.5$

$\therefore 3.5 < x < 7$

$\therefore x \in]3.5, 7[$

(2)



Solution

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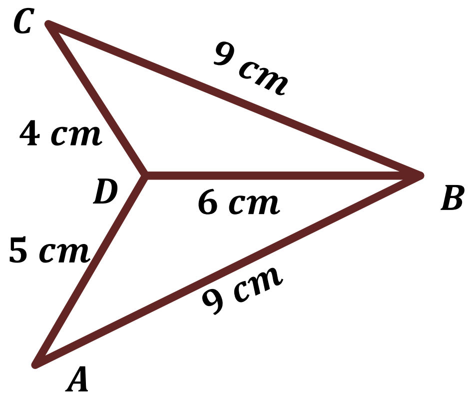
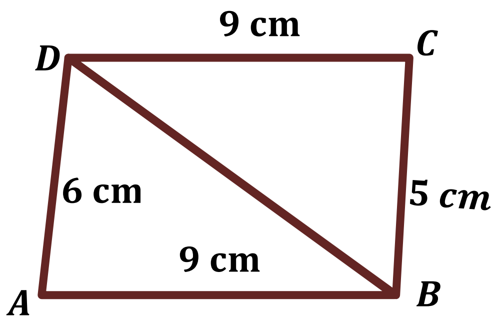
Inequality in two Triangles (3)

Example 1 :

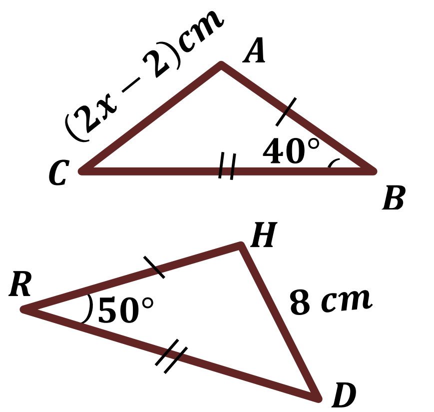
(1) In the figure opposite, compare: $m(\angle CBD)$ and $m(\angle ABD)$

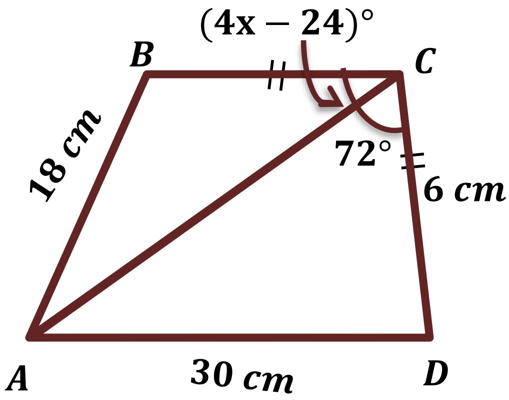
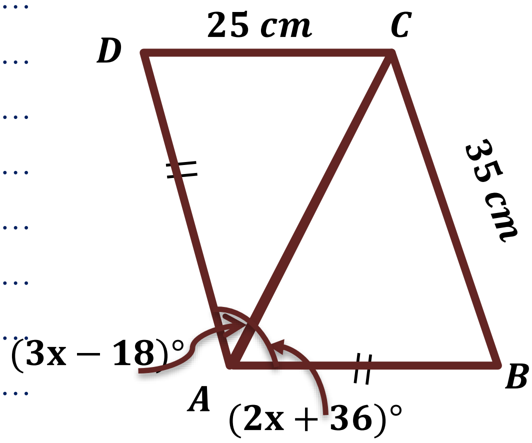


(2) In the figure opposite, compare: m $(\angle ABD)$, m $(\angle CDB)$



(3) In the figure opposite, find the set of possible values of x, where x is a real number.

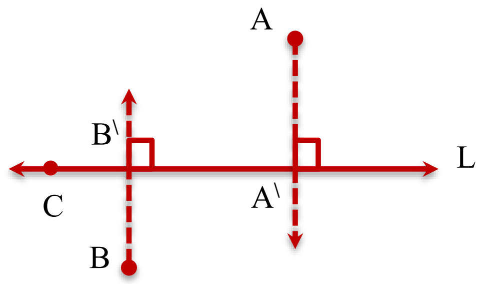


(4)	In the figure opposite, find the set of possible values of x, where x is a real number. 
(5)	In the figure opposite, find the set of possible values of x, where x is a real number. 

Unit 3 Lesson 3 — Projections

The projection of a point on a straight line

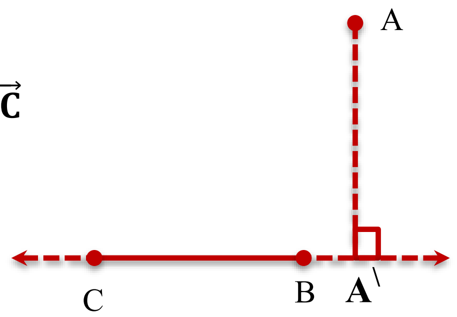
- The projection of a point on a straight line is the point of intersection of the perpendicular segment from this point to the straight line.
- If the point lies on the straight line, its projection on it is the same point.



Example 1 :

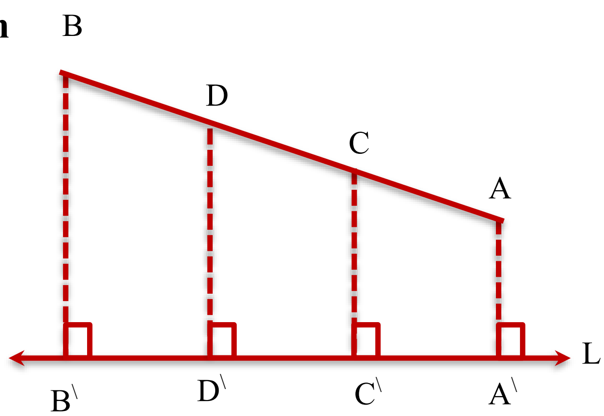
(1) In the opposite figure :

The point A\
is the projection of the point A on the straight line $\overleftrightarrow{BC}$



The projection of a line segment on a straight line

- The projection of a line segment on a given straight line is the line segment whose two endpoints are the projections of the two endpoints of the main line segment on this straight line.

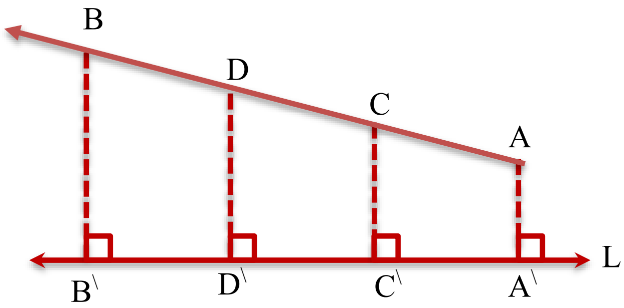


we notice that :

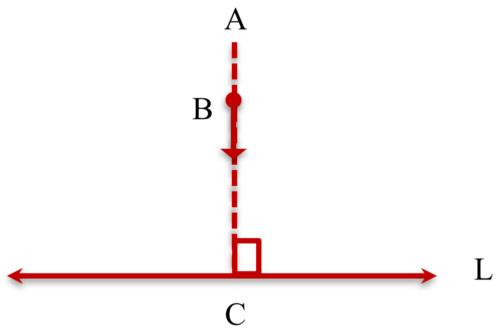
The length of the projection of a line segment on a given straight line $\leq$ the length of the line segment.

The projection of a ray on a straight line

- The projection of a ray on a straight line not perpendicular to it is a ray ⊂ this straight line.

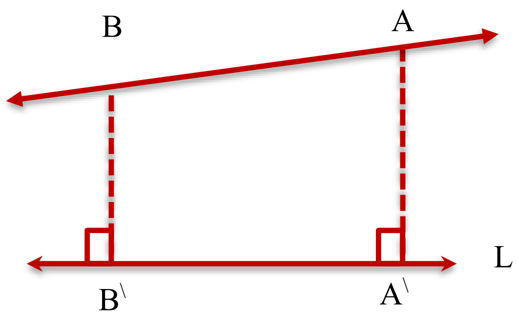


Example 2 :

(1)	In the opposite figure: If $\overrightarrow{AB} \perp$ the straight line L, then the projection of $\overrightarrow{AB}$ on the straight line L is the point C  The projection of a ray on a straight line perpendicular to it is a point belonging to the straight line.

The projection of a straight line on another straight line

- In the opposite figure : The projection of $\overleftrightarrow{AB}$ on the straight line L is the straight line $\overleftrightarrow{AB}$ which is the straight line L itself.



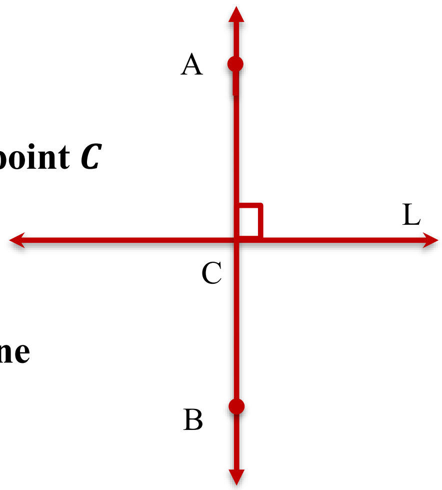
- The projection of a straight line on another straight line not perpendicular to it is that another straight line.

Example 3 :

(1) In the opposite figure :

If $\overleftrightarrow{AB} \perp$ the straight line L ,

then the projection of $\overleftrightarrow{AB}$ on the straight line L is the point C

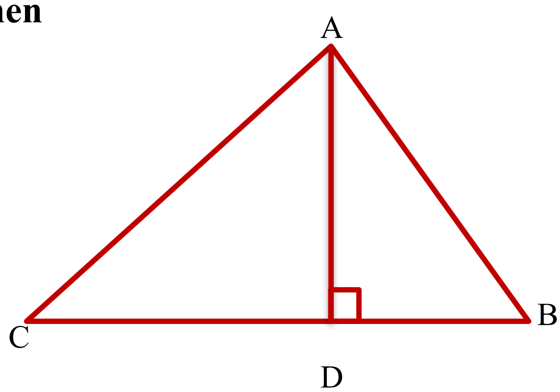


The projection of a straight line on another straight line perpendicular to it is the point of intersection of the two straight lines.

Example 4 :

(1) In the opposite figure:

ΔABC is right-angled at A and $\overline{AD} \perp \overline{BC}$, then



(1) The projection of $\overline{AB}$ on $\overleftrightarrow{BC}$ is $\overline{DB}$

(2) The projection of $\overline{AC}$ on $\overleftrightarrow{BC}$ is $\overline{DC}$

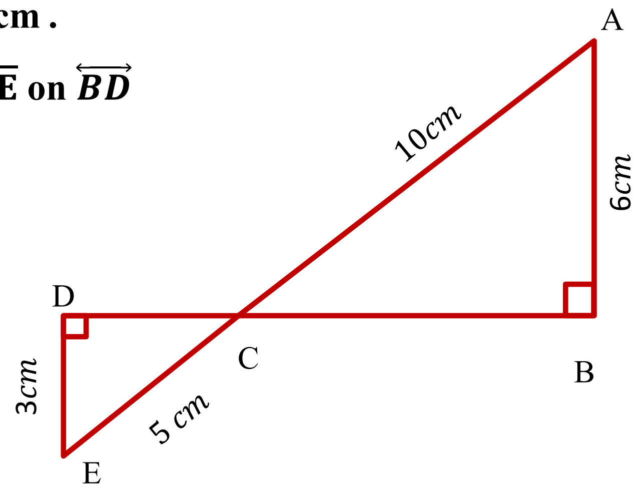
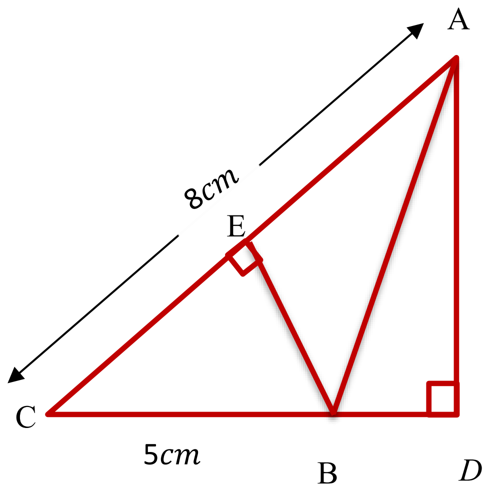
(3)The projection of $\overline{BC}$ on $\overleftrightarrow{AC}$ is $\overline{AC}$

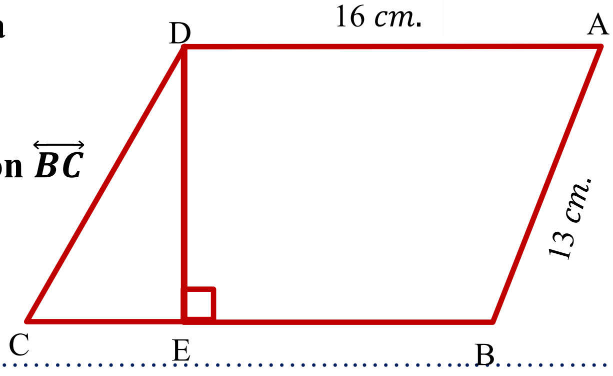
(4)The projection of $\overline{BC}$ on $\overleftrightarrow{AB}$ is $\overline{BA}$

(5)The projection of $\overline{AC}$ on $\overleftrightarrow{AD}$ is $\overline{AD}$

(6)The projection of $\overline{AD}$ on $\overleftrightarrow{BC}$ is The point D

(7)The projection of $\overline{AB}$ on $\overleftrightarrow{AD}$ is $\overline{AD}$

(2)	In the opposite figure : $\overline{BD} \cap \overline{AE} = \{C\}$, $m(\angle B) = m(\angle D) = 90°$, $AB = 6$ cm., $AC = 10$ cm. and $DE = 3$ cm , $EC = 5$ cm . Find : The length of the projection of $\overline{AE}$ on $\overleftrightarrow{BD}$  **Solution** ……………………………………………………………………………… ……………………………………………………………………………… ……………………………………………………………………………… ……………………………………………………………………………… ……………………………………………………………………………… ……………………………………………………………………………… ……………………………………………………………………………… ………………………………………………………………………………
(3)	In the opposite figure : ABC is a triangle in which $AB = BC = 5$ cm. , $AC = 8$ cm. , $\overrightarrow{AD} \perp \overrightarrow{CB}$ and $\overline{BE} \perp \overline{AC}$  Complete the following : (1) The projection of $\overline{AB}$ on $\overline{BC}$ is ……… (2) The length of the projection of $\overrightarrow{AB}$ on $\overleftrightarrow{AC}$ = ……… (3) The projection of $\overline{AB}$ on $\overleftrightarrow{AD}$ ≡ the projection of …………… on $\overleftrightarrow{AD}$ (4) The length of the projection of $\overline{BE}$ on $\overleftrightarrow{AC}$ = ……… (5) The area of △ ABC = ………

(4)	In the opposite figure: ABCD is a parallelogram in which : AD = 16 cm. and AB = 13 cm. If $\overline{DE} \perp \overline{BC}$ and the area of the parallelogram ABCD = 192 cm^2, Find : The length of the projection of $\overline{DC}$ on $\overleftrightarrow{BC}$  …………………………………………………………………………

Projections (4)

1 : Choose the correct answer from the given ones :

1	The projection of a point on a given straight line is ___						
(a)	a point	(b)	a line segment	(c)	a ray	(d)	a straight line
2	The projection of a line segment on a straight line not perpendicular to it is ___						
(a)	a ray	(b)	a point	(c)	a line segment	(d)	a straight line
3	The projection of a line segment on a straight line perpendicular to it is ___						
(a)	a point	(b)	a line segment	(c)	a ray	(d)	a straight line
4	The projection of a ray on a straight line not perpendicular to it is ___						
(a)	a ray	(b)	a point	(c)	a line segment	(d)	a straight line
5	The length of the projection of a line segment on a given straight line ___ the length of the line segment itself.						
(a)	$\leq$	(b)	$>$	(c)	$\geq$	(d)	$=$
6	The length of the projection of a line segment on a straight line perpendicular to it is ___						
(a)	greater than the length of the main line segment.	(b)	equal to the length of the main line segment.	(c)	greater than or equal to the length of the main line segment.	(d)	equal to zero.
6	he length of the projection of a line segment on a straight line parallel to it ___ the length of the main line segment.						
(a)	$<$	(b)	$>$	(c)	$=$	(d)	$\neq$

Example 2 : Find the area of each of the following figures :

(1)

In the opposite figure:

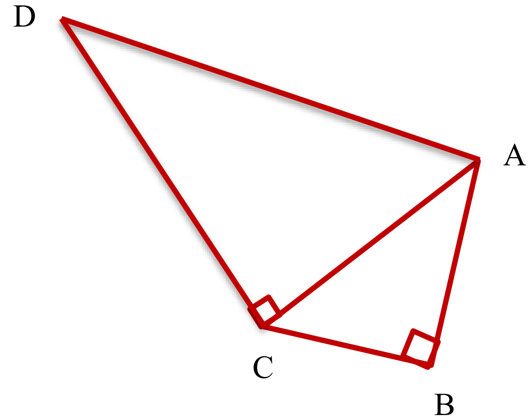
$m(\angle B) = m(\angle ACD) = 90^\circ$

Complete:

(1) The projection of $\overline{AD}$ on $\overleftrightarrow{CD}$ is ___

(2) The projection of $\overline{AC}$ on $\overleftrightarrow{CD}$ is ___

(3) The projection of $\overline{AC}$ on $\overleftrightarrow{AB}$ is ___



(2)

In the opposite figure:

$\triangle$ ABC is right-angled at B, $D \in \overline{BC}$ and $\overline{DE} \perp \overline{AC}$

Complete each of the following :

(1) The projection of $\overline{AD}$ on $\overleftrightarrow{BC}$ = ___

(2) The projection of $\overline{AD}$ on $\overleftrightarrow{AC}$ = ___

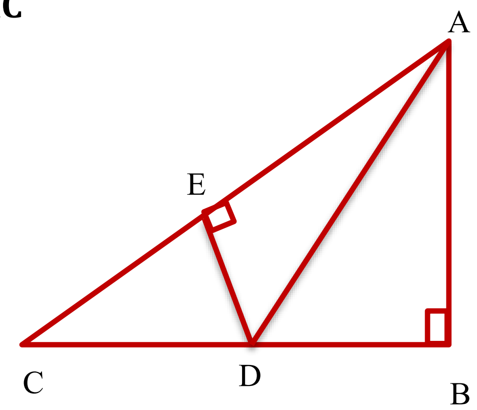
(3) The projection of $\overline{DE}$ on $\overleftrightarrow{AC}$ = ___

(4) The projection of the point C on $\overleftrightarrow{AB}$ = ___

(5) The projection of the point A on $\overleftrightarrow{CD}$ = ___

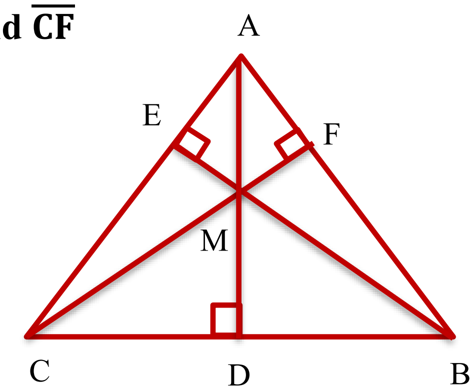
(6) The projection of the point D on $\overleftrightarrow{AC}$ = ___

(7) The projection of $\overline{AB}$ on $\overleftrightarrow{CD}$ = ___



(3)

In the opposite figure :ABC is a triangle, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ are three perpendicular line segments drawn from the vertices to the opposite sides and they are intersecting at M



Complete the following :

(1) The projection of $\overline{AB}$ on $\overleftrightarrow{BC}$ is ___ , the projection of $\overline{BC}$ on $\overleftrightarrow{AB}$ is ___

(2) The projection of $\overline{AC}$ on $\overleftrightarrow{BC}$ is ___ , the projection of $\overline{BC}$ on $\overleftrightarrow{AC}$ is ___

(3) The projection of $\overline{AC}$ on $\overleftrightarrow{AB}$ is ___ , the projection of $\overline{AB}$ on $\overleftrightarrow{AC}$ is ___

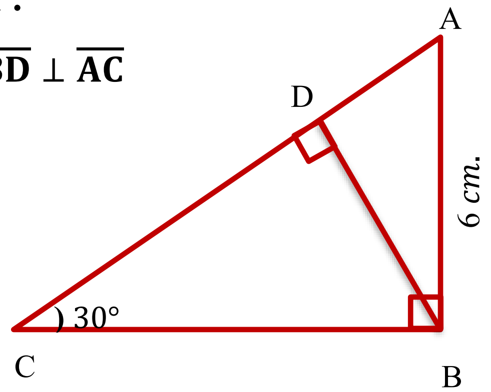
(4) In the opposite figure: ABC is a triangle in which :

$m(\angle B) = 90^\circ, m(\angle C) = 30^\circ$, AB = 6 cm. and $\overline{BD} \perp \overline{AC}$

Find :

(1) The length of the projection of $\overline{AB}$ on $\overleftrightarrow{AC}$

(2) The length of the projection of $\overline{BC}$ on $\overleftrightarrow{AC}$



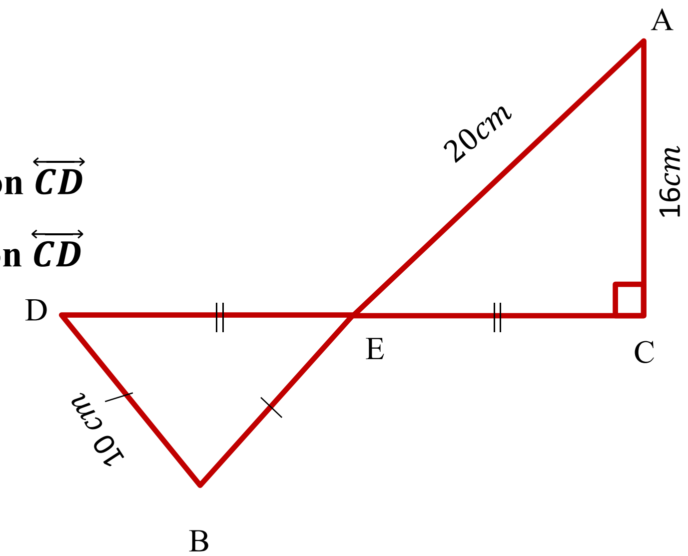
(5) In the opposite figure: $\overline{AB} \cap \overline{CD} = \{E\}$, E is the midpoint of $\overline{CD}$,

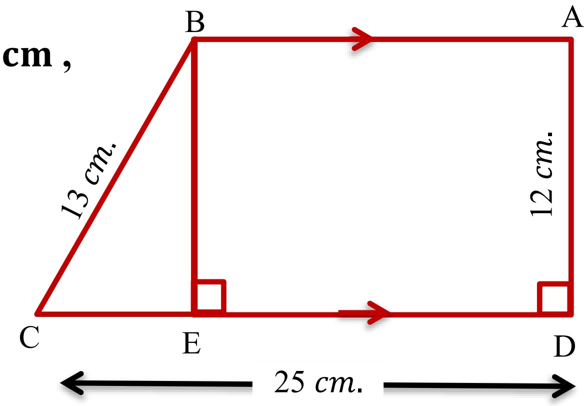
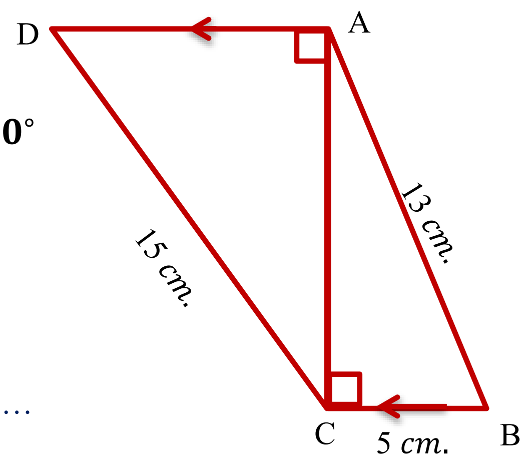
AC = 16 cm, AE = 20 cm.

and BD = BE = 10 cm. Find :

(1) The length of the projection of $\overline{BD}$ on $\overleftrightarrow{CD}$

(2) The length of the projection of $\overline{AB}$ on $\overleftrightarrow{CD}$



(6)	In the opposite figure:ABCD is a trapezium in which : $\overline{AB}//\overline{DC}$, $\overline{AD} \perp \overline{DC}$, AD = 12 cm , BC = 13 cm , and DC = 25 cm. If $\overline{BE} \perp \overline{DC}$, find : (1) The length of the projection of $\overline{BC}$ on $\overleftrightarrow{DC}$ (2) The length of the projection of $\overline{AB}$ on $\overleftrightarrow{DC}$ (3) The length of the projection of $\overline{DC}$ on $\overleftrightarrow{AB}$ (4) The area of the trapezium ABCD  …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… ……………………………………………………………………………
(7)	In In the opposite figure : $\overline{AD}//\overline{BC}$, AB = 13 cm., BC = 5 cm. CD = 15 cm. and m(∠ACB) = m(∠DAC) = 90° Find : (1) The length of the projection of $\overline{AB}$ on $\overleftrightarrow{AC}$ (2) The length of the projection of $\overline{CD}$ on $\overleftrightarrow{AD}$  …………………………………………… …………………………………………… …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… ……………………………………………………………………………

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(8)

In the opposite figure :ABCD is a trapezium in which :

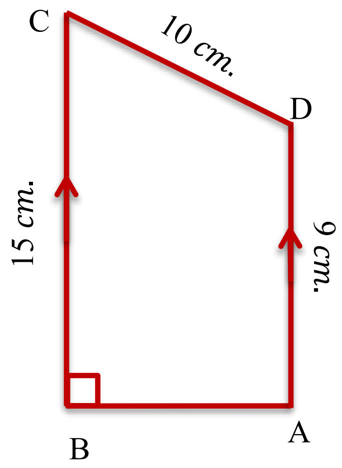
$\overline{AD} // \overline{BC}$ and $m(\angle ABC) = 90°$

If $AD = 9$ cm., $DC = 10$ cm. and $CB = 15$ cm.

Find :

(1) The length of the projection of $\overline{DC}$ on $\overleftrightarrow{BC}$

(2) The length of the projection of $\overline{DC}$ on $\overleftrightarrow{AB}$



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(9)

Complete the following :

(1) If $X \in \overleftrightarrow{AB}$, then the projection of X on $\overleftrightarrow{AB}$ is ___

(2) If $\overline{AB} \perp \overline{BC}$, then the projection of $\overline{AB}$ on $\overleftrightarrow{BC}$ is ___

(3) In $\triangle$ ABC, if $m(\angle B) = 90°$, then the projection of C on $\overleftrightarrow{AB}$ is ___

(4) ABC is a right-angled triangle at A , then the projection of $\overline{BA}$ on $\overleftrightarrow{AC}$ is ___

Unit 3
Lesson 3

Euclidean theorem

Theorem 1

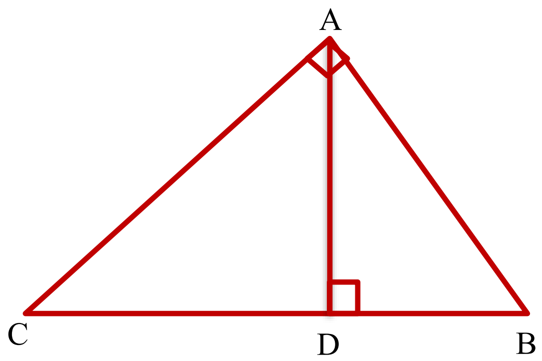
In the right-angled triangle, the area of the square on a side of the right angle is equal to the area of the rectangle whose dimensions are the length of the projection of this side on the hypotenuse and the length of the hypotenuse.

Euclid

- In the opposite figure :

If $\triangle$ ABC is right-angled at A

, D $\in \overline{BC}$ where $\overline{AD} \perp \overline{BC}$



Notice that:

- BD is the length of the projection of $\overline{AB}$ on $\overleftrightarrow{BC}$
- CD is the length of the projection of $\overline{AC}$ on $\overleftrightarrow{BC}$

Corollary

- $(AD)^2 = BD \times DC$
- $(AB)^2 = BD \times BC$
- $(AD)^2 = BD \times DC$
- $AD = \frac{AB \times AC}{BC}$

Example 1 :

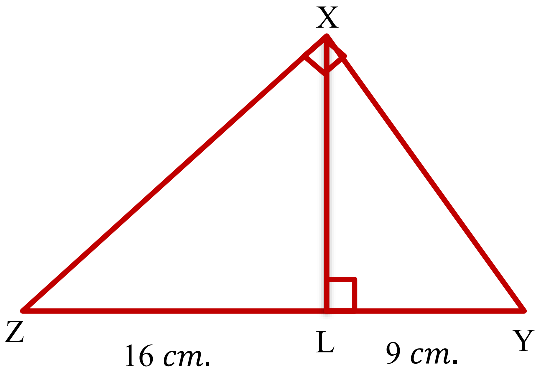
In the opposite figure : XYZ is a right-angled triangle at X, $\overline{XL} \perp \overline{YZ}$ such that : $L \in \overline{YZ}$, YL = 9 cm. and LZ = 16 cm.

Find :

(1) The length of $\overline{XY}$

(2) The length of $\overline{XZ}$

(3) The length of $\overline{XL}$



Solution

(1)

$\because$ ΔXYZ is right-angled at X, $\overline{XL} \perp \overline{YZ}$

$\therefore (XY)^2 = YL \times YZ$ (Euclidean theorem)

$\therefore (XY)^2 = 9 \times 25 = 225$

$\therefore$ XY = 15 cm. (First req.)

Similarly :

$(XZ)^2 = ZL \times ZY$ (Euclidean theorem)

$\therefore (XZ)^2 = 16 \times 25 = 400$ $\therefore$ XZ = 20 cm. (Second req.)

, $\because (XL)^2 = LY \times LZ$ (Corollary)

$\therefore (XL)^2 = 9 \times 16 = 144$

$\therefore$ XL = 12 cm. (Third req.)

Another solution to find the length of $\overline{XL}$

$XL = \frac{XY \times XZ}{YZ} = \frac{15 \times 20}{25} = 12$ cm.

And we can find the length of $\overline{XL}$ using any of the two right-angled triangles XLZ or XLY using Pythagoras' theorem as follows :

In the right-angled triangle XLY

$(XL)^2 = (XY)^2 - (YL)^2 = (15)^2 - (9)^2 = 225 - 81 = 144$

XL = 12 cm.

(2)

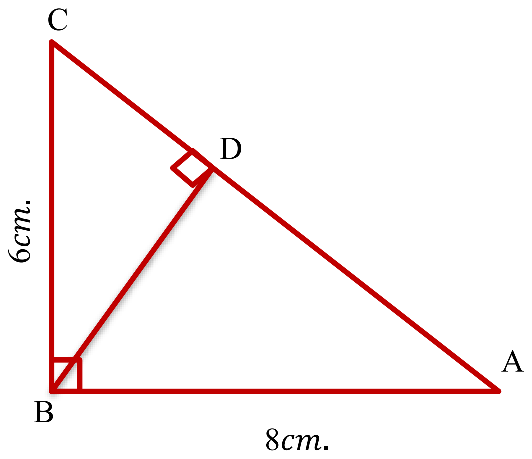
In the opposite figure : $\triangle$ ABC is right-angled at B and D $\in \overline{AC}$ such that $\overline{BD} \perp \overline{AC}$, AB = 8 cm. and CB = 6 cm.

Find :

(1) AC (2) DB

(3) The length of the projection of $\overline{BC}$ on $\overleftrightarrow{AC}$

(4)The length of the projection of $\overline{AB}$ on $\overleftrightarrow{AC}$



Solution

$\because \triangle$ ABC is right-angled at B

$\therefore (AC)^2 = (AB)^2 + (BC)^2$ (Pythagoras' theorem)

$\therefore (AC)^2 = 64 + 36 = 100$ $\therefore$ AC = 10 cm. (First req.)

$, \because \overline{BD} \perp \overline{AC}, m(\angle ABC) = 90^\circ$

$\therefore DB = \frac{AB \times BC}{AC} = \frac{8 \times 6}{10} = 4.8$ cm. (Second req.)

, $\because$ the projection of $\overline{BC}$ on $\overleftrightarrow{AC}$ is $\overline{DC}$

$, (BC)^2 = CD \times CA$ (Euclidean theorem)

$\therefore 36 = CD \times 10$

$\therefore CD = \frac{36}{10} = 3.6$ cm. (Third req.)

, $\because$ the projection of $\overline{AB}$ on $\overleftrightarrow{AC}$ is $\overline{AD}$

$, (AB)^2 = AD \times AC$ (Euclidean theorem)

$\therefore 64 = AD \times 10$

$\therefore AD = \frac{64}{10} = 6.4$ cm. (Fourth req.)

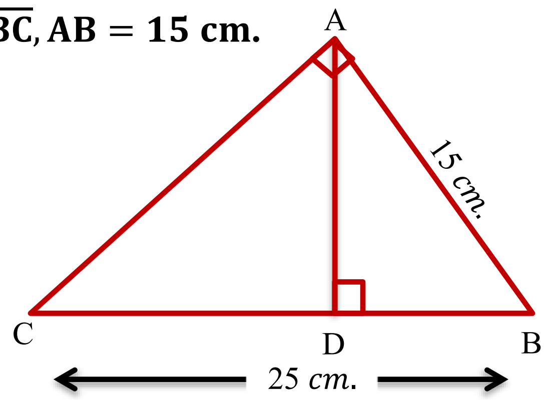
(3) In the opposite figure : ABC is a triangle in which,

$m(\angle BAC) = 90^\circ$ and $D \in \overline{BC}$ such that $\overline{AD} \perp \overline{BC}$, $AB = 15$ cm.

and $BC = 25$ cm.

Find : (1) AC (2) DB

(3) AD (4) CD



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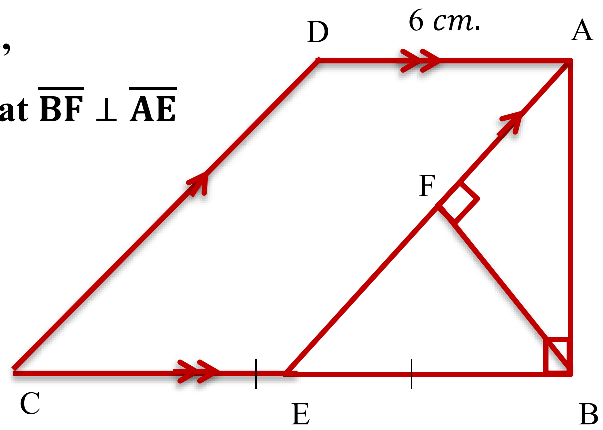
(4) In the opposite figure: ABCD is a trapezium whose area equals 72 cm^2. in which :

$\overline{AD} // \overline{BC}$, $m(\angle ABC) = 90^\circ$ and $AD = 6$ cm.,

E is the midpoint of $\overline{BC}$ and $F \in \overline{AE}$ such that $\overline{BF} \perp \overline{AE}$

and $\overline{AE} // \overline{DC}$

Find : The length of $\overline{BF}$



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Euclidean theorem (5)

Example 1

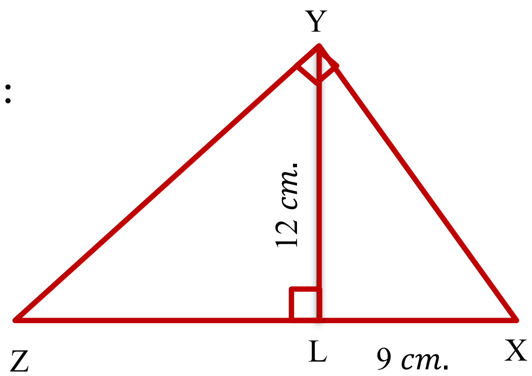
(1) In the opposite figure : XYZ is a triangle in which : $m(\angle XYZ) = 90^\circ$ and $L \in \overline{XZ}$ such that $\overline{YL} \perp \overline{XZ}$,

XL = 9 cm . and YL = 12 cm . Find :

(1) The length of $\overline{XY}$

(2) The length of $\overline{LZ}$

(3) The length of $\overline{ZY}$



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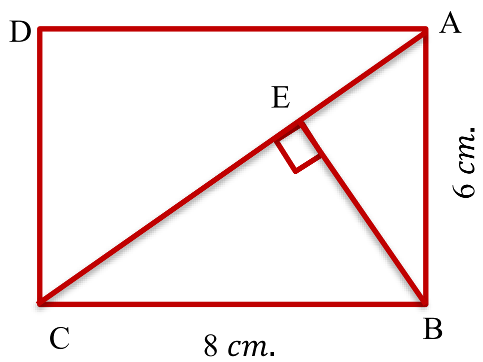
(2) In the opposite figure :ABCD is a rectangle in which :

AB = 6 cm. , BC = 8 cm. and $E \in \overline{AC}$

such that $\overline{BE} \perp \overline{AC}$ Find the length of each of :

(1) $\overline{BE}$ (2) $\overline{EC}$



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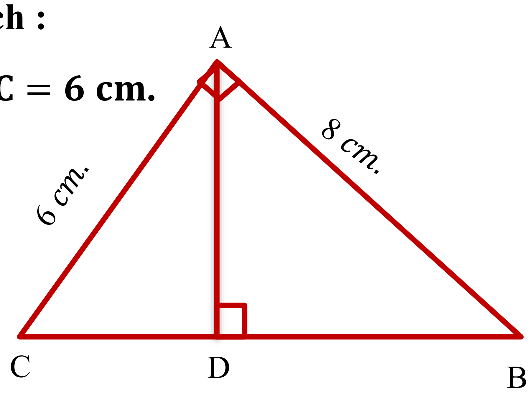
(3)

In the opposite figure : ABC is a triangle in which :

$m(\angle BAC) = 90°, \overline{AD} \perp \overline{BC}$, AB = 8 cm. and AC = 6 cm.

Find : BD, CD and AD



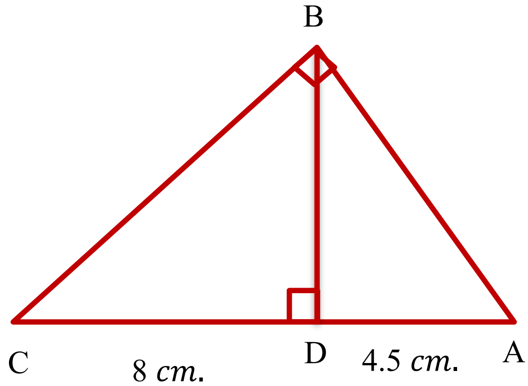
(4)

In the opposite figure :

$\triangle$ ABC is right-angled at B and $\overline{BD} \perp \overline{AC}$

If AD = 4.5 cm. and DC = 8 cm.

Find : The length of each of $\overline{AB}, \overline{BC}$ and $\overline{BD}$



(5)

In the opposite figure : ABCD is a quadrilateral where :

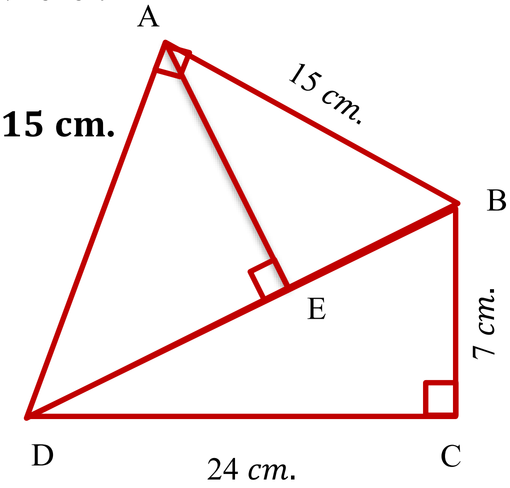
$m(\angle BCD) = m(\angle BAD) = 90°$,

$\overline{AE} \perp \overline{BD}$, BC = 7 cm., CD = 24 cm. and AB = 15 cm.

Find : (1) The length of each of $\overline{BD}$ and $\overline{AD}$

(2) The length of the projection of $\overline{AB}$ on $\overline{AD}$

(3) The length of the projection of $\overline{AD}$ on $\overleftrightarrow{AE}$



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(6)

In the opposite figure : △ AXE is right-angled at X

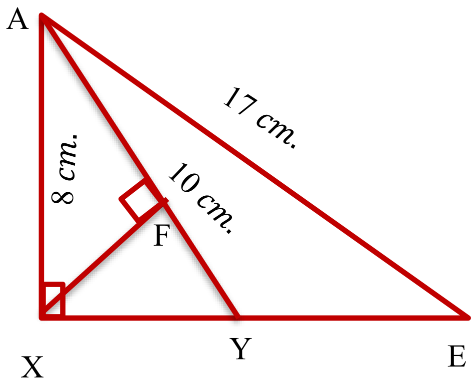
and $\overline{XF} \perp \overline{AY}$ where $Y \in \overline{XE}, F \in \overline{AY}$,

AX = 8 cm., AY = 10 cm. and AE = 17 cm.

Find : (1) The length of the projection of $\overline{AY}$ on $\overleftrightarrow{XE}$

(2) The length of $\overline{XF}$ (3) The length of $\overline{AF}$

(4) The area of △ AXE



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(7)

In the opposite figure :

XYZ is a triangle in which : $m(\angle Y) = 90^\circ$

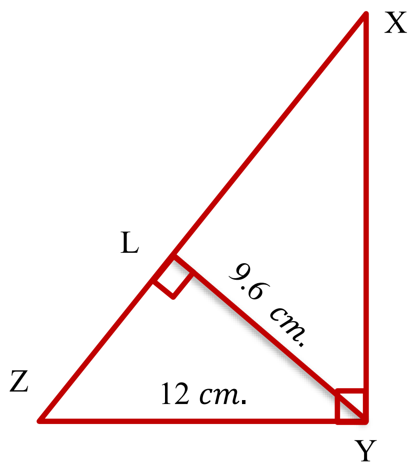
and $\overrightarrow{YL} \perp \overline{XZ}$ where $L \in \overline{XZ}$

If $YZ = 12$ cm. and $YL = 9.6$ cm.

Find : 1 The length of the projection of $\overline{YZ}$ on $\overleftrightarrow{XZ}$

2 The length of the projection of $\overline{XY}$ on $\overleftrightarrow{XZ}$

3 The length of the projection of $\overline{XZ}$ on $\overleftrightarrow{XY}$



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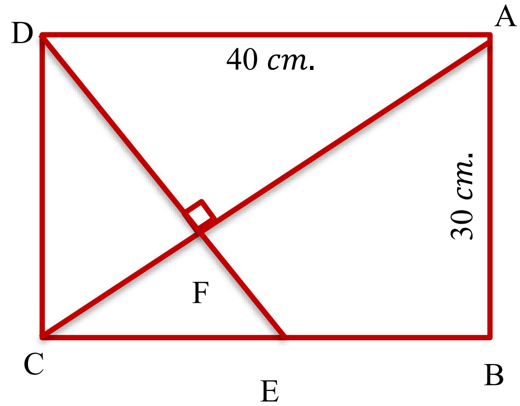
(8)

In the opposite figure : ABCD is a rectangle in which :

$AB = 30$ cm , $AD = 40$ cm.

$\overline{DE} \perp \overline{AC}$ intersects $\overline{AC}$ at F and intersects $\overline{BC}$ at E

Find : The length of each of $\overline{AF}, \overline{DF}$ and $\overline{EC}$



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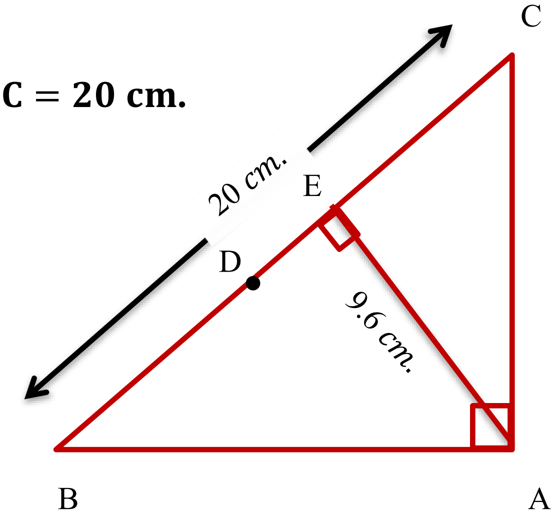
(9) In the opposite figure : $\triangle$ CAB is right-angled at A ,

E $\in \overline{BC}$ such that $\overline{AE} \perp \overline{BC}$,

D is the midpoint of $\overline{BC}$, AE = 9.6 cm. and BC = 20 cm.

Find : The length of each of $\overline{AB}$ and $\overline{AC}$



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(10) In the opposite figure : ABCD is a parallelogram ,

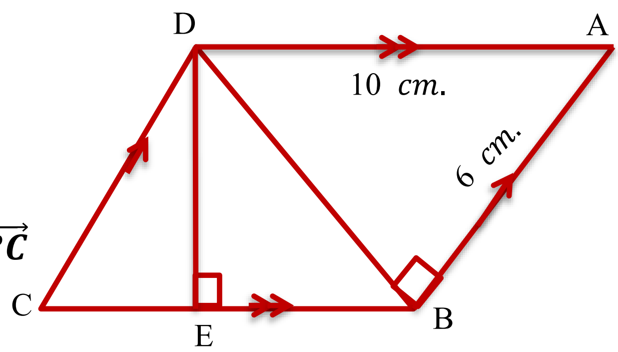
AB = 6 cm. , AD = 10 cm., $\overline{BD} \perp \overline{AB}$

and $\overline{DE} \perp \overline{BC}$, Find :

(1) The area of the parallelogram ABCD

(2) The length of the projection of $\overline{DB}$ on $\overleftrightarrow{BC}$

(3) The length of $\overline{DE}$



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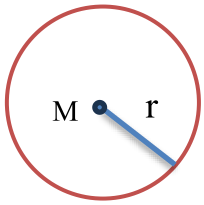
Unit 3
Lesson 4

circle

circle

- A circle is the set of all points in a plane that are at the same distance from a fixed point called the **center of the circle**.

  This fixed distance is called the **radius (r)**.



- **Center of the Circle (Center)**

  It is the fixed point inside the circle, usually denoted by **M**.

- **Radius (r)**

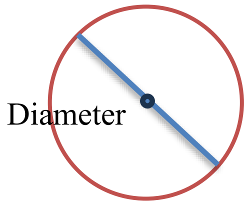
  A line segment that joins the center of the circle to any point on the circumference.

  Note: All radii of a circle are equal in length.

- **Diameter (d)**

  A line segment that passes through the center and joins two points on the circumference. It is the longest chord in the circle.

$$\textbf{Diameter } d = 2r$$



- **Circumference of the Circle**

  The curved line that forms the boundary of the circle.

$$\textbf{Circumference} = \pi d = 2\pi r$$

$\pi = \frac{22}{7}$

$.\pi \approx 3.14$

- **Area of the Circle**

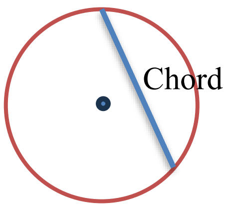
  The amount of surface enclosed by the circle.

$$\text{Area} = \pi r^2$$

- **Chord**

  A chord is a line segment that joins two points on the circumference of the circle.

  ✓ It may pass through the center or may not.



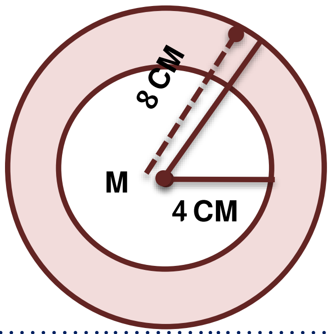
- If it passes through the center → it becomes a **diameter**.
- If it does not pass through the center → it remains a **chord**.

Therefore: **Every diameter is a chord, but not every chord is a diameter.**

EX 1 :

(1)	The radius of a circle is $r = 7$ cm. Find its circumference $\left(\pi = \frac{22}{7}\right)$. **Solution:** $C = 2\pi r$ $C = 2 \times \frac{22}{7} \times 7 = 44$ cm
(2)	The diameter of a circle is $d = 14$ cm. Find its circumference in terms of π. **Solution:** $C = \pi d$ $C = \pi d$ $C = \pi \times 14 = 14\pi$ cm
(3)	The circumference of a circle is $C = 88$ cm. Find the radius $\left(\pi = \frac{22}{7}\right)$. **Solution:** $C = 2\pi r$ $88 = 2 \times \frac{22}{7} \times r$ $88 = \frac{44}{7} r$ Multiply by 7: $616 = 44r$ $r = \frac{616}{44} = 14$ cm
(4)	The radius of a circle is $r = 5$ cm. Find the area in terms of π. **Solution:** $A = \pi r^2$ $A = \pi \times 5^2 = 25\pi$ cm^2
(5)	The area of a circle is 49π cm^2. Find the radius. **Solution:** $A = \pi r^2$ $49\pi = \pi r^2$ Divide by π: $r^2 = 49$ $r = 7$ cm

(6)	The area of a circle is 154 cm^2. Find the radius $\left(\pi = \frac{22}{7}\right)$. **Solution:** $A = \pi r^2$ $154 = \frac{22}{7} r^2$ Multiply by 7: $1078 = 22\, r^2$ $r^2 = \frac{1078}{22} = 49$ $r = 7$ cm
(7)	The circumference of a circle is 44 cm. Find the area $\left(\pi = \frac{22}{7}\right)$. **Solution:** First, find the radius: $C = 2\pi r$ $44 = 2 \times \frac{22}{7} \times r = \frac{44}{7} r$ $r = 7$ cm Then find the area: $A = \pi r^2 = \frac{22}{7} \times 7^2 = \frac{22}{7} \times 49 = 154\ cm^2$
(8)	The diameter of a circle is 10cm. Find the area $(\pi = 3.14)$. **Solution:** $r = \frac{d}{2} = \frac{10}{2} = 5$ $A = \pi r^2 = 3.14 \times 25 = 78.5\ cm^2$
(9)	A circle with radius 8 cm. Find its circumference in terms of π. …………………………………………………………………… ……………………………………………………………………
(10)	A circle with diameter 18 cm. Find its area in terms of π. …………………………………………………………………… …………………………………………………………………… ……………………………………………………………………

(11)	**The circumference of a circle is $C = 176\ CM$. Find the radius $\left(\pi = \frac{22}{7}\right)$.** ………………………………………………………… ………………………………………………………… ………………………………………………………… …………………………………………………………
(12)	**The area of a circle is $A = 81\pi$ cm^2. Find the diameter.** ………………………………………………………… ………………………………………………………… ………………………………………………………… …………………………………………………………
(13)	**The circumference of a circle is 31.4cm ($\pi = 3.14$). Find the radius, then the area.** ………………………………………………………… ………………………………………………………… ………………………………………………………… …………………………………………………………
(14)	In the figure opposite: Two concentric circles have radii of 4 cm and 8 cm. Find the area of the shaded region to the nearest tenth.  ………………………………………………………… ………………………………………………………… ………………………………………………………… ………………………………………………………… ………………………………………………………… ………………………………………………………… ………………………………………………………… ………………………………………………………… …………………………………………………………

circle (6)

Example 1:

(1)	A circle is the set of points that are the same distance from a fixed point called the …………………..
(2)	The distance between the center of the circle and any point on its circumference is called the …………………..
(3)	The diameter equals ………………… the radius.
(4)	The longest chord in a circle is the …………………..
(5)	The circumference of a circle equals …………………..
(6)	The area of a circle equals …………………..
(7)	The unit of measurement of the circumference of a circle is …………………..
(8)	The unit of measurement of the area of a circle is …………………..
(9)	If the radius of a circle is doubled, its circumference …………………..
(10)	If the radius of a circle is doubled, its area …………………..
(11)	Every diameter is a …………………., but not every chord is a …………………..
(12)	If the radius of a circle is 5 cm, then its diameter is ………………… cm.
(13)	If the circumference of a circle is equal to the perimeter of a square, then the length of the side of the square is …………………..

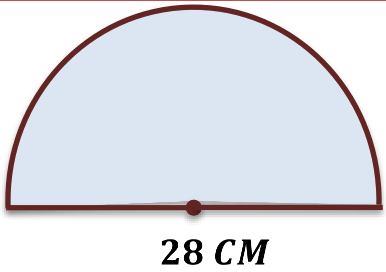
Example 2 :

(1)	A circle has a radius of 11 cm. Find its circumference. $\left(\pi = \frac{22}{7}\right)$ ………………………………………………………………………………………… …………………………………………………………………………………………
(2)	A circle has a radius of 4.5 cm. Find its area. $\left(\pi = \frac{22}{7}\right)$ ………………………………………………………………………………………… …………………………………………………………………………………………

(3)	A circle has a diameter of 26 cm. Find its circumference in terms of π. ....
(4)	The circumference of a circle is 55 cm. Find its radius. $\left(\pi = \frac{22}{7}\right)$ ....
(5)	The area of a circle is 121π cm^2. Find its radius. ....
(6)	A circle has a diameter of 9.6 cm. Find its area. $(\pi = 3.14)$ ....
(7)	A circle has a radius of 13 cm. Find its circumference and its area in terms of π ....
(8)	The circumference of a circle is 62.8 cm. Find its diameter. $(\pi = 3.14)$ ....
(9)	The area of a circle is 201.06 cm^2. Find its radius. $(\pi = 3.14)$ ....

(3) Calculate the perimeter and the area of the following shaded figures, rounding the result to the nearest tenth (given that $\pi = \frac{22}{7}$).

(1)



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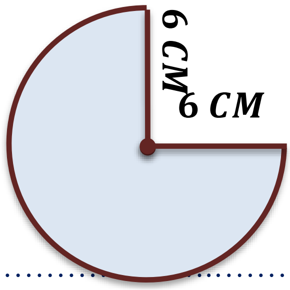

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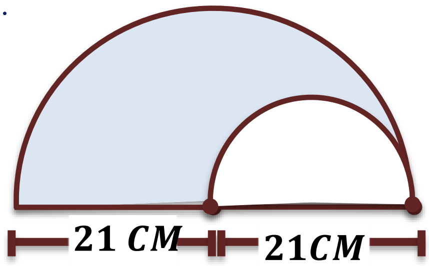

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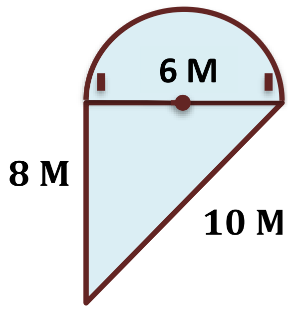


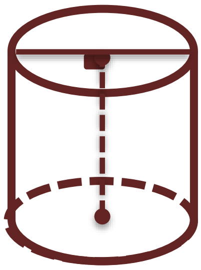
مدرسة جيل 2000 للغات

Unit 3 Lesson 5 Right circular cylinder

Right circular cylinder

It is a solid that has two congruent and parallel bases, each of which is a circular surface, and its lateral surface is a curved surface called the cylindrical surface.

- **Lateral surface area of a cylinder**
  = (circumference of the base) × (height)
  Lateral Area (L.A) $= 2\pi rh$
- **Total surface area of a cylinder**
  = lateral surface area + twice the area of the base
  Total Surface Area (S.A) $= 2\pi rh + 2\pi r^2$
- **Volume of a cylinder**
  = (area of the base) × (height)
  Volume (V) $= \pi r^2 h$



Example 3 :

(1) **A right circular cylinder has a height of 10 cm and a volume of 1540 cm³. Find the total surface area of the cylinder ($\pi = \frac{22}{7}$).**

Solution:

$\because \quad V = \pi r^2 h$

$\therefore 1540 = \frac{22}{7} \times r^2 \times 10$

$\therefore 1540 = \frac{220}{7} r^2$

$\therefore r^2 = 1540 \times \frac{7}{220} = 49$

$\therefore r = \sqrt{49} = 7$ cm

$\therefore$ A $= 2\pi r$ h $+ 2\pi r^2 = 2 \times \frac{22}{7} \times 7 \times 10 + 2 \times \frac{22}{7} \times 7^2 = 440 + 308 = 748$ cm^2

(2) A right circular cylinder has a volume of 90π cm³ and a height of 10 cm. Find the diameter of its base.

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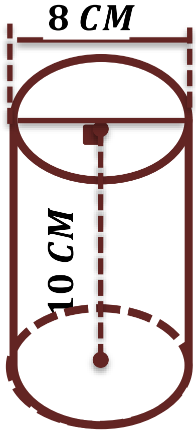
(3) In the figure opposite, find the volume, lateral surface area, and total surface area of the right circular cylinder in terms of π.



Solution:

Given: Diameter of the base = 8 cm, height $h = 10$cm

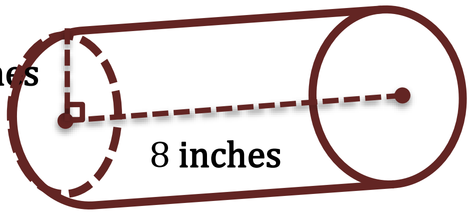
$r = \frac{8}{2} = 4$ cm , $h = 10$ cm

Volume: $V = \pi r^2 h = \pi(4)^2(10) = 160\pi$

Lateral area: $L.A = 2\pi r h = 2\pi(4)(10) = 80\pi$

Total surface area: $S.A = 2\pi r(h + r) = 2\pi(4)(10 + 4) = 112\pi$

(4) In the figure opposite: Calculate the volume, lateral area, and total surface area of the right circular cylinder, rounding the results to the nearest tenth.



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Right circular cylinder (7)

Example 1 :

(1)	A right circular cylinder has a base radius of 14 cm and a height of 20 cm. Find: the volume and total surface area of the cylinder. ……………………………………………………………………………… ……………………………………………………………………………… ……………………………………………………………………………… ……………………………………………………………………………… ………………………………………………………………………………
(2)	Find the lateral surface area of a right circular cylinder with volume 924 cm^3 and height 6 cm. ……………………………………………………………………………… ……………………………………………………………………………… ………………………………………………………………………………
(3)	Find the total surface area of a right circular cylinder with volume 7536 cm^3 and height 24 cm ($\pi = 3.14$). ……………………………………………………………………………… ……………………………………………………………………………… ………………………………………………………………………………
(4)	Which has a larger volume: • A right circular cylinder with radius 7 cm and height 10 cm, or • A cube with side length 11 cm? ……………………………………………………………………………… ……………………………………………………………………………… ……………………………………………………………………………… ……………………………………………………………………………… ………………………………………………………………………………

(5)	The lateral surface area of a right circular cylinder is 52 cm^2, and the diameter of its base is 8 cm. Find the volume of the cylinder. ……………………………………………………………………………………… ……………………………………………………………………………………… ……………………………………………………………………………………… ……………………………………………………………………………………… ……………………………………………………………………………………… ……………………………………………………………………………………… ………………………………………………………………………………………
(6)	A right circular cylinder has volume 36π cm^3 and height 4 cm, and the radius of its base equals the side length of a cube. Find the total surface area of the cube. ……………………………………………………………………………………… ……………………………………………………………………………………… ……………………………………………………………………………………… ……………………………………………………………………………………… ……………………………………………………………………………………… ………………………………………………………………………………………
(7)	Find the height of a right circular cylinder if its height equals the radius of its base, and its volume is 72π cm^3. ……………………………………………………………………………………… ……………………………………………………………………………………… ……………………………………………………………………………………… ……………………………………………………………………………………… ……………………………………………………………………………………… ……………………………………………………………………………………… ………………………………………………………………………………………

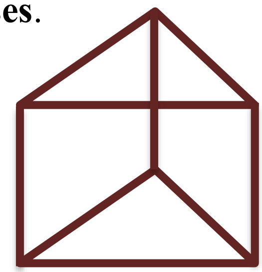
Unit 3 Lesson 5 — Right prism

learn

A **right prism** is a three-dimensional solid that has two **congruent and parallel bases**, each of which is a polygon, and its **lateral edges are perpendicular to the bases**.

All the **lateral faces** of a right prism are **rectangles**.

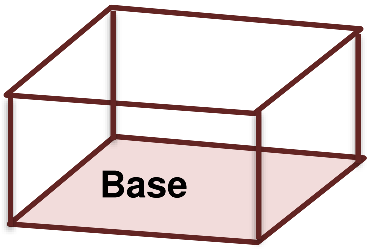
The **height** of a right prism is the length of any lateral edge.



- **Volume of a right prism (V)** = Base area × Height
- **Lateral area of a right prism (L.A.)** = Perimeter of the base × Height
- **Total surface area of a right prism (S.A.)** = Lateral area + 2 × Base area

learn

- If **all the faces of a right prism** are rectangles, as in the case of the corresponding **rectangular parallelepiped**, then **any face can serve as the base of the prism**, and when calculating the **lateral area**, the base must be specified.
- A **right circular cylinder** is considered a **special case of a right prism**, where the base is a **regular polygon with an infinite number of sides** (thus forming a circle).



Remember that:

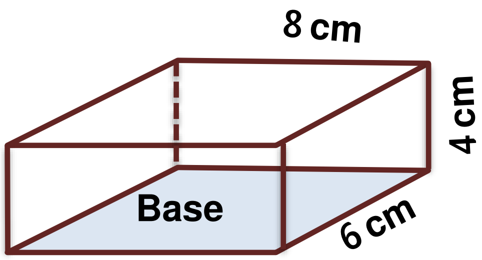
- **Perimeter of any shape: sum of all side lengths**
- **Perimeter of a regular polygon: side × number of sides**
- **Area of a triangle:** $A = \frac{1}{2} \times \text{base} \times \text{height}$
- **Area of a rectangle:** $A = \text{length} \times \text{width}$
- **Perimeter of a rectangle:** $P = 2 \times (\text{length} + \text{width})$
- **Area of a square:** $A = \text{side}^2$
- **Area of a trapezoid:** $A = \frac{\text{base}_1 + \text{base}_2}{2} \times \text{height}$

Example 1: Find the total surface area (S.A.) and volume (V) of the following solids:

(1)

Prism with rectangular base

Base length = 8 cm, width = 6 cm, height = 4 cm



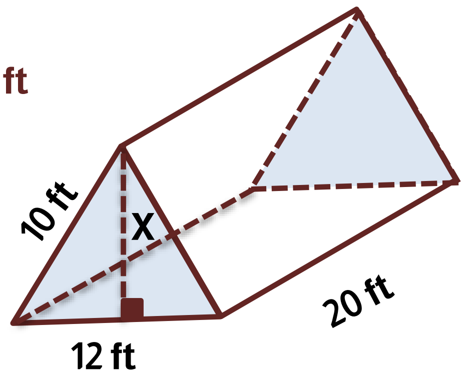
Solution:

- Perimeter of base: $P = 2 \times (8 + 6) = 28$ cm
- Lateral area: $L.A. = P \times h = 28 \times 4 = 112$ cm²
- Base area: $A_{\text{base}} = 8 \times 6 = 48$ cm²
- Total surface area: $S.A. = L.A. + 2 \times A_{\text{base}} = 112 + 2 \times 48 = 208$ cm²
- Volume: $V = A_{\text{base}} \times h = 48 \times 4 = 192$ cm³

(2)

Prism with an isosceles triangular base

Base = 12 ft, equal sides = 10 ft, height of prism = 20 ft

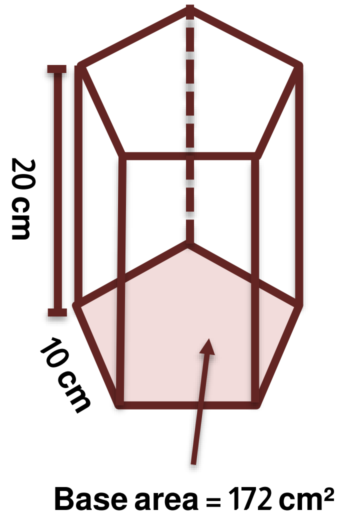
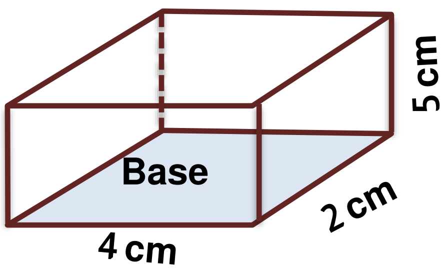
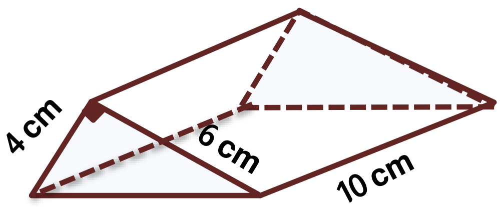


Solution:

- Drop a perpendicular from the apex to the base to find the triangle height using Pythagoras' theorem:

  $x = \sqrt{10^2 - 6^2} = \sqrt{100 - 36} = \sqrt{64} = 8$ ft

- Perimeter of base: $P = 12 + 10 + 10 = 32$ ft
- Lateral area: $L.A. = P \times h = 32 \times 20 = 640$ ft²
- Base area: $A_{\text{base}} = \frac{1}{2} \times 12 \times 8 = 48$ ft²
- Total surface area: $S.A. = L.A. + 2 \times A_{\text{base}} = 640 + 2 \times 48 = 736$ ft²
- Volume: $V = A_{\text{base}} \times h = 48 \times 20 = 960$ ft³

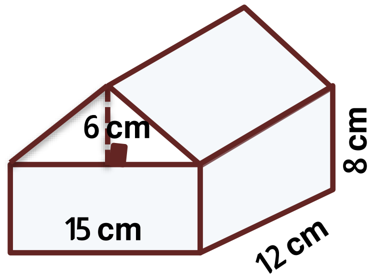
(3)	Prism with regular pentagonal base Side length = 10 cm, height = 20 cm Solution: • Perimeter of base: $P = 5 \times 10 = 50$ cm • Lateral area: $L.A. = P \times h = 50 \times 20 = 1,000$ cm² • Base area (given or calculated): $A_{\text{base}} = 172$ cm² (for the regular pentagon) • Total surface area: $S.A. = L.A. + 2 \times A_{\text{base}} = 1,000 + 2 \times 172 = 1,344$ cm² • Volume: $V = A_{\text{base}} \times h = 172 \times 20 = 3,440$ cm³ 
(4)	……………………………………………… ……………………………………………… ……………………………………………… ……………………………………………… ……………………………………………… ……………………………………………… ……………………………………………………………………… ……………………………………………………………………… ……………………………………………………………………… ……………………………………………………………………… 
(5)	……………………………………………… ……………………………………………… ……………………………………………… ……………………………………………… ……………………………………………… ……………………………………………… ……………………………………………………………………… ……………………………………………………………………… ……………………………………………………………………… 

Example 2: Find the volume (V) of the following solids:

(1)

Solution:

Prism with a pentagonal base



- **Base is composed of a rectangle and a triangle**

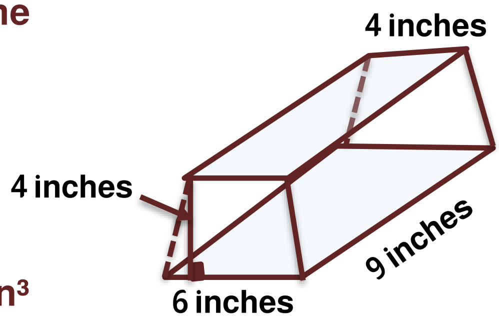
$A_{\text{base}} = A_{\text{rectangle}} + A_{\text{triangle}} = 15 \times 8 + \frac{1}{2} \times 15 \times 6 = 120 + 45 = 165$ **cm²**

$V = A_{\text{base}} \times h = 165 \times 12 = 1{,}980$ **cm³**

(2)

Prism with a trapezoidal base , Find its volume

Solution:



- **Base area:**

$A_{\text{base}} = \frac{(b_1+b_2)}{2} \times h = \frac{6+4}{2} \times 4 = 20$ **in²**

- **Volume:** $V = A_{\text{base}} \times H = 20 \times 9 = 180$ **in³**

(3)

Rectangular prism

- **Base: 9 × 5 cm, total surface area = 566 cm² , Find its volume**

Solution:

- **Base area:**

$$A_{\text{base}} = 9 \times 5 = 45 \text{ cm}^2$$

- **Perimeter of base:**

$$P = 2(9 + 5) = 28 \text{ cm}$$

- **Total surface area:**

$$S.A. = L.A. + 2 \times A_{\text{base}} = (28 \times h) + 2 \times 45 = 566$$
$$= 28h + 90 = 566$$
$$28h = 476$$
$$h = 17 \text{ cm}$$

- **Volume:**

$$V = A_{\text{base}} \times h = 45 \times 17 = 765 \text{ cm}^3$$

(4)	**Cube-based prism** • **Base: square, side = 4 cm, volume = 192 cm³ , Find Total surface area .** **Solution:** • **Base area:** $A_{base} = 4 \times 4 = 16$ cm² • **Height:** $V = A_{base} \times h \Rightarrow h = \frac{192}{16} = 12$ cm • **Perimeter of base:** $P = 4 \times 4 = 16$ cm • **Lateral area:** $L.A. = P \times h = 16 \times 12 = 192$ cm² • **Total surface area:** $S.A. = L.A. + 2 \times A_{base} = 192 + 2 \times 16 = 224$ cm²
(5)	**Prism with right triangular base** • **Hypotenuse = 5 m, one leg = 4 m, height of prism = 10 m , Find its volume** **Solution:** • **Using Pythagoras' theorem, find the other leg:** other leg $= \sqrt{5^2 - 4^2} = \sqrt{25 - 16} = 3$ m • **Base area (triangle):** $A_{base} = \frac{1}{2} \times 4 \times 3 = 6$ m² • **Volume:** $V = A_{base} \times h = 6 \times 10 = 60$ m³
(6)	**A quadrilateral prism has a rectangular base with dimensions 12 cm and 8 cm. Its total surface area is 276 cm². Find its volume.** …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… ……………………………………………………………………………

(7)	The base of a triangular prism is a right triangle. The length of its hypotenuse is 25 cm, and the length of one of its legs is 24 cm. If the height of the prism is 10 m, find its volume. …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… ……………………………………………………………………………………
(8)	A right pentagonal prism has a height of 15 cm and a volume of 330 cm^3. Find the area of its base. …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… ……………………………………………………………………………………
(9)	A right pentagonal prism has a regular pentagonal base with an area of 7 cm^2, and the area of one of its lateral rectangular faces is 16 cm^2. Find the total surface area of the prism. …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… ……………………………………………………………………………………

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(10)	A metal cube has a total edge length of 144 cm. It is melted and reshaped into 16 identical prisms, each having a square base with side length 6 cm. Find the height of each prism, assuming no metal is lost during melting and reshaping. Solution: Since the total length of the edges of the cube is 144 cm and a cube has 12 edges: Edge length of the cube $= \frac{144}{12} = 12$ cm Volume of the cube $= 12^3 = 1,728$ cm^3 The volume of the 16 identical prisms equals the volume of the cube: Volume of one prism $= \frac{1,728}{16} = 108$ cm^3 Area of the base of one prism $= 6 \times 6 = 36$ cm^2 Height of the prism $(h) = \frac{\text{Volume}}{\text{Base area}} = \frac{108}{36} = 3$ cm
(11)	A right pentagonal prism made of metal has a base area of 66 cm² and a height of 12 cm. It is melted and converted into identical right circular cylinders, each with a height of 7 cm and a base radius of 3 cm. Find the number of right circular cylinders. …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… ……………………………………………………………………………………

The Right Prism (8)

Example (1): Find the volume of each of the following solids:

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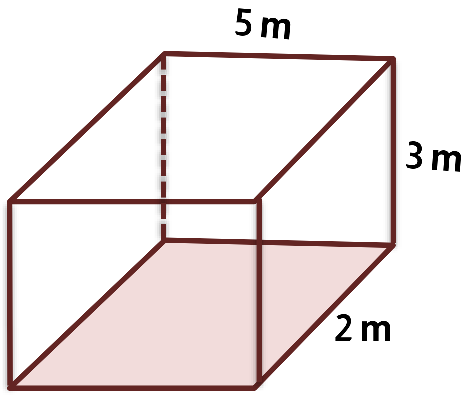
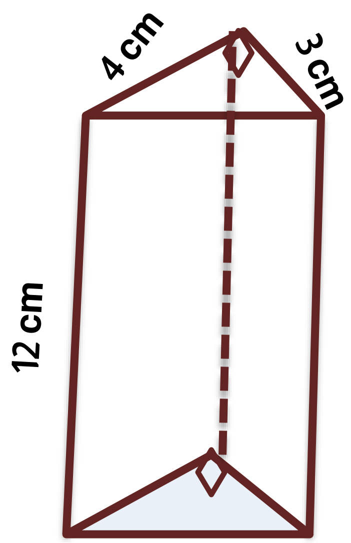
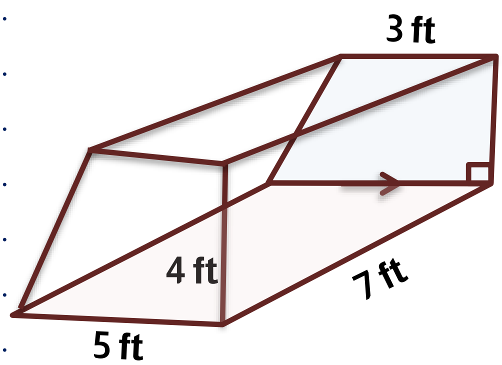


(2)



(3)



Example (2): Find the lateral area of each of the following solids:

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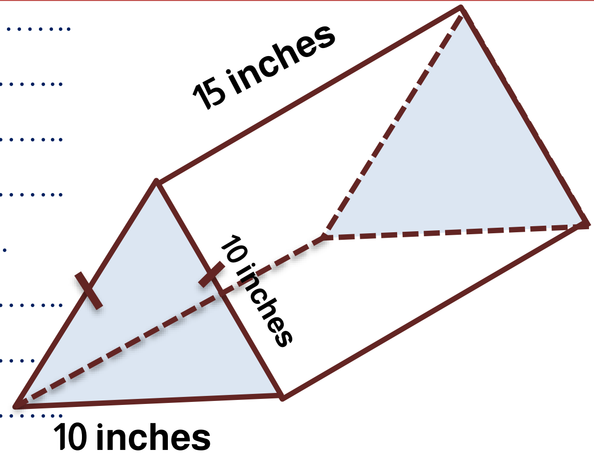

(2)

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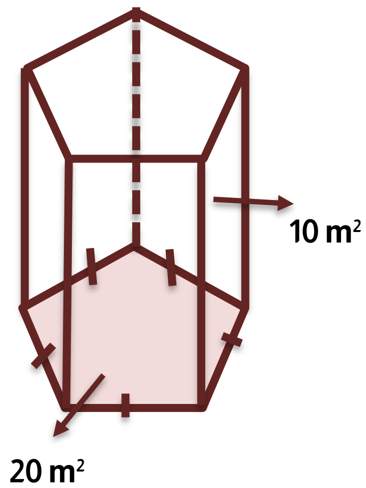

Example (3): Find the volume of the pentagonal prism for each of the following solids:

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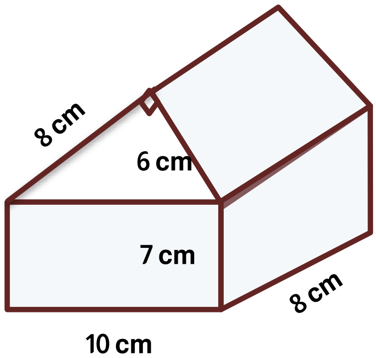

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Example (4) :	
(1)	A quadrilateral prism has a square base with side length 6 cm and a height of 4 cm. Find its volume. …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… ……………………………………………………………………………………
(2)	The base of a triangular prism is a right triangle with hypotenuse 17 m and one leg 8 m. If the prism's height is 12 m, find its volume. …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… ……………………………………………………………………………………
(3)	A rectangular prism has a base of 9 cm , 8 cm and a volume of 720 cm^3. Find its lateral area. …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… ……………………………………………………………………………………

(4)	A rectangular prism has a base of 5 cm , 4 cm and a total surface area of 130 cm^2. Find its volume. ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… …………………………………………………………………………………………
(5)	A triangular gift box has a base that is an equilateral triangle with side length 20 cm and prism height 10 cm. Find both the lateral area and volume. ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… …………………………………………………………………………………………
(6)	A metal right triangular prism has a right triangle base with legs 6 cm and 8 cm, and is cut into cubes with edge length 2 cm. Find the number of cubes. ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… …………………………………………………………………………………………

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(7)	A right pentagonal prism made of metal has a base area of 105 cm^2 and height 11 cm, and is melted into identical right circular cylinders with height 2 cm and base radius 3.5 cm. Find the number of cylinders, assuming no metal is lost. …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… ……………………………………………………………………………………
(8)	A right circular cylinder has volume 45π cm^3 and height 5 cm, and its base radius equals the edge length of a cube. Find the total surface area of the cube. …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… ……………………………………………………………………………………

Unit 3 Lesson 6 — Distance between two points

learn

- The distance between the two points M and N equals

$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Generally :

The distance between two points =

$\sqrt{\text{square of the difference between } x\text{-coordinates} + \text{square of the difference between } y\text{-coordinates}}$

- For example : If $A(3, 6)$ and $B(-1, 4)$, then the length of $\overline{AB}$ =

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-1 - 3)^2 + (4 - 6)^2} = \sqrt{(-4)^2 + (-2)^2}$

$= \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5}$ length unit.

Example 1 :

(1)	The distance between the two points $(6, 0)$ and $(0, 8)$ **solu** The required distance $= \sqrt{(0 - 6)^2 + (8 - 0)^2} = \sqrt{(-6)^2 + (8)^2} = \sqrt{36 + 64}$ $= \sqrt{100} = 10$ length unit.
(2)	The distance between the point $A(\sqrt{2}, 4)$ and the origin point equals ....
(3)	The distance between the point $(-7, -3)$ and y-axis equals ....
(4)	ABCD is a rectangle in which $A(-1, -3)$ and $C(2, 1)$, then the length of $\overline{BD}$ = ...

(5)	If the distance between the two points $(a, 5)$ and $(3a - 1, 1)$ equals 5 length units , find the value of : a **Solution** $\because \sqrt{(3a - 1 - a)^2 + (1 - 5)^2} = 5$ $\therefore \sqrt{(2a - 1)^2 + (-4)^2} = 5$ "Squaring the two sides" $\therefore (2a - 1)^2 + 16 = 25$ $\therefore (2a - 1)^2 = 9$ "Taking the square root of the two sides" $\therefore 2a - 1 = \pm 3$ $\therefore 2a - 1 = 3$ $2a - 1 = -3$ $\therefore 2a = 4$ $2a = -2$ $\therefore a = 2$ $\therefore a = -1$
(6)	If $A(2, 5)$ and $B(-1, 1)$, find the length of : $\overline{AB}$ …………………………………………………………………………………… ……………………………………………………………………………………
(7)	If ABC is a triangle where $A(0, 0), B(3, 4)$ and $C(-4, 3)$, find the perimeter of $\triangle$ ABC **Solution** $\because$ The perimeter of $\triangle ABC = AB + BC + CA$ $AB = \sqrt{3^2 + 4^2}$ $= \sqrt{9 + 16} = \sqrt{25} = 5$ length unit. $BC = \sqrt{(-4 - 3)^2 + (3 - 4)^2}$ $= \sqrt{(-7)^2 + (-1)^2}$ $= \sqrt{49 + 1} = \sqrt{50} = 5\sqrt{2}$ length unit. $CA = \sqrt{(-4)^2 + (3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$ length unit. $\therefore$ The perimeter of $\triangle ABC = 5 + 5\sqrt{2} + 5 = (10 + 5\sqrt{2})$ length unit.

(8)	Prove that : $\triangle$ ABC is an equilateral triangle where : A(6, 0), B(2, 0) and $C(4, 2\sqrt{3})$, then find its area.

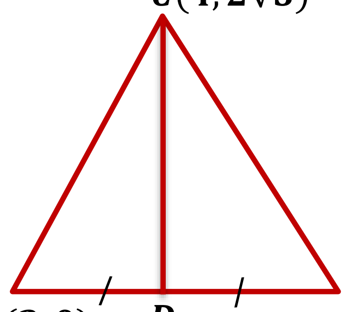
Solution

$AB = \sqrt{(6-2)^2 + (0-0)^2} = \sqrt{16+0} = \sqrt{16} = 4$ length unit.

$BC = \sqrt{(2-4)^2 + (0-2\sqrt{3})^2} = \sqrt{4+12} = \sqrt{16} = 4$ length unit.

$AC = \sqrt{(6-4)^2 + (0-2\sqrt{3})^2} = \sqrt{4+12} = \sqrt{16} = 4$ length unit.

$\therefore AB = BC = AC$



$\therefore \triangle$ ABC is equilateral

Let M be the midpoint of the base $\overline{AB}$

$\therefore \overline{CM} \perp \overline{AB}$

AM = 2 length unit

$\therefore$ By using Pythagoras' theorem , we find that :

$\therefore$ The height $MC = \sqrt{(AC)^2 - (AM)^2} = \sqrt{4^2 - 2^2} = \sqrt{12} = 2\sqrt{3}$ length unit

$\therefore$ The area of $\triangle ABC = \frac{1}{2} \times AB \times MC = \frac{1}{2} \times 4 \times 2\sqrt{3} = 4\sqrt{3}$ square unit.

(9)	Prove that : $\triangle$ ABC is an isosceles triangle where : A(3, 3), B(5, 9) and $C(-1, 7)$

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Remark

- To prove that three given points are collinear (i.e. They lie on one straight line) we can find the distance between each two of these points, then prove that the greatest distance equals the sum of the two other distances.

Example 2 :

(1)	Prove that: The points $A(-2,7), B(-3,4)$ and $C(1,16)$ are collinear. **Solution** $\because AB = \sqrt{(-2+3)^2+(7-4)^2} = \sqrt{1+9} = \sqrt{10}$ length unit. $BC = \sqrt{(-3-1)^2+(4-16)^2} = \sqrt{16+144} = \sqrt{160} = 4\sqrt{10}$ length unit. $AC = \sqrt{(-2-1)^2+(7-16)^2} = \sqrt{9+81} = \sqrt{90} = 3\sqrt{10}$ length unit. $AC = AB + AC$ $\therefore$ A, B and C are collinear.
(2)	The distance between the point $A(\sqrt{2},4)$ and the origin point equals ....

Remark

- To prove that the points A, B and C are the vertices of a triangle, we can find AB, BC and *AC*, then prove that the sum of the smaller two lengths is greater than the third length.
- To determine the type of the triangle ABC according to its angle measures (where $\overline{AC}$ is the longest side of the triangle ABC)
  , we compare between $(AC)^2$ and $(AB)^2+(BC)^2$ as the following :
  (1) If $(AC)^2 > (AB)^2+(BC)^2$, then the triangle is obtuse-angled at B
  (2) If $(AC)^2 = (AB)^2+(BC)^2$, then the triangle is right-angled at B
  (3) If $(AC)^2 < (AB)^2+(BC)^2$, then the triangle is acute-angled.

Example 3 :

(1)	**Prove that: The triangle whose vertices are $A(3,2), B(-4,1)$ and $C(2,-1)$ is right-angled , then find its area.** **Solution** $\because AB = \sqrt{(3+4)^2 + (2-1)^2} = \sqrt{49+1} = \sqrt{50}$ length unit. $BC = \sqrt{(-4-2)^2 + (1+1)^2} = \sqrt{36+4} = \sqrt{40}$ length unit. $AC = \sqrt{(3-2)^2 + (2+1)^2} = \sqrt{1+9} = \sqrt{10}$ length unit. $\because (AC)^2 + (BC)^2 = 10 + 40 = 50$ $(AB)^2 = 50$ $\therefore (AC)^2 + (BC)^2 = (AB)^2$ $\therefore \triangle$ ABC is right-angled at C $\therefore$ The area of the triangle ABC $= \frac{1}{2} AC \times BC = \frac{1}{2} \times \sqrt{10} \times \sqrt{40} = 10$ square unit
(2)	**If $A(-1,-1), B(2,3)$ and $C(6,0)$** **, prove that: $\triangle$ ABC is right-angled at B , then find its area.** ....

Remark

If ABCD is a quadrilateral :

(1) To prove that ABCD is a parallelogram, we prove that : $AB = CD$, $BC = AD$

(2) To prove that ABCD is a rhombus, we prove that : $AB = BC = CD = DA$

(3) To prove that ABCD is a rectangle, we prove that : $AB = CD, BC = AD, AC = BD$

(4) To prove that ABCD is a square, we prove that : $AB = BC = CD = DA, AC = BD$

Example 4 :

(1)

If $A(3,-2), B(-5,0), C(0,-7)$ and $D(8,-9)$,

prove that : ABCD is a parallelogram.

Solution

$\because AB = \sqrt{(3+5)^2+(-2-0)^2} = \sqrt{64+4} = \sqrt{68}$ length unit.

$BC = \sqrt{(-5-0)^2+(0+7)^2} = \sqrt{25+49} = \sqrt{74}$ length unit.

$CD = \sqrt{(0-8)^2+(-7+9)^2} = \sqrt{64+4} = \sqrt{68}$ length unit.

$DA = \sqrt{(8-3)^2+(-9+2)^2} = \sqrt{25+49} = \sqrt{74}$ length unit.

$\therefore AB = CD$, $BC = DA$ $\therefore$ ABCD is a parallelogram.

(2)

Prove that: The points $A(-1,4), B(1,1), C(-1,-2)$ and $D(-3,1)$ are the vertices of a rhombus , then find its area.

Solution

$\because AB = \sqrt{(-1-1)^2+(4-1)^2} = \sqrt{4+9} = \sqrt{13}$ length unit

$BC = \sqrt{(1+1)^2+(1+2)^2} = \sqrt{4+9} = \sqrt{13}$ length unit.

$CD = \sqrt{(-1+3)^2+(-2-1)^2} = \sqrt{4+9} = \sqrt{13}$ length unit.

$DA = \sqrt{(-3+1)^2+(1-4)^2} = \sqrt{4+9} = \sqrt{13}$ length unit.

$\therefore AB = BC = CD = DA$ $\therefore$ The quadrilateral ABCD is a rhombus.

$\because AC = \sqrt{(-1+1)^2+(4+2)^2} = \sqrt{0+36} = \sqrt{36} = 6$ length unit.

$BD = \sqrt{(1+3)^2+(1-1)^2} = \sqrt{16+0} = \sqrt{16} = 4$ length unit.

$\therefore$ The area of the rhombus ABCD $= \frac{1}{2} \times 6 \times 4 = 12$ square unit.

(3)	Prove that: The points $A(-1,3), B(5,1), C(6,4)$ and $D(0,6)$ are the vertices of a rectangle, then find its area. ………………………………………………………………………………… ………………………………………………………………………………… ………………………………………………………………………………… ………………………………………………………………………………… ………………………………………………………………………………… ………………………………………………………………………………… ………………………………………………………………………………… ………………………………………………………………………………… ………………………………………………………………………………… ………………………………………………………………………………… …………………………………………………………………………………

Remark

- The axis of symmetry of a line segment is the straight line that is perpendicular to it at its midpoint.
- Any point on the axis of symmetry of a line segment is at equal distances from its terminals.

The converse is true, i.e. If a point is at equal distances from the two terminals of a line segment, then this point lies on the axis of this line segment.

Example 5 :

(1)	If $A(1,-1)$ and $B(1,3)$, prove that: The point $C(-1,1)$ lies on the axis of symmetry of $\overline{AB}$ **Solution** $\because CA = \sqrt{(-1-1)^2 + (1+1)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$ length unit. $CB = \sqrt{(-1-1)^2 + (1-3)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$ length unit. $\therefore CA = CB$ $\therefore$ The point C lies on the axis of symmetry of $\overline{AB}$

.

Remark

- If $A \in$ the circle M , then the radius length of this circle $(r) = MA$
- To prove that : Three points as A, B and C lie on the same circle of centre M we prove that: $MA = MB = MC$
- Remember that :
- The circumference of the circle $= 2\pi r$
- The area of the circle $= \pi r^2$

Example 6 :

(1)	The diameter length of the circle of centre $A(-2, 3)$ and passing through $B(2, -1)$ equals . **Solution** $r =$ the length of $\overline{AB} = \sqrt{(2+2)^2 + (-1-3)^2}$ $= \sqrt{(4)^2 + (-4)^2} = \sqrt{32} = 4\sqrt{2}$ length unit. $\therefore$ The diameter length $= 2r = 2 \times 4\sqrt{2} = 8\sqrt{2}$ length unit.
(2)	Prove that : The points $A(-6, 2), B(0, 8)$ and $C(-8, 4)$ lie on the circle whose centre is $M(-4, 6)$ and find its area where $\pi \approx 3.14$ **Solution** $\because MA = \sqrt{(-6+4)^2 + (2-6)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$ length units. $MB = \sqrt{(0+4)^2 + (8-6)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$ length units. $MC = \sqrt{(-8+4)^2 + (4-6)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$ length units. $\therefore MA = MB = MC$ $\therefore$ The points A, B and C lie on the circle M whose radius length $r = 2\sqrt{5}$ length units. $\therefore$ The area of the circle $M = \pi r^2 \approx 3.14 \times (2\sqrt{5})^2 \approx 62.8$ square units.

(3)	**Prove that: The points $A(-2, 0)$, $B(5, 1)$ and $C(6, -6)$ lie on the circle whose centre is $M(2, -3)$ and find the circumference of the circle in terms of π .** ……………………………………………………………………………… ……………………………………………………………………………… ……………………………………………………………………………… ……………………………………………………………………………… ……………………………………………………………………………… ……………………………………………………………………………… ……………………………………………………………………………… ……………………………………………………………………………… ……………………………………………………………………………… ……………………………………………………………………………… ………………………………………………………………………………

Distance between two points (9)

EX (1)

(1)	If $A(3,1), B(1,2)$ and $C(5,4)$, prove that: $BC = 2AB$ ………………………………………………………………………… ………………………………………………………………………… ………………………………………………………………………… ………………………………………………………………………… …………………………………………………………………………
(2)	Prove that: The points $A(4,3), B(1,1)$ and $C(-5,-3)$ are collinear. ………………………………………………………………………… ………………………………………………………………………… ………………………………………………………………………… …………………………………………………………………………
(3)	If $A(-2,2)$ and $B(1,-1)$, then prove that the point $C(3,4)$ lies on the axis of symmetry of $\overline{AB}$ ………………………………………………………………………… ………………………………………………………………………… ………………………………………………………………………… ………………………………………………………………………… …………………………………………………………………………
(4)	Show which of the following sets of points are collinear : (1) $A(1,4), B(3,-2)$ and $C(-3,16)$ ………………………………………………………………………… ………………………………………………………………………… ………………………………………………………………………… ………………………………………………………………………… .

(5)	Show the type of $\triangle$ ABC such that $A(-2,4), B(3,-1)$ and $C(4,5)$ according to its side lengths. …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… ……………………………………………………………………………
(6)	Show the type of each of the following triangles according to its angles if its vertices are : (1) $A(2,1), B(4,-2)$ and $C(7,5)$ …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… (2) $A(3,5), B(-1,1)$ and $C(5,-5)$ …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… ……………………………………………………………………………
(7)	Prove that the triangle whose vertices are : $A(5,-5), B(-1,7)$ and $C(15,15)$ is right-angled at B , then find its area . …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… ……………………………………………………………………………

(8)	If the points $A(5,0), B(7,2\sqrt{3})$ and $C(3,2\sqrt{3})$ are three points in a Cartesian coordinates plane, prove that: $\triangle$ ABC is equilateral and find its area. …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… ……………………………………………………………………………
(9)	In each of the following, prove that the points A, B, C and D are vertices of a parallelogram where : (1) $A(-1,1), B(0,5), C(5,6)$ and $D(4,2)$ …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… (2) $A(-2,4), B(5,-3), C(7,1)$ and $D(0,8)$ …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… …………………………………………………………………………… ……………………………………………………………………………

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(10)	**Prove that: The points $A(0,1), B(4,5), C(1,8)$ and $D(-3,4)$ are vertices of a rectangle and find its diagonal length.** ....
(11)	**Prove that : The points $A(3,3), B(0,3), C(0,0)$ and $D(3,0)$ in the Cartesian coordinates plane are vertices of a square and calculate the length of its diagonal and its area.** ....
(12)	**ABCD is a quadrilateral where $A(5,3), B(6,-2), C(1,-1)$ and $D(0,4)$ Prove that: ABCD is a rhombus, then find its area.** ....

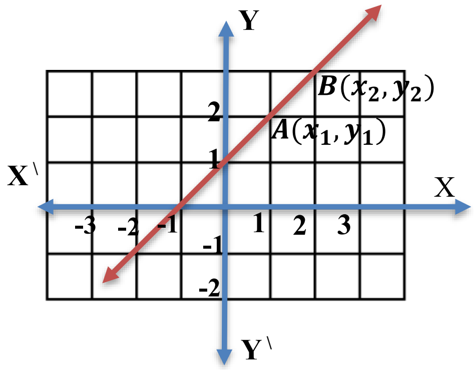
(13)	**Prove that: The points $A(-2, 5), B(3, 3)$ and $C(-4, 2)$ are non-collinear and if $D(-9, 4)$ Prove that : The figure ABCD is a parallelogram.** …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… ……………………………………………………………………………………
(14)	**Prove that: The points $A(3, -1), B(-4, 6)$ and $C(2, -2)$ are located on a circle whose centre is $M(-1, 2)$, then find the circumference of the circle where $\pi = 3.14$** …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… ……………………………………………………………………………………
(15)	**If the distance between the point $(x, 5)$ and the point $(6, 1)$ equals $2\sqrt{5}$ length units. , find : the value of x** …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… ……………………………………………………………………………………

Unit 3
Lesson 6

Slope of straight line

The cube

If a point moves on a straight line L from the location $A(x_1, y_1)$ to the location B (x_2, y_2), then :



- The change in the x-coordinates $= x_2 - x_1$
  It is called (the horizontal change).
- The change in the y -coordinates $= y_2 - y_1$
  It is called (the vertical change).
- The ratio of the change in the y-coordinates to the change in the X-coordinates is called the slope of the straight line (S).

- Definition

The slope of the straight line $= \frac{\text{the change in } y\text{-coordinates}}{\text{the change in } x\text{-coordinates}} = \frac{\text{the vertical change}}{\text{the horizontal change}}$

i.e. $\cdot$ S $= \frac{y_2 - y_1}{x_2 - x_1}$, where $x_1 \neq x_2$

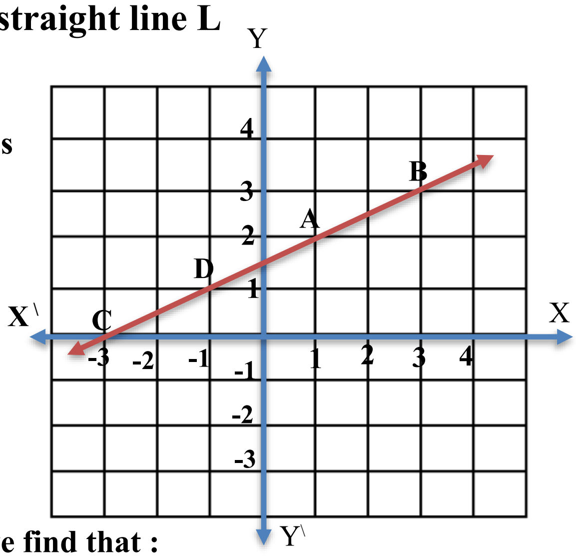
- S is undefined if $x_1 = x_2$

Example 1 :

(1)

In the opposite figure: Find the slope of the straight line L

Solution



We determine two points on the straight line such as

$\mathbf{A = (1,2)}$ **and** $\mathbf{B = (3,3)}$

$\because S = \frac{y_2 - y_1}{x_2 - x_1}$

$\therefore S = \frac{3-2}{3-1} = \frac{1}{2}$

find its slope as the points $C(-3,0)$ **and** $D(-1,1)$ **we find that :**

$S = \frac{y_2-y_1}{x_2-x_1} = \frac{1-0}{-1-(-3)} = \frac{1}{2}$ **(the same result)**

i.e. The slope of the straight line is constant for any two selected points on it.

(2)

Find the slope of the straight line passing through each pair of points in the following:

(1) $(2,4),(4,5)$, $S = \frac{y_2-y_1}{x_2-x_1} = \frac{5-4}{4-2} = \frac{1}{2}$

(2) $(1,3),(4,2)$, $S = \frac{y_2-y_1}{x_2-x_1} = \frac{2-3}{4-1} = \frac{-1}{3}$

(3) $(-2,-3),(-4,1)$, $S = \frac{y_2-y_1}{x_2-x_1} = \frac{1-(-3)}{-4-(-2)} = \frac{4}{-2} = -2$

(3)

Find the slope of the straight line passing through each pair of points in the following :

(1) $(2,1),(3,4)$ ……………………………………………………………………………

……………………………………………………………………………………………

(2) $(3,-5),(-4,2)$ ……………………………………………………………………

……………………………………………………………………………………………

(3) $(-3,-1),(1,0)$ ……………………………………………………………………

……………………………………………………………………………………………

(4) $(-6,3),(-4,2)$ ……………………………………………………………………

……………………………………………………………………………………………

learn

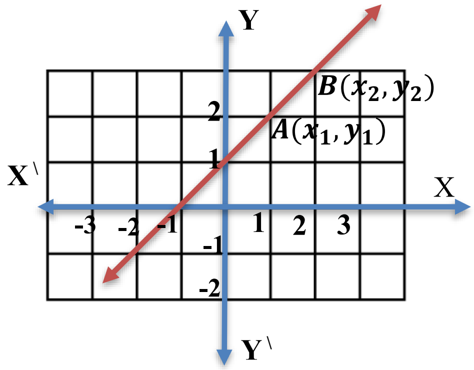
Remarks

- If a point moves on a straight line from the location A(x_1, y_1) to the location B(x_2, y_2), where $x_2 > x_1$, then

(1) If $y_2 > y_1$

i.e. y increases as X increases, then the slope of the straight line is a positive number.

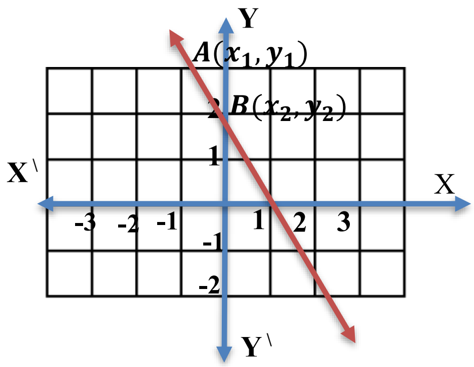
i.e. S > 0



(2) If $y_2 < y_1$

i.e. y decreases as X increases, then the slope of the straight line is a negative number.

i.e. $S < 0$

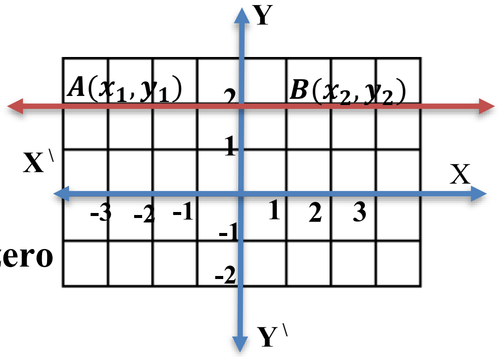


(3) If $y_2 = y_1$

i.e. y is constant as x changes, then the slope of the straight line = zero

i.e. $S = 0$

i.e. The slope of the straight line parallel to χ-axis = zero

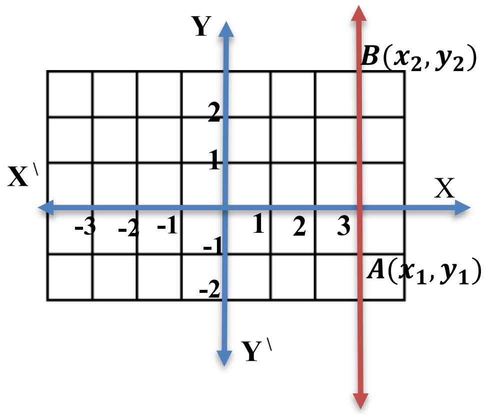


(4) If $X_2 = X_1$

, then the slope of the straight line is undefined because there is no change in the x-axis.

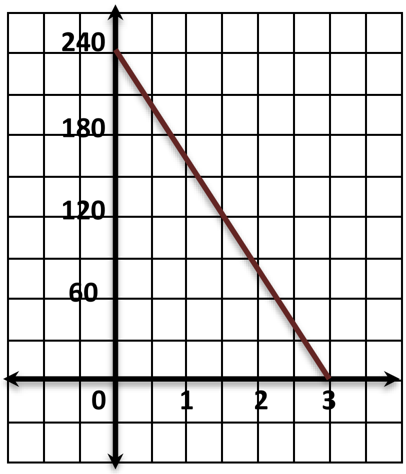
i.e. $x_2 - x_1 = 0$

i.e. The slope of the straight line parallel to y-axis is undefined.



Example 1 :

(1)	If the slope of the straight line passing through the two points $(-3,4)$ and $(1,y)$ is 2 , find the value of y **Solution** $\because S=\frac{y_2-y_1}{x_2-x_1}$ $\therefore 2=\frac{y-4}{1-(-3)}$ $\therefore 2=\frac{y-4}{4}$ $\therefore y-4=2\times 4$ $\therefore y-4=8$ $\therefore y=12$
(2)	Prove that the points $A(2,3), B(4,2)$ and $C(8,0)$ are collinear. **Solution** $\because S=\frac{y_2-y_1}{x_2-x_1}$ $\therefore$ The slope of $\overleftrightarrow{AB}=\frac{2-3}{4-2}=-\frac{1}{2}$, the slope of $\overleftrightarrow{BC}=\frac{0-2}{8-4}=\frac{-2}{4}=-\frac{1}{2}$, $\because$ the slope of $\overleftrightarrow{AB}=$ the slope of $\overleftrightarrow{BC}$ and the point B is common. $\therefore$ The points A, B and C are collinear.
(3)	If the points A, B and C are collinear where $A(3,2), B(5,-1)$ and $C(1,k)$, find the value of k **Solution** $\because S=\frac{y_2-y_1}{x_2-x_1}$ $\therefore$ The slope of $\overleftrightarrow{AB}=\frac{-1-2}{5-3}=\frac{-3}{2}$ the slope of $\overleftrightarrow{BC}=\frac{k-(-1)}{1-5}=\frac{k+1}{-4}$ $\because$ A, B and C are collinear, the slope of the straight line is constant for any two points on it. $\therefore$ The slope of $\overleftrightarrow{AB}=$ the slope of $\overleftrightarrow{BC}$ $\therefore \frac{-3}{2}=\frac{k+1}{-4}$ $\therefore 2(k+1)=-3\times(-4)$ $\therefore 2k+2=12$ $\therefore 2k=10$

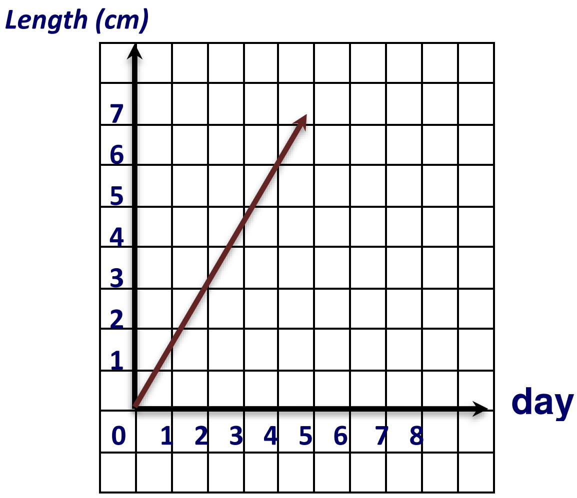
(4)	If the slope of the straight line passing through the two points $(3,-1),(7,a)$ is $\frac{3}{4}$, find the value of a ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… …………………………………………………………………………………………
(5)	Prove that: $C(-1,2) \in \overleftrightarrow{AB}$, where $A(1,3)$ and $B(3,4)$ ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… ………………………………………………………………………………………… …………………………………………………………………………………………
(6)	The figure opposite represents the relationship between distance (d) in kilometers and time (t) in hours for the return trip of a car from Damietta to Cairo. Find the constant speed of the car.  **Solution:** Consider the points: $(0,240)=(t_1,d_1)$ and $(3,0)=(t_2,d_2)$ The slope of the straight line represents the constant speed (V) of the car: $V=\frac{d_2-d_1}{t_2-t_1}=\frac{0-240}{3-0}=\frac{-240}{3}=-80$ Constant speed of the car $=80$ km/h Note: The negative sign indicates that the journey is in the return direction. A constant speed means covering equal distances in equal intervals of time, which equals the change in distance d divided by the change in time t.

(7) The figure opposite shows the relationship between number of days (x-axis) and plant height in centimeters (y-axis).

Find the growth rate of this plant.

Assuming the growth rate continues at the same pace, what will its height be after 10 days?

Solution:



Growth rate $= \frac{\text{change in height}}{\text{change in days}}$

$= \text{slope of the line } l$

Using points on the line: $A(2,3)$, $B(4,6)$

$m = \frac{6-3}{4-2} = \frac{3}{2}$

Growth rate $= \frac{3}{2}$ cm/day

Height after 10 days: $10 \times \frac{3}{2} = 15$ cm

(8) A road rises at a rate of 2 meters for every 25 meters horizontally. What is the elevation at the end of a 300 m long road?

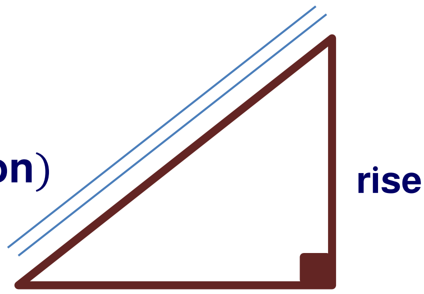
Solution:

slope of road $= \frac{\text{rise}}{\text{horizontal distance}}$

$= \frac{2}{25} = \frac{x}{300}$ (let x = end elevation)

$x = \frac{2 \times 300}{25} = 24$

Elevation at the end of the road $= 24$ m



Note: The slope of a straight line is used as a measure of the steepness of roads:

slope of road $= \frac{\text{rise}}{\text{horizontal distance}}$

Slope of straight line (10)

Example 1 : Find the slope of the straight line passing through the two points in each of the following :

(1)	$A(1,3), B(3,4)$ …………………………………………………………………………
(2)	$A(3,-1), B(3,2)$ …………………………………………………………………………
(3)	$E(-3,-1), O(0,0)$ …………………………………………………………………………
(4)	$N(4,-2), K(-1,-7)$ …………………………………………………………………………
(5)	$A(-6,-9), B(-1,-1)$ …………………………………………………………………………

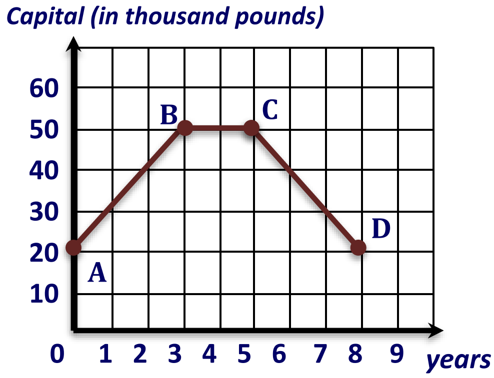
Example 2 :

(1)	If $A(2,-1), B(10,3)$ and $C(2,3)$, find the slope of each of $\overleftrightarrow{AB}, \overleftrightarrow{BC}$ and $\overleftrightarrow{CA}$, then mention the type of the triangle according to the measures of its angles. ………………………………………………………………………… ………………………………………………………………………… ………………………………………………………………………… ………………………………………………………………………… ………………………………………………………………………… …………………………………………………………………………
(2)	If the slope of the straight line which passes through the two points $(1,3)$ and $(3,k)$ equals 3 , find the value of k ………………………………………………………………………… ………………………………………………………………………… ………………………………………………………………………… …………………………………………………………………………

(3)	If $A(-1,4), B(x,2)$ and the slope of $\overleftrightarrow{AB}$ equals -2 , find the value of x ………………………………………………………………………… …………………………………………………………………………
(4)	If the straight line which passes through the two points $(-2,y)$ and $(3,-1)$ has a slope -0.6 , find the value of y ………………………………………………………………………… ………………………………………………………………………… …………………………………………………………………………
(5)	Find the value of k such that the straight line passing through the two points $(3,4)$ and $(2,\mathrm{k})$ is parallel to $\mathcal{X}$-axis. ………………………………………………………………………… ………………………………………………………………………… ………………………………………………………………………… …………………………………………………………………………
(6)	Find the value of χ such that the straight line which passes through the two points $(2x,3)$ and $(6,7)$ is parallel to y-axis. ………………………………………………………………………… ………………………………………………………………………… …………………………………………………………………………
(7)	Find the value of y such that the straight line passing through the two points $(3,6)$ and $(-2,3\mathrm{y})$ is perpendicular to y -axis. ………………………………………………………………………… ………………………………………………………………………… …………………………………………………………………………
Example 4 : In each of the following, prove that the points A,B and C are collinear :	
(1)	$A(1,1), B(2,2), C(-3,-3)$ ………………………………………………………………………… …………………………………………………………………………
Example 6 :	
(1)	Find the value of y such that the points $(4,1), (-2,7)$ and $(3,y)$ are collinear. ………………………………………………………………………… …………………………………………………………………………

(2) The figure opposite shows the change in a company's capital over 8 years.

- Find the company's capital at the start of its business.
- Find the slope of each line segment $\overleftrightarrow{CD}$, $\overleftrightarrow{BC}$, $\overleftrightarrow{AB}$.



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(3) Which of the two slopes is steeper?

Omar climbs a rocky slope that rises 12 meters over 6 meters horizontally, and Khaled climbs another slope that rises 25 meters over 12 meters horizontally. Which slope is steeper?

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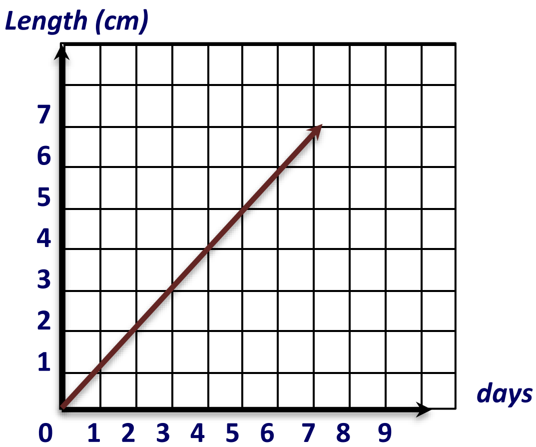
(4) The figure opposite shows the relationship between number of days (x-axis) and plant height in centimeters (y-axis).

- Find the growth rate of this plant.



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Unit 4
Lesson 1

Samples

Learn

First: Statistics

Statistics is the science of collecting, organizing, and analyzing data to use it in making correct decisions.

1 Data Collection

Sources of data collection:

- Primary sources: Data collected directly by ourselves, such as:
  - Personal interviews
  - Questionnaires
  - Observation
- Secondary sources: Data that already exists and was collected previously, such as:
  - Publications of the Central Agency for Public Mobilization and Statistics
  - The Internet
  - School or institution records

2 Methods of Data Collection

- Census (Complete Enumeration)
  - Data is collected from all members of the statistical population.
  - Used when the population size is small.

Advantages:

- Very accurate results

Disadvantages:

- Time-consuming
- Expensive and requires significant effort

Example: Counting the number of students in a class to determine how many passed.

- Sampling Method
  - Data is collected from a part of the population.
  - Used when the population is large.

Advantages:

- Saves time, effort, and cost

Disadvantages:

- Results are approximate and not completely accurate

Example: Selecting a group of students to know their opinion about a school trip.

learn

3 Types of Samples

◆ **Random Sample**

Every individual has an equal chance of being selected.

Example: Randomly selecting student names from a box.

◆ **Non-Random Sample (Biased Selection)**

Selection is done in an unfair or biased way.

Example: Selecting only first-grade students to ask about the school trip.

4 Simple Random Sample

Used in homogeneous populations.

Methods of selection:

- Writing individuals' names on cards and drawing them randomly
- Using random numbers (calculator or software)

Example: Selecting 40 students from a school with 800 students.

Important note: The suitable sample size for any survey is, on average, 10% of the population size.

5 Stratified Random Sample

Used in heterogeneous populations.

- Divide the population into strata (layers), then take a sample from each stratum proportionally.

Basic formula:

$$\text{Sample size from stratum} = \frac{\text{Number of individuals in stratum}}{\text{Total population size}} \times \text{Sample size}$$

learn

- **Simple Random Sample**
  Used with homogeneous populations and can be selected in two ways depending on the population size:

- **Method 1 (if the population size is small)**
  1. Prepare a paper card for each individual in the population, writing the name and number of the individual on it. All cards should be identical in color and size.
  2. Fold all cards in the same way so that the numbers are not visible, then place them in a box and mix well.
  3. Select the sample by randomly drawing a card from the box without looking inside, then reshuffle the cards and draw the next card. Repeat until the required sample size is reached.

- Example: This method is suitable for selecting a sample of 5 players from a team of 35 players.

- **Method 2 (if the population size is large)**
  1. Number all individuals in the population under study.
  2. Select the sample using the random number function on a scientific calculator by pressing the following keys in order:

ON → SHIFT → Ran# → =

This generates a decimal number between 0.000 and 0.999.

- If there is one decimal place, add two zeros to make it part of a thousand (e.g., 0.5 → 0.500).
- If there are two decimal places, add one zero to the right (e.g., 0.72 → 0.720).

Ignore the decimal point and select the individual corresponding to the resulting number.

- Repeat pressing (=) to generate numbers until reaching the desired sample size.
- Exclude numbers greater than the population size and numbers already selected.

- Example: This method is suitable for selecting a sample of 50 students from a school of 600 students.

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Example 1:

(1)

A school has 400 students and wants to improve the library services by knowing students' opinions through a questionnaire.

If the questionnaire is given to a random sample representing 10% of the total students:

Solution:

- Total students = 400
- Required sample percentage = 10%

$$\text{Number of students in the sample} = 400 \times \frac{10}{100} = 40 \text{ students}$$

So, we need to select a random sample of 40 students.

Method of selection using a calculator:

- Idea: We do not ask all 400 students; instead, we take a representative sample.
- Why 40 students? Because the sample = 10% of total students, and 10% of 400 = 40.
- Why use a calculator? It gives random numbers, ensuring fairness and no bias.
- When to reject a number? If it is greater than 400 or already selected.
- Final result: The chosen sample is random, fair, and represents all students.

Example using a calculator:

- Press: ON → SHIFT → Ran# → =
- Decimal 0.258 → student number 258 selected
- Decimal 0.37 → student number 370 selected
- Decimal 0.534 → rejected (because 534 > 400)
- Continue until 40 students are selected.

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(2)	A school has 300 students in 1st grade and 200 students in 2nd grade, required sample size = 50. **Solution** • Sample 1st grade $= \frac{300}{500} \times 50 = 30$ students • Sample 2nd grade $= \frac{200}{500} \times 50 = 20$ students
(3)	A school has: 250 students in primary, 200 students in preparatory, and 150 students in secondary. A teacher wanted to conduct a stratified sample of 60 students representing all educational stages. Calculate the sample size of each stratum in the sample. ....
(4)	In a company, employees are distributed by job grades as follows: 500 employees in grade one 300 employees in grade two 200 employees in grade three If a stratified sample of 100 employees representing all grades proportionally is required, calculate the number of employees in each grade. ....

Samples (1)

Example 1: Find the slope of the straight line passing through the two points in each of the following:	
(1)	The methods of data collection are .... and ....
(2)	Sources of data collection are divided into .... sources and .... sources.
(3)	The sampling method is characterized by availability of .... and ....
(4)	The most suitable method for a complete census of the population is ....

2: Choose the correct answer from the given options.							
1	Which of the following are secondary sources for data collection?						
(a)	Personal interviews	(b)	Questionnaires	(c)	Observation and measurement	(d)	Employee database
2	The sampling method is suitable for all of the following EXCEPT:						
(a)	Checking factory production	(b)	Testing desert sand	(c)	Knowing the population count	(d)	Testing a patient's blood
3	Which of the following are primary sources for data collection?						
(a)	Questionnaires	(b)	School students' data from last year	(c)	Statistical center publications	(d)	Employee records of an institution
4	A factory has 125 workers: 75 technicians and 50 engineers. A stratified sample of 50 people was taken, representing each stratum according to its size. The number of engineers in this sample is .... engineers.						
(a)	30	(b)	20	(c)	25	(d)	15
5	A hospital has 40 doctors and 200 nurses. A stratified sample of 36 people was taken, representing each stratum according to its size. The number of nurses in this sample is						
(a)	6	(b)	18	(c)	24	(d)	30
6	A school has 1,000 students, 300 boys and 700 girls. It was decided to take a proportional stratified sample of 20 students to evaluate the library services. The number of boys that should be selected in this sample is						
(a)	4	(b)	6	(c)	2	(d)	3

Example 3 :

(1)	Which of the following statistical data sources are primary and which are secondary? 1. Searching the internet for the results of the Egyptian national football team in previous Africa Cup of Nations tournaments. …………………………………………………………………… 2. Surveying your classmates about their favorite school subject. …………………………………………………………………… 3. Surveying your classmates about the place they want to go on the next trip. 4. Counting the number of seats in each classroom in your school. …………………………………………………………………… 5. Researching the number of students who passed each subject in your school last year's first term, based on school records. ……………………………………………………………………
(2)	Write the appropriate method (census or sampling) to collect data in each of the following cases: 1. The number of attendees at a university seminar. ……………………… 2. The rainfall rate in a certain governorate. ……………………… 3. The number of cars passing over a bridge in one hour. ……………………… 4. Determining the salinity level in a lake. ……………………… 5. The average prices of vegetables this week. ……………………… 6. The results of testing the soil in a particular field. ………………………
(3)	A school with 350 students conducted a survey to find out which of the products sold in the school cafeteria are preferred by the students (hot drink – pizza – iced juice – hot meal). For this survey, a sample of 10% of the students was taken. Each student was given a number, and a random sample was selected. Determine, using a calculator, the numbers of the students in this sample. …………………………………………………………………… …………………………………………………………………… …………………………………………………………………… …………………………………………………………………… …………………………………………………………………… …………………………………………………………………… …………………………………………………………………… …………………………………………………………………… …………………………………………………………………… ……………………………………………………………………

(4)	**In a factory, there are 180 workers, 120 technicians, and 80 engineers. A stratified sample of 57 people was taken.** **Find the number of individuals in each stratum in the sample.** ....
(5)	**It is required to draw a stratified random sample representing each stratum according to its size from a population of 5,000 individuals, divided into four strata. The first stratum has 1,500 individuals, and the number of individuals from the first stratum in the sample is 225.** **Find the total sample size.** ....

Unit 4 Lesson 2 | Probability and Expected Future Events

learn

- Theoretical probability is based on the principle of equally likely outcomes. It is calculated as the ratio of the number of outcomes favorable to the event to the total number of possible outcomes.
- Probability of an event $(A) = \frac{\text{Number of outcomes in } A}{\text{Total number of outcomes}}$

Example 1 :

(1) A sample of 20 students was taken from 600 students in a school and asked about their favorite subject. Their answers were as shown in the table.

Subject	Frequency
Mathematics	5
Arabic Language	8
Science	4
Social Studies	3

1- If a student is chosen at random, what is the probability that the student prefers Mathematics?

$$P(\text{Mathematics}) = \frac{5}{20} = \frac{1}{4} = 0.25$$

The expected number of students who prefer Mathematics in the school:

$$\text{Expected number} = 600 \times \frac{1}{4} = 150 \text{ students}$$

2- What is the expected number of students who prefer Social Studies in the school?

$$P(\text{Social Studies}) = \frac{3}{20} = 0.15$$

$$\text{Expected number} = 600 \times 0.15 = 90 \text{ students}$$

(2) A sample of 30 students was taken from 300 students in a school and asked about their favorite sport. It was found that 10 students prefer handball and 20 students prefer football. If a student is chosen at random:

- What is the probability that the student prefers football?
- What is the expected number of students who prefer football in the school

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Example 2:

(1)

A factory producing electrical appliances makes two types of televisions. To study production adjustments according to market demand, a random sample of 5 sales outlets was selected, each selling 40 TVs. The data are as follows:

Outlet No	1	2	3	4	5
Type 1 TVs sold	15	17	28	30	20
Type 2 TVs sold	25	23	12	10	20

1. Which type is more demanded? What would you advise the factory?
2. If the factory's total production is 2,000 TVs, how many Type 1 TVs are expected according to this study?

Solution:

- Total sales of Type 1 TVs: 15 + 17 + 28 + 30 + 20 = 110 TVs
- Total sales of Type 2 TVs: 25 + 23 + 12 + 10 + 20 = 90 TVs
- ∴ Type 1 is more demanded; advise the factory to increase production of Type 1.

$$\text{Probability of selling Type 1} = \frac{\text{Number of Type 1 TVs sold}}{\text{Total number of TVs sold of both types}}$$

$$= \frac{110}{200} = 0.55$$

$$\text{Expected production of Type 1} = 0.55 \times 2,000 = 1,100 \text{ TVs}$$

(2)

A random sample of 50 students in a school was surveyed about their favorite sport. It was found that 35 students prefer football, 20 students prefer swimming, and 15 students prefer both football and swimming. If a student is chosen randomly:

First: Calculate the probability of each of the following events:

1. A: The chosen student prefers at least one of the two sports.
2. B: The chosen student prefers only one of the two sports.
3. C: The chosen student prefers neither sport.

Second: If the total number of students in the school is 800:

1. Find the expected number of students who prefer swimming only.
2. Find the expected number of students who prefer at most one sport (football or swimming).

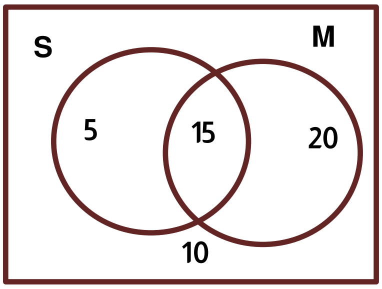
Solution:

- Students who prefer football only = 35 – 15 = 20 students
- Students who prefer swimming only = 20 – 15 = 5 students
- Students who prefer neither sport = 50 – (20 + 5 + 15) = 10 students

Let M be the set of students who prefer football,

and 5 be the set of students who prefer swimming only.

The numbers of students in the survey can be represented using a Venn diagram.



First:

1. Probability of event (A): $P(A) = \frac{20+15+5}{50} = \frac{40}{50} = 0.8$

"Prefers at least one of the two sports" means the student prefers one sport only or both sports.

2. Probability of event (B): $P(B) = \frac{20+5}{50} = \frac{25}{50} = 0.5$

"Prefers only one of the two sports" means the student prefers exactly one sport, not both.

3. Probability of event (C): $P(C) = \frac{10}{50} = 0.2$

Students who prefer neither sport.

Second:

1. Probability that a student prefers swimming only = $\frac{5}{50}$ = 0.1

   Expected number of students who prefer swimming only $= 0.1 \times 800$
   $=$ 80 students

2. Probability that a student prefers at most one sport = $\frac{20+5+10}{50} = \frac{35}{50}$ = 0.7

   "Prefers at most one sport" means the student prefers one sport only or neither sport.

   Expected number of students who prefer at most one sport $= 0.7 \times 800$
   $= 560$ students

(3)

A random sample of 60 people was surveyed about the football teams they support. It was found that 40 people support Al Ahly, 28 support Barcelona, and 12 support both teams. If one person is chosen randomly from the sample, calculate the probability of each of the following events:

- **A: The person supports at least one of the two teams.**
- **B: The person supports only one of the two teams.**
- **C: The person supports both teams.**
- **D: The person supports Al Ahly only.**

(4)

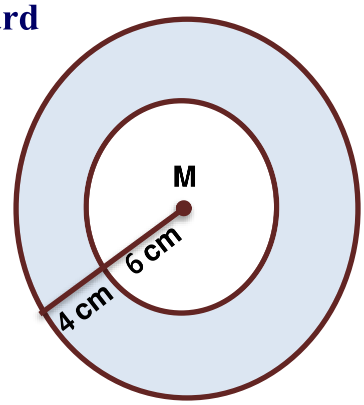
In a basketball team practice, the coach asked players to shoot at the target board. If all players hit the board:

1. **What is the probability that a player hits the shaded area of the board?**
2. **If 250 players shoot in this practice, what is the expected number of players who will hit the shaded area?**

Solution:

The probability that a player hits the shaded area is given by:

$$P(\text{shaded}) = \frac{\text{Area of the shaded part}}{\text{Total area of the board}}$$



- **Area of the larger circle:**
  $A_1 = \pi r^2 = \pi(10)^2 = 100\pi \text{ cm}^2$
- **Area of the smaller circle:**
  $A_2 = \pi r^2 = \pi(6)^2 = 36\pi \text{ cm}^2$
- **Area of the shaded region:**
  $A_3 = 100\pi - 36\pi = 64\pi \text{ cm}^2$
- **Probability of hitting the shaded part:**

$$P = \frac{64\pi}{100\pi} = 0.64 = \frac{16}{25}$$

For the rectangular board scenario:

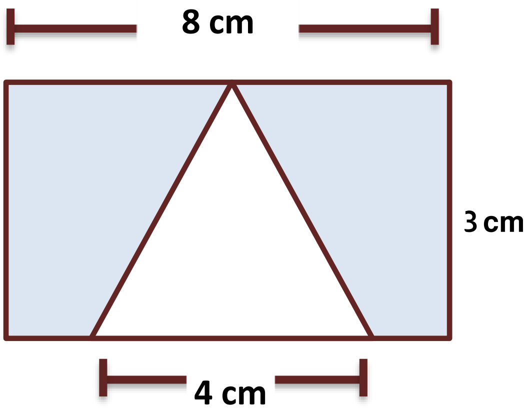
1. **Probability that a student hits the shaded region = 0.64**
2. **Expected number of hits if 52 students shoot =**

$$52 \times 0.64 \approx 33 \text{ students}$$

(5)

In the given figure: If a student shoots an arrow at the rectangular board and hits it:

1. **Find the probability that the student hits the shaded area.**
2. **If 52 students each shoot one arrow, find the expected number of times the shaded area is hit.**



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Probability and Expected Future Events(2)

Example 1: Find the slope of the line passing through the two points in each of the following:

(1) A sample of 100 students is selected to survey their favorite sports from a total of 900 students, as shown in the table:

Favorite Game	Number of Students
Football	45
Basketball	30
Tennis	15
Handball	10

1. Find the probability that a student prefers basketball.
2. Find the expected number of students who prefer basketball from the total students.
3. Find the expected number of students who prefer football from the total students.

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(2) A classroom has some students wearing glasses and some not. If a student is chosen at random and the probability that the student wears glasses is 0.1:

1. Find the probability that the student does not wear glasses.
2. If the classroom has 40 students, find the expected number of students who wear glasses.

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(3)	**A car insurance company pays 3,500 EGP for a car involved in an accident. If the probability of a car being in an accident is 0.006, and there are 9,000 policyholders, find the expected total compensation the company will pay** …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… ……………………………………………………………………………………
(4)	**A classroom has 36 students, 27 passed math, 12 passed Arabic, and 9 passed both exams. If a student is chosen at random:** • **(1) Find the probability the student passed math.** • **(2) Find the probability the student passed Arabic.** • **(3) Find the probability the student failed Arabic.** • **(4) Find the probability the student failed both math and Arabic.** **Also, if the total school population is 540 students, find the expected number of students who passed math only.** …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… …………………………………………………………………………………… ……………………………………………………………………………………

(5) **Two TV factories produce 210 TVs per day, with some defective, according to the table:**

	Factory 1	Factory 2	Total
Defective	12	....	30
Non-defective	....	....	....
Total	100	....	210

1- Complete the table.

2- Find the probabilities of the following events:

- A: the TV chosen is defective
- B: the TV chosen is from Factory 2
- C: the TV chosen is defective and from Factory 2
- D: the TV chosen is defective or from Factory 2

3- If next month the two factories will produce 7,000 TVs, find the expected number of non-defective TVs that month according to this study.

(6) On a certain day, 60 students watched videos on a platform:

- 35 watched educational videos
- 25 watched entertainment videos
- 10 watched both types

1. Find the probability that a student watched educational or entertainment videos.
2. If the total school population is 2,400 students, find the expected number of students who watched educational videos.

(7) You have a board shaped like two squares, and a person throws an arrow at it:

1. Find the probability of hitting the shaded area.
2. If 90 people each throw an arrow at the board, find the expected number of times the shaded area is hit.