

SCIENTIFIC SECTION

SECONDARY 2

PURE MATHEMATICS



Geel 2000 Language Schools

Math Department

First Term

2024 / 2025

Name

.....

Class

.....

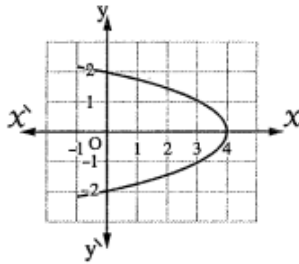
(Unit 1)

Real functions

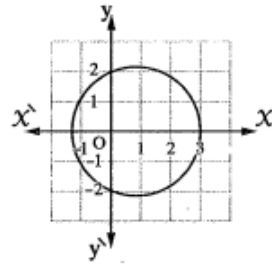
(Domain, Range and monotony)

1-

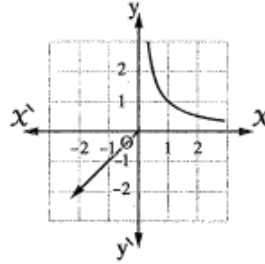
(1) Which of the following figures represents a function of  $X$ ?



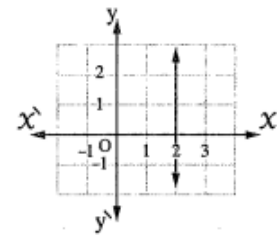
(a)



(b)



(c)



(d)

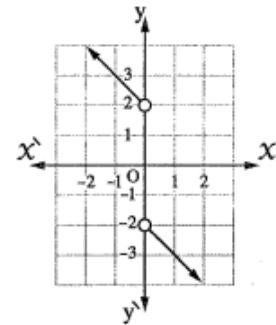
(2) The opposite figure represents a function of  $X$  whose domain is .....

(a)  $\mathbb{R}$

(b)  $\mathbb{R} - ]-1, 2[$

(c)  $\mathbb{R} - [-1, 2]$

(d)  $\mathbb{R} - \{0\}$



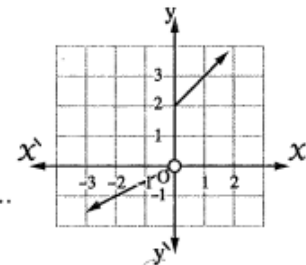
(3) The opposite figure represents a function of  $X$  whose range is .....

(a)  $\mathbb{R} - [0, 2]$

(b)  $\mathbb{R} - \{0\}$

(c)  $\mathbb{R} - [0, 2[$

(d)  $\mathbb{R} - ]0, 2[$



(4)  $f(x) = \sqrt{4 - x^2}$ , then the domain of the function  $f = \dots\dots\dots$

(a)  $[-2, 2]$

(b)  $] -2, 2[$

(c)  $[-2, 2[$

(d)  $] -2, 2]$

## Operations on functions (composition functions)

2-

Choose the correct answer from those given :

(1)  $f(x) = \frac{1}{x}$ ,  $g(x) = \sqrt{x}$ , then the domain of  $(f \cdot g) = \dots\dots\dots$

- (a)  $\mathbb{R} - \{0\}$       (b)  $\mathbb{R}$       (c)  $\mathbb{R}^+$       (d)  $[0, \infty[$

(2)  $f(x) = x + 1$ ,  $g(x) = x^2$ , then  $(f \circ g)(2) = \dots\dots\dots$

- (a) 3      (b) 4      (c) 5      (d) 9

(3) The domain of the function  $f : f(x) = \sqrt{5-x}$  equals  $\dots\dots\dots$

- (a)  $\mathbb{R} - \{5\}$       (b)  $\mathbb{R}^+$       (c)  $]-\infty, 5]$       (d)  $[5, \infty[$

(4)  $f(x) = \sqrt{x}$ ,  $g(x) = x^2$ , then the domain of  $(f \circ g) = \dots\dots\dots$

- (a)  $[0, \infty[$       (b)  $\mathbb{R}$       (c)  $\mathbb{R}^+$       (d)  $\mathbb{R}^-$

3-

If  $f(x) = \frac{1}{x}$ ,  $g(x) = x + 3$ , find :

(1)  $(f \circ g)(x)$

(2)  $(g \circ f)(x)$

and state the domain in each case.

4-

If  $f(x) = \frac{1}{x}$ ,  $g(x) = x + 3$ , find :

(1)  $(f \circ g)(x)$

(2)  $(g \circ f)(x)$

and state the domain in each case.

5-

If  $f(x) = \frac{1}{x}$  ,  $g(x) = 2x$  , find each of the following :

(1)  $(f + g)(x)$

(2)  $(f \cdot g)(x)$

(3)  $\left(\frac{g}{f}\right)(x)$

Showing the domain.

## Properties of functions

6-

Find the type of each function whether it is even , odd or otherwise :

$$(1) f(x) = \frac{x^3}{|x|+2}$$

$$(2) f(x) = \sin x^2 - \sin^2 x$$

7-

[b] If  $f(x) = x - 1$  ,  $g(x) = \sqrt{x}$  , then find  $(g \circ f)(x)$  and determine its domain , then find  $(g \circ f)(5)$

8-

From the following functions , the one-to-one function is .....

$$(a) f_1(x) = x + 2 \quad (b) f_2(x) = x^2 \quad (c) f_3(x) = |x| \quad (d) f_4(x) = 5$$

9-

Find the domain of  $f : f(x) = \frac{3x-5}{x-2}$  , then prove that  $f$  is one-to-one.

10-

Draw the graph of the function  $f : f(x) = \begin{cases} x|x| & \text{when } x < 0 \\ \frac{x^4}{|x|} & \text{when } x > 0 \end{cases}$

, then deduce its domain and discuss its type whether it is even , odd or otherwise.

## Graphical (basic and piecewise)

11-

Graph the function  $f : f(x) = 4 - (x - 2)^2$ , then deduce its range, its monotony and whether the function is odd, even or otherwise.

12-

Graph the function  $f : [-2, 6] \longrightarrow \mathbb{R}$  where  $f(x) = \begin{cases} 4 - x & , -2 \leq x < 1 \\ x & , 1 \leq x \leq 6 \end{cases}$

and from the graph deduce its range and discuss its monotonicity.

13-

Graph the function  $f : f(x) = \begin{cases} x - 1 & , 2 < x \leq 4 \\ -1 & , -2 \leq x \leq 2 \end{cases}$

from the graph determine its range.

14-

[b] If  $f : [-4, 3] \longrightarrow \mathbb{R}$ ,  $f(x) = \begin{cases} 4 & \text{when } x < 0 \\ (x - 1)^2 + 1 & \text{when } 0 \leq x \leq 3 \end{cases}$

, graph the function  $f$ , then from the graph, **deduce** :

(1) The range.

(2) The monotonicity.

(3) The type (even, odd, otherwise).

## Geometric transformations

15-

Use the graph of the function  $f$  where  $f(x) = x^2$  to represent the function  $g$  where  $g(x) = (x-1)^2 + 2$  and from the graph determine the range of the function  $g$  and discuss its monotonicity and tell whether it is even, odd or otherwise.

16-

Use the curve of the function  $f$  where  $f(x) = x^3$  to represent each of the following functions :

( 1 )  $f_1(x) = (x+1)^3$

( 2 )  $f_2(x) = x^3 + 1$

17-

If  $f(x) = x^2 - 1$  ,  $g(x) = x + 1$

Graph the function  $\frac{f}{g}$  , show its domain and its range.

18-

Graph the curve of the function  $f : f(x) = x|x|$  , then discuss its monotony and its type whether its even, odd or otherwise.

## Solving absolute value equations and inequalities

19-

Find in  $\mathbb{R}$  the solution set for each of the following :

(1)  $|2x - 5| \leq 3$

20-

Find in  $\mathbb{R}$  the S.S. of the following equation algebraically :  $\sqrt{x^2 - 4x + 4} = 4$

21-

Find in  $\mathbb{R}$  the S.S. of the following inequality algebraically :  $|x - 3| \geq 5$

22-

Find the solution set in  $\mathbb{R}$  of each of the following :

(1)  $|2x - 3| + |6 - 4x| \leq 0$

(2)  $|2x - 4| = x + 3$

23-

Find graphically in  $\mathbb{R}$  the solution set of the inequality :  $|5 - x| > 3$

24-

Find in  $\mathbb{R}$  the solution set of the inequality :  $\sqrt{4x^2 - 12x + 9} > 5$

25-

Find in  $\mathbb{R}$  the solution set of each of the following :

(1)  $|x + 2| + 5 = 9$

(2)  $|2x - 3| \leq 5$

26-

Solve in  $\mathbb{R}$  the equation :  $2|x - 2| - |2 - x| = 3$



(Unit 2 )

Rational exponents and exponential equations

1-

Find the value of  $x$  which satisfies :  $2^{x+1} = 25$  by using calculator.

2-

Find in  $\mathbb{R}$  the solution set of each of the following :

$$\left(\frac{1}{2}\right)^{x+1} + \left(\frac{1}{2}\right)^{x+3} + \left(\frac{1}{2}\right)^{x+5} = 84$$

3-

Simplify :

$$\frac{16^{x+\frac{1}{4}} \times 9^{x+2}}{8^{x-1} \times 18^{x+2}}$$

4-

Find the S.S. of each of the following in  $\mathbb{R}$  :

$$4^x + 2^{x+1} = 8$$

5-

Put in the simplest form :  $\frac{9^{4n+1} \times 4^{2-2n}}{3^{9n+1} \times 48^{1-n}}$

6-

If  $f(2x) = 3^x$ , solve in  $\mathbb{R}$  the equation :  $f(2x+4) + f(2x) = 90$

## Applications

7-

A patient gets 40 mg. of medicine , his body gets rid of 10 % of this medicine every hour.

( 1 ) Write the exponential function which represents the quantity of medicine left after t hours.

( 2 ) Estimate this quantity of medicine left in the body after 4 hours.

8-

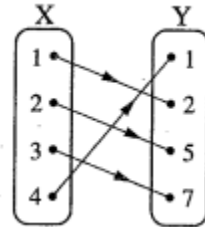
The number of cows in a cattle farm is 80 cows and the reproduction rate of these cows is 18 % annually. Find the number of cows after 4 years , given by  $C = a (1 + r)^t$  where t is the number of years , a is the starting amount and r is the annual percentage of reproduction.

## The invers function

9-

The opposite figure represents a function  $f : X \longrightarrow Y$ , then  $f^{-1}(2) = \dots\dots\dots$

- (a) 5                      (b) 1  
(c) 3                      (d) 4



10-

If  $f(x) = 3 + \sqrt{2x-1}$ , find  $f^{-1}(x)$  and find the domain of  $f^{-1}$  and its range.

11-

If  $f(x) = x - 3$ , then find the inverse function of  $f$

12-

Find the inverse function of  $f$  where  $f(x) = x^3 + 1$

13-

If  $f(x) = 5x$ , find  $f^{-1}(x)$  and represent it graphically.

## Logarithmic function and its properties

14-

Without using calculator , find the value of :  $\log 25 + \frac{\log 8 \times \log 16}{\log 64}$

15-

Find in  $\mathbb{R}$  the S.S of the equation :

$$\log_7 X + \log_7 (X + 6) = 1$$

16-

If  $Xy = 16$  , prove that :  $3 \log_2 X + 4 \log_2 y - \log_2 Xy^2 = 8$

17-

Find in  $\mathbb{R}$  the solution set of each of the following :

$$\log_5 (X^2 - 25) - \log_5 (X - 5) = 2$$

18-

Find the S.S. in  $\mathbb{R}$  of the equation :

$$(\log_3 X)^2 - 2 \log_3 X - 3 = 0$$

19-

Simplify : ( 1 )  $\frac{1}{\log_a ab} + \frac{1}{\log_b ab}$

20-

Find the S.S. of the following equation in  $\mathbb{R}$  :  $\log_2 X + \log_2 (X + 1) = 1$

21-

**Without using calculator , find the value of the following :**

$$2 \log 25 + \log \left( \frac{1}{3} + \frac{1}{5} \right) + 2 \log 3 - \log 30$$

22-

**Solve the equation in  $\mathbb{R}$  :  $\log_2 X + \log_2 (X + 1) = 1$**

23-

**Solve in  $\mathbb{R}$  the following equation :  $\log_4 X = 1 - \log_4 (X - 3)$**

24-

**Find in  $\mathbb{R}$  the solution set for each of the following :**

$$\log_3 X + \log_X 3 = 2$$

25-

**( 1 ) Find in  $\mathbb{R}$  the solution set of the equation :  $\log_3 X + \log_X 3 = 2$**

**( 2 ) Prove that :  $\frac{\log 729 - \log 64}{\log 9 - \log 4} = 3$**

26-

**| Prove that :  $\log_2 \frac{4}{11} - \log_2 \frac{7}{130} + \log_2 \frac{77}{65} = \log_5 125$**

27-

**| Find in  $\mathbb{R}$  the S.S. of the equation :**

$$\log_4 X = 1 - \log_4 (X - 3)$$

28-

**Prove that :**  $\log_b a \times \log_c b \times \log_d c \times \log_a d = 1$

(Unit 3) Introduction of limits of functions

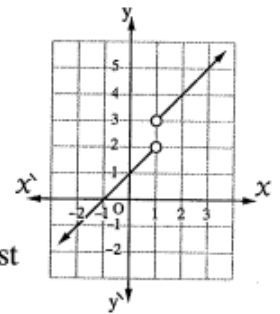
1-

Choose the correct answer from those given :

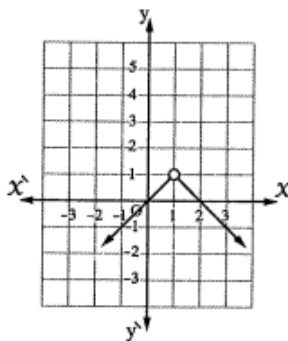
(1) The opposite figure represents the graph of the function  $f$ , then

$$\lim_{x \rightarrow 1} f(x) = \dots\dots\dots$$

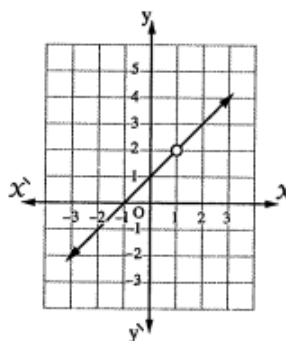
- (a) 2                      (b) 3                      (c) 1                      (d) not exist



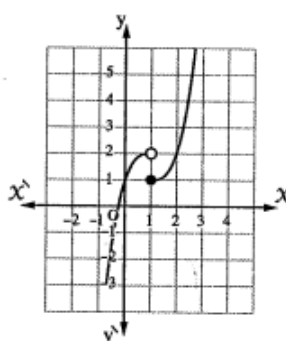
(2) Which of the following functions has no limit at  $x = 1$  ?



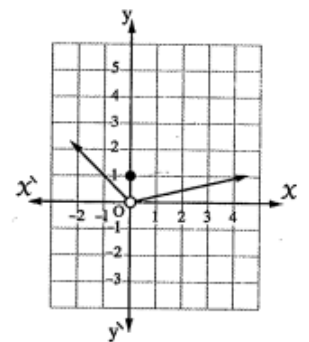
(a)



(b)



(c)

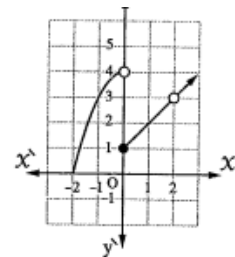


(d)

2-

In the opposite figure, find :

- (1)  $f(\text{zero}^+)$                       (2)  $f(\text{zero}^-)$   
 (3)  $f(2)$                               (4)  $\lim_{x \rightarrow 2} f(x)$



## Finding the limit of the function algebraically

3-

Choose the correct answer from those given :

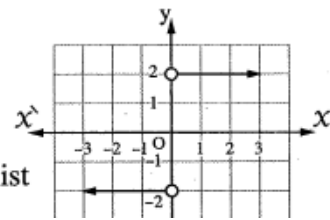
(1)  $\lim_{x \rightarrow 0} \frac{1+x}{4x-1} = \dots\dots\dots$   
 (a) -1                      (b)  $\frac{1}{4}$                       (c)  $-\frac{1}{4}$                       (d) 1

(2)  $\lim_{x \rightarrow 3} \frac{x^2-9}{x-3} = \dots\dots\dots$   
 (a) -6                      (b) zero                      (c) 3                      (d) 6

(3) The opposite figure represents  $f(x)$

Then :  $\lim_{x \rightarrow 2} f(x) = \dots\dots\dots$

- (a) 0                      (b) -2                      (c) 2                      (d) not exist



4-

Find :

$$\lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x - 3}$$

5-

find :

$$\lim_{x \rightarrow -1} \left( \frac{5x^2 + 5x}{3x^2 - 3} \right)$$

6-

$$\lim_{x \rightarrow 0} \frac{x^2 + x}{x} = \dots\dots\dots$$

7-

Find :  $\lim_{x \rightarrow 1} \frac{x^3 - 2x^2 + 1}{x^2 + x - 2}$

8-

Find :

$$(2) \lim_{x \rightarrow 2} \frac{x^2 - 5x + 6}{x^3 - 8}$$

9-

Find :

$$\lim_{x \rightarrow 5} \frac{x - 5}{2x - 3}$$

10-

Find : (

$$\lim_{x \rightarrow 2} \frac{x^3 + 8}{x + 2}$$

11-

$$\lim_{x \rightarrow -3} \frac{x^2 + 4x + 3}{x^2 - 9}$$

12-

Find : (1)

$$\lim_{x \rightarrow -2} \frac{x^3 + 8}{x + 2}$$

The law

13-

$$\lim_{x \rightarrow 4} \frac{\sqrt{x-3}-1}{x-4}$$

14-

Find : (1)  $\lim_{x \rightarrow -2} \frac{(x+3)^5 - 1}{x+2}$

15-

Find : (1)  $\lim_{x \rightarrow 2} \frac{x^3 + 8}{x+2}$

16-

Find : (1)  $\lim_{x \rightarrow 3} \frac{x^5 - 243}{x^2 - 9}$

17-

Find :  $\lim_{x \rightarrow 5} \frac{(x-3)^5 - 32}{x-5}$

18-

Find : (1)  $\lim_{x \rightarrow -3} \frac{x^4 - 81}{x^5 + 243}$

19-

Find : (1)  $\lim_{x \rightarrow -2} \frac{\sqrt[3]{x} + \sqrt[3]{2}}{x+2}$

20-

Find : (1)  $\lim_{x \rightarrow \sqrt{5}} \frac{x^7 - 125\sqrt{5}}{x^4 - 25}$

21-

$\lim_{x \rightarrow 5} \frac{(x-3)^5 - 32}{x^2 - 5x}$

22-

$\lim_{x \rightarrow -3} \frac{x^4 - 81}{x^5 + 243}$

23-

$\lim_{x \rightarrow -3} \frac{x^4 - 81}{x^5 + 243}$

24-

$\lim_{x \rightarrow 2} \frac{x^7 - 128}{x^5 - 32}$

25-

Find : (1)  $\lim_{x \rightarrow 2} \frac{2x^4 - 32}{x^2 - 4}$

26-

**Find : (1)**  $\lim_{x \rightarrow \frac{1}{3}} \frac{27x^4 - \frac{1}{3}}{3x - 1}$

## Limit of the function at infinity

26-

**Find : (1)**  $\lim_{x \rightarrow \infty} \frac{2x^3 - 5x}{x^4 + 3}$

27-

$$\lim_{x \rightarrow \infty} \frac{2x^3 - 9}{|3x|^3 + 7}$$

28-

$$\lim_{x \rightarrow \infty} \frac{5x^{-3} + 4x^{-2} - 3}{7x^{-3} - 2x^{-2} + 8}$$

29-

**Find : (1)**  $\lim_{x \rightarrow \infty} (\sqrt{4x^2 - 2x + 1} - 2x)$

30-

$$\lim_{x \rightarrow \infty} \frac{2x^2 - x + 1}{x^3 - x^2 + 1}$$

31-

**Find : (1)**  $\lim_{x \rightarrow \infty} \frac{2x^3 - 3}{3x^2 + 1}$

32-

$$\lim_{x \rightarrow \infty} \frac{x^{-2} + 3}{x^{-3} + 6}$$

33-

**Find : (1)**  $\lim_{x \rightarrow \infty} \frac{\sqrt{9x^2 + 3}}{6x - 1}$

34-

$$\lim_{x \rightarrow \infty} \frac{5 - 6x - 3x^2}{2x^2 + x + 4}$$

35-

$$\lim_{x \rightarrow \infty} \frac{-x}{\sqrt{4 + x^2}}$$

36-

**Find : (1)**  $\lim_{x \rightarrow \infty} \frac{x^3 - 4x + 5}{(2x - 1)^3}$

37-

**Find : (1)**  $\lim_{x \rightarrow \infty} \frac{5 - 6x - 3x^2}{2x^2 + x + 4}$

38-

**Find : (1)**  $\lim_{x \rightarrow \infty} \frac{3x^2 + x - 1}{8x^2 - 3}$

39-

$$\lim_{x \rightarrow \infty} \frac{2x^{-1} - 3x^{-2}}{4 + x^{-1}}$$

40-

$$\lim_{x \rightarrow \infty} \frac{3x + \sqrt{4x^2 + 5}}{5x - 3}$$

41-

$$\lim_{x \rightarrow \infty} (\sqrt{x^2 + 5x} - x)$$

42-

$$\lim_{x \rightarrow \infty} \frac{x^3 - 2}{|x|^3 + 1}$$

43-

**Find : (1)**  $\lim_{x \rightarrow \infty} \frac{x^3 - 4x + 5}{(2x - 1)^3}$

44-

Find : (1)  $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 5x} - x)$

45-

$$\lim_{x \rightarrow \infty} \frac{1}{x} \sqrt{3 + 4x^2}$$

## Limits of trigonometric functions

46-

$$\lim_{x \rightarrow 0} \frac{1 - \cos^2 2x}{3x^2}$$

47-

$$\lim_{x \rightarrow 0} \frac{x - x \cos x}{\sin^2 3x}$$

48-

$$\lim_{x \rightarrow 0} \frac{\sin 3x - \sin 2x}{5x}$$

49-

**Find : (1)**  $\lim_{x \rightarrow 0} \frac{2x + \sin 3x}{\tan 5x}$

50-

**Find :**  $\lim_{x \rightarrow 0} \frac{x^3 + \sin 3x}{5x \cos 2x}$

51-

**Find : (1)**  $\lim_{x \rightarrow 0} \frac{x - x \cos x}{\sin^2 3x}$

52-

$$\lim_{x \rightarrow 0} \frac{x^2 + \sin 3x}{5x \cos 2x}$$

53-

$$\lim_{x \rightarrow 0} \frac{\sin^2 3x - \tan 2x^2}{x^2 \cos 4x}$$

54-

$$\lim_{x \rightarrow 0} \frac{x \tan 2x \cos 3x}{x^2 + \sin^2 3x}$$

55-

**Find : (1)**  $\lim_{x \rightarrow 1} \frac{\sin(x-1)}{x^2 + x - 2}$

56-

**Find : (1)**  $\lim_{x \rightarrow 0} \frac{x - x \cos x}{\sin^2 3x}$

57-

**Find : ( 1 )**  $\lim_{x \rightarrow 0} \frac{1 - \cos x + \sin x}{1 - \cos x - \sin x}$

58-

$$\lim_{x \rightarrow 0} \frac{\sin 3x - \tan 2x}{5x}$$

59-

$$\lim_{x \rightarrow 0} \frac{\sin 2x + 5 \sin 3x}{x}$$

60-

$$\lim_{x \rightarrow 0} \frac{x \tan x + \sin^2 3x}{2x^2 + \sin 3x^2}$$

## Existence of the limit of a piecewise function

61-

Find : (1)  $\lim_{x \rightarrow \pi} f(x)$  where  $f(x) = \begin{cases} \frac{2 \sin x}{\pi - x} & , \quad x < \pi \\ 1 - \cos x & , \quad x > \pi \end{cases}$

(2)  $\lim_{x \rightarrow \infty} x \tan x^{-1}$

62-

$$f(x) = \begin{cases} \frac{x \tan x + \sin^2 3x}{5x^2} \\ 2 \cos 2x \end{cases}$$

63-

Discuss the existence of the limit of the function  $f$  where

$$f(x) = \begin{cases} \frac{1 - \cos x}{x} & , \quad x > 0 \\ 2 \sin x & , \quad x \leq 0 \end{cases} \text{ at } x = 0$$

64-

**Find :**  $\lim_{x \rightarrow 3} f(x)$  if  $f(x) = \begin{cases} x^2 + 1 & , \quad x < 3 \\ 3x + 1 & , \quad x > 3 \end{cases}$

65-

| If  $f(x) = \begin{cases} x^2 - 2x & , \quad -2 < x < 1 \\ 3x - 4 & , \quad 1 \leq x < 4 \end{cases}$

**discuss the existence of each of the following :**

(1)  $\lim_{x \rightarrow -2} f(x)$

(2)  $\lim_{x \rightarrow 1} f(x)$

66-

If  $\lim_{x \rightarrow 2} f(x) = 7$  where  $f(x) = \begin{cases} x^2 + 3m & , \quad x < 2 \\ 5x + k & , \quad x > 2 \end{cases}$ , find m and k

67-

Discuss the existence of  $\lim_{x \rightarrow 0} f(x)$  where

$$f(x) = \begin{cases} \frac{5x^2 + \tan 2x^2}{\sin^2 x} & , \quad x > 0 \\ 7 \cos 3x & , \quad x < 0 \end{cases}$$

68-

Find : (1)  $\lim_{x \rightarrow \pi} f(x)$  where  $f(x) = \begin{cases} \frac{2 \sin x}{\pi - x} & , x < \pi \\ 1 - \cos x & , x > \pi \end{cases}$

(2)  $\lim_{x \rightarrow \infty} x \tan x^{-1}$

69-

If  $f(x) = \begin{cases} \frac{x^2 - 7x + 12}{x - 3} & , x > 3 \\ a x - 7 & , x < 3 \end{cases}$

,  $\lim_{x \rightarrow 3} f(x) = -1$ , then find the value of a

## Continuity

70-

Discuss the continuity of the function  $f$

$$\text{where } f(x) = \begin{cases} 8 - x & , \quad x \geq 3 \\ x + 2 & , \quad x < 3 \end{cases} \quad \text{at } x = 3$$

71-

**Discuss the continuity of the function  $f$  where :**

$$f(x) = \begin{cases} \frac{x^2 + x - 2}{x + 2} & , \quad x > -2 \\ 3x + 5 & , \quad x \leq -2 \end{cases} \quad \text{at } x = -2$$

72-

**Discuss the continuity of the function  $f$  on its domain where :**

$$f(x) = \begin{cases} 1 + \sin x & , \quad \text{when } 0 \leq x \leq \frac{\pi}{2} \\ 1 - \cos 2x & , \quad \text{when } x > \frac{\pi}{2} \end{cases}$$

73-

**Discuss the continuity of the function  $f$  :**

$$f(x) = \begin{cases} \frac{\sin(x-2)}{x^2-4} & , \quad x < 2 \\ 1 - \frac{3}{x^2} & , \quad x \geq 2 \end{cases} \text{ at } x = 2$$

74-

Discuss the continuity of  $f$  :  $f(x) = \begin{cases} \frac{x^2-x-6}{x-3} & , \quad x < 3 \\ 2x-1 & , \quad x \geq 3 \end{cases}$  at  $x = 3$

75-

**Discuss the continuity of the function  $f$  where**

$$f(x) = \begin{cases} \frac{\sin x}{2x-2\pi} & , \quad x < \pi \\ \frac{1}{2} \cos x & , \quad x \geq \pi \end{cases} \text{ at } x = \pi$$

76-

**Discuss the continuity of  $f$  at  $x = 1$**

where  $f(x) = \begin{cases} x^2 + 3 & , \quad x \geq 1 \\ \frac{x^2 + 2x - 3}{x-1} & , \quad x < 1 \end{cases}$

77-

**Find the value of a which makes the function f**

$$\text{where } f(x) = \begin{cases} \frac{x^2 - 5x + 6}{x^3 - 8} & , \quad x \neq 2 \\ \frac{-2}{|a|} & , \quad x = 2 \end{cases} \text{ continuous at } x = 2$$

78-

**Find the value of k which makes the function f continuous at  $x = 2$  where**

$$f(x) = \begin{cases} \frac{x^2 + x - 6}{x^3 - 8} & , \quad x \neq 2 \\ \frac{2}{|k|} & , \quad x = 2 \end{cases}$$

79-

**Discuss the continuity of  $f : f(x) = \begin{cases} x^2 + 3 & , \quad x \geq 1 \\ \frac{x^2 + 2x - 3}{x - 1} & , \quad x < 1 \end{cases}$  at  $x = 1$**

80-

**Discuss the continuity of the function f**

$$\text{where } f(x) = \begin{cases} x^2 + 3 & , \quad x \geq 1 \\ \frac{x^2 + 2x - 3}{x - 1} & , \quad x < 1 \end{cases} \text{ at } x = 1$$

81-

**Find the value of the constant a if the function f :**

$$\text{where } f(x) = \begin{cases} \frac{(x+3)^4 - 81}{x} & , \quad x \neq 3 \\ a & , \quad x = 3 \end{cases} \text{ is continuous at } x = 3$$

82-

**Discuss the continuity of the function f where :**

$$f(x) = \begin{cases} \frac{\sin x}{\pi - x} & , \quad x \neq \pi \\ 1 & , \quad x = \pi \end{cases} \text{ at } x = \pi$$

### (Unit 4) The sine rule

1-

Solve  $\Delta ABC$  in which  $m(\angle B) = 35^\circ$ ,  $m(\angle C) = 70^\circ$ , and the diameter length of its circumcircle = 32 cm.

2-

In  $\Delta ABC$ , if  $m(\angle A) = 35^\circ$ ,  $a = 17$  cm. and  $b = 20$  cm.

**Prove that :**  $\Delta ABC$  has two solutions, then find them.

3-

Find the perimeter of  $\Delta ABC$  in which  $m(\angle A) = 57^\circ 13'$ ,  $c = 8.7$  cm.  
and  $m(\angle B) = 64^\circ 18'$

4-

ABC is a triangle in which :  $m(\angle A) = 35^\circ$  ,  $a = 8$  cm. and  $b = 6$  cm. **Find** :  $m(\angle B)$

5-

ABC is a triangle in which :  $b = 12$  cm. ,  $m(\angle B) = 75^\circ$  and  $m(\angle C) = 45^\circ$  **Find** :

( 1 ) a

( 2 ) The area of  $\Delta ABC$

( 3 ) The radius length of the circumcircle of the triangle ABC

6-

ABC is a triangle in which  $m(\angle A) : m(\angle B) : m(\angle C) = 3 : 4 : 3$

If  $a = 5$  cm. , find the perimeter of  $\Delta ABC$

7-

Solve the triangle ABC in which  $a = 8$  cm. ,  $m(\angle A) = 60^\circ$  and  $m(\angle B) = 40^\circ$

8-

Solve the acute-angled triangle ABC in which  $a = 21$  cm. ,  $b = 25$  cm. and the diameter length of its circumcircle = 28 cm.

9-

Find the shortest side length in  $\Delta ABC$  , in which :  $m(\angle A) = 43^\circ$  ,  $m(\angle B) = 70^\circ$  and  $c = 9$  cm. Find the area of the triangle ABC.

10-

ABC is a triangle in which :  $AC = 4.7$  cm. ,  $m(\angle B) = 34^\circ$  and  $m(\angle C) = 66^\circ$   
Find the length of  $\overline{BC}$  , then find the area of its circumcircle.

11-

ABC is a triangle in which  $m(\angle A) = 40^\circ$  ,  $a = 5$  cm. and  $b = 7$  cm.  
Find  $m(\angle B)$  approximating to the nearest minute.

12-

Solve the triangle ABC in which  $a = 5$  cm. ,  $b = 7$  cm. and  $m(\angle C) = 65^\circ$

13-

ABC is a triangle in which  $m(\angle A) = 85^\circ$  ,  $m(\angle B) = 55^\circ$  and  $c = 5$  cm.  
Find the area of the circumcircle of  $\Delta ABC$

### The cosine rule

14-

The perimeter of the triangle ABC is 52 cm. ,  $a = 13$  cm. and  $b = 17$  cm.  
Find the measure of the greatest angle.

15-

Solve the triangle ABC in which  $a = 5$  cm. ,  $b = 7$  cm. and  $m(\angle C) = 65^\circ$

16-

Solve the triangle ABC in which  $a = 9$  cm. ,  $b = 7$  cm. and  $c = 5$  cm. , then find its area.

17-

In  $\Delta ABC$  , if  $a = 4$  cm. ,  $b = 5$  cm. and  $c = 6$  cm. , **prove that** :  $\cos C = \cos 2 A$   
 , then find the circumference of the circumcircle of  $\Delta ABC$

18-

Solve the triangle  $ABC$  in which  $a = 9$  cm. ,  $b = 15$  cm. and  $m(\angle C) = 106^\circ$

19-

Solve the triangle  $ABC$  in which  $a = 15$  cm. ,  $b = 13$  cm. and  $c = 14$  cm.

20-

$ABC$  is a triangle in which  $a = 27$  cm. ,  $b = 35$  cm. and  $c = 18$  cm.

Find the measure of the greatest angle.

## Solution of the triangle

21-

In  $\Delta ABC$  ,  $\cos A = \frac{2}{5}$  ,  $b = 2.5$  cm. and  $c = 2$  cm.

**Prove that :**  $\Delta ABC$  is an isosceles triangle and find its area.

22-

In  $\Delta ABC$  , if  $m(\angle A) = 35^\circ$  ,  $a = 17$  cm. and  $b = 20$  cm.

**Prove that :**  $\Delta ABC$  has two solutions , then find them.

23-

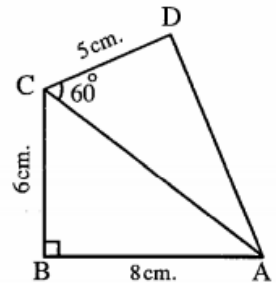
**In the opposite figure :**

ABCD is a quadrilateral in which

$AB = 8$  cm. ,  $BC = 6$  cm. ,  $m(\angle B) = 90^\circ$

,  $DC = 5$  cm. and  $m(\angle ACD) = 60^\circ$

Find the area of the circumcircle of  $\Delta ADC$



24-

In  $\Delta ABC$  , show whether it has only 1 solution , 2 solutions or non , given your answer to the nearest decimal :

( 1 )  $m(\angle B) = 110^\circ$  ,  $b = 8$  cm. and  $c = 5$  cm.

( 2 )  $m(\angle A) = 60^\circ$  ,  $a = 7$  cm. and  $b = 9$  cm.

25-

Show whether the triangle ABC has one , two or no solution , given that :  $m(\angle A) = 100^\circ$  ,  $a = 12$  cm. and  $b = 15$  cm.

26-

ABC is a triangle in which  $m(\angle A) = 52^\circ$  ,  $a = 21$  cm. and  $b = 26$  cm.

**Prove that :**  $\Delta ABC$  has two solutions , then find them.

27-

ABC is a triangle in which :  $\frac{\sin A}{3} = \frac{\sin B}{4} = \frac{\sin C}{5}$

and its perimeter = 24 cm. Find its area.