MATHEMATICS
PREPARATORY ONE
FIRST TERM

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Percentage, Ratio, and Proportion

Number Sets

Operations on Numbers

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UNIT 1

Numbers and Their Operations

Unit Lessons

Percentage, Ratio, and Proportion
1-1 Proportion
1-2 Applications of Ratio and Proportion

Number Sets
1-3 Sets and Their Operations

Operations on Numbers
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1-5 Operations on Rational Numbers

Artificial Intelligence (AI) is one of the latest fields and forms of technological development.
It can be used to enhance teaching and learning techniques.

▶ Can we develop (AI) applications to solve mathematical problems in our daily life?

Issues and Life Skills		Values	
- Mathematical Communication	- Mathematical Understanding	- Respect	- Responsibility
- Creative/Critical Thinking	- Information Technology (IT)	- Perseverance	- Endurance
- Sustainable Development	- Globalization	- Belonging	- Justice

Lesson (1) Proportion

Proportion: is the equality of at least two ratios or rates.

$\frac{1}{5}=\frac{2}{10}$ or $\frac{20km}{1hour}=\frac{40km}{2hours}$

Writing the proportion:

If : a , b , c , d are proportional , Then $\frac{a}{b}=\frac{c}{d}$, a : b = c : d

(a and d are called Extremes) , (b and c are called Means)

Example : 1 , 2 , 3 , 6 are proportional ($\frac{1}{2}=\frac{3}{6}$)

• Cross multiplication property :

The product of extremes = The product of means

1 × 6 = 2 × 3

(1) Determine which of the following ratios represents a proportion.

A) $\frac{3}{8},\frac{6}{10}$ B) $\frac{2}{7},\frac{10}{35}$

(2) Solve the following proportional to find the value of x :

A) $\frac{4}{12}=\frac{20}{x}$

B) $\frac{3}{x-3}=\frac{12}{8}$

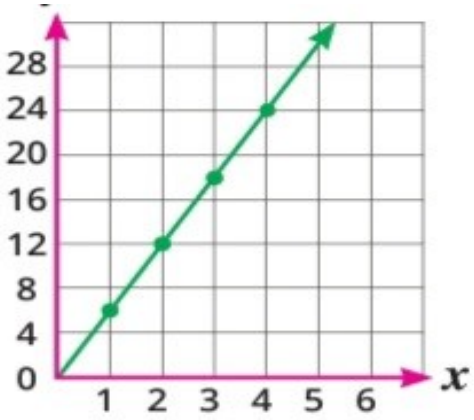
C) $\frac{x}{9}=\frac{15}{45}$

(3) An object weighs 90 Newtons on Earth. If you know that its weight on the moon is 15 Newtons. How much does another object weigh on the moon if its weight on Earth is 60 Newtons?

Solution : $\frac{90}{15} = \frac{60}{X}$

The weight of the moon = $\frac{15 \times 60}{90} = 10$ Newtons

(6) The table opposite shows the distance covered by Moez in meters in a running exercise. Determine whether the distance in meters is proportional to the time in seconds.



Distance covered	
meters	Time
6	1
12	2
18	3
24	4

(7) In each of the following, are the quantities proportional? If they are proportional, write the proportion.

A) 5 , 8 , 15 , 24

B) 1 , 27 , 16 , 18

C) 2 , 3 , 16 , 24

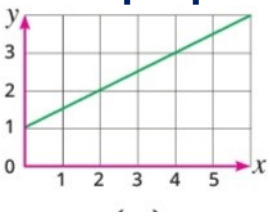
Homework

(1) Omar bought 8 apples for 60 pounds. How many apples of the same type can he buy for 105 pounds?

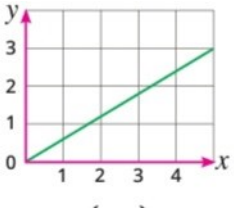
(2) A car uses 5 liters of gasoline to travel a distance of 40 km. How much gasoline does the car need to travel a distance of 128 km if it drives at the same rate?

(3) Choose the correct answer :

A) Which of the following relationships represents a proportion? ....



(a)



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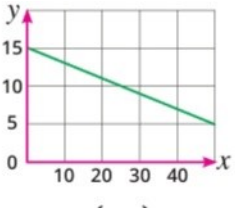
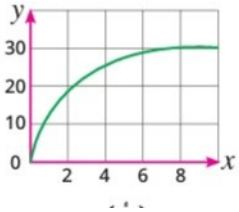


(c)



(d)

B) If : $\frac{3}{7}=\frac{12}{x}$, then x = ……..

(a) 4 (b) 5 (c) 21 (d) 28

C) The price of 6 kg of apple is 60 LE , Then the price of 8 kg is ….. LE

(a) 80 (b) 70 (c) 60 (d) 10

(4) Solve the following proportional to find the value of x :

A) $\frac{7}{8}=\frac{14}{x}$

B) $\frac{4}{7}=\frac{x}{35}$

C) $\frac{8}{x+1}=\frac{16}{10}$

Lesson (2) Applications of Ratio and Proportion

First : Scale Drawing:

is a ratio between Length in drawing and real length .

Scale Drawing = Length in drawing : real length

* Both lengths should have the same units.

*Kaber took a picture of his sister. If her height in the picture is 5 cm, her height in reality is 120 cm. This means 5cm in picture represents 120 cm in reality .

Solution : Drawing : real

5 : 120 (÷ 5)

1 : 24 (Minimization)

Each 1 cm in picture represents 24 cm in reality

Notice that :

Km $\overset{\times 1000}{\underset{\div 1000}{\rightleftarrows}}$ m $\overset{\times 10}{\underset{\div 10}{\rightleftarrows}}$ dm $\overset{\times 10}{\underset{\div 10}{\rightleftarrows}}$ cm $\overset{\times 10}{\underset{\div 10}{\rightleftarrows}}$ mm

Remark

If the drawing scale is

- Less than 1 (< 1), then it refers to minimization (reduction)
  (length in drawing < length in reality)
  For example : maps and geometric figures.
- Greater than 1 (> 1), then it refers to enlargement (magnification)
  (length in drawing > length in reality)
  For example : a picture for a small insect.

(1) If the length in the drawing is 2 cm and the actual length is 6 meters, what is the scale drawing of the drawing?

.....

.....

.....

(2) If the actual distance between Cairo and Damietta is 200 km and the distance between them on the map is 5 cm, find the scale drawing of this map?

.....

.....

.....

(3) A model of the Cairo Tower was made on a scale of 1: 200 If the height of the Cairo Tower is 187 meters, what is the height of the tower in the model in centimeters?

Solution : Drawing : real

1 : 200

..... : 187 ×100

Height = $\frac{1\times 18{,}700}{200} = 93.5cm$

(4) If the scale of the map is 1: 600,000 and the distance between two points on the map is 4 cm. Find the actual distance between the two points.

.....

.....

.....

(5) If the distance between two cities on the map is 10 cm , and the real distance between them is 120 km. Find the scale drawing.

.....

.....

.....

(6) If the distance between two cities 24 km, find the distance between them in cm on the map drawing with scale 1 : 400,000 ?

.....

.....

.....

(7) Complete :

A) Drawing scale = ... : .....

B) If drawing scale > 1 , it represents .....

C) If drawing scale < 1 , it represents .....

(8) If the length of an insect in the picture is 4 cm and its real length is 2 mm, find the drawing scale?

.....

.....

.....

(9) Choose the correct answer :

A) If the length in drawing is 2 cm and the real length is 6 m then the drawing scale is ……..

(a) 1 : 3 (b) 1 : 30 (c) 1 : 300 (d) 1 : 3,000

B) If real length is 90 m, the drawing scale is 1 :10,000 the length in drawing is ……. cm

(a) 0.09 (b) 90 (c) 9 (d) 0.9

C) The price of 6 kg of apple is 60 LE , Then the price of 8 kg is ….. LE

(a) 80 (b) 70 (c) 60 (d) 10

Second: (Proportion Division)

(1) The ratio between two numbers is 2 : 3 , if the sum of two number is 150 find the two numbers ?

First	:	second	:	sum
2	:	3	:	5
	:		:	150

First = $\frac{2\times150}{5}=60$

second = $\frac{2\times150}{5}=90$

(2) The ratio between two numbers is 3 : 5 , if the greater number is 100 . What is the smaller number ?

……

……

……

(3) Three persons shared 7,200 LE in the ratio of 3 : 4 : 5 ,
Calculate the share of each one ?

.....

.....

.....

(4) Ali earns twice as much as Khaled. If the sum of their wages Is 36,000 LE . What is the wage for each of them ?

.....

.....

.....

(5) If the number of students in a school is 576 , and the ratio of girls to boys is 5 : 3 , find the increase of the number of girls than boys?

.....

.....

.....

(6) A piece of land with an area of 36 feddans .was divided between two persons in the ratio of 7 : 2 . find the share of each one ?

.....

.....

(7) If X : Y = 2 : 7 , and y - x = 5,000 , find X and Y ?

.....

.....

.....

(8) Choose the correct answer :

A) If we divided 120 between two persons in the ratio 2 : 3
Then the smallest number is ….

(a) 24 (b) 48 (c) 96 (d) 72

B) If we divided 600 between two persons in the ratio 3 : 5
Then the greater number is ….

(a) 225 (b) 250 (c) 375 (d) 400

Third : (Percentage Application)

(1) A T.V after deduction rate of 16 % is 12,600 LE ,
what is the TV price before deduction ?

Before discount	:	discount	:	After discount
100	:	16	:	84
.....	:		:	12,600

Before deduction =

$$\frac{12{,}600 \times 100}{84} = 15{,}000 LE$$

(2) A meal costs 150 LE , and 18 % of the cost is added as the meal's profit . What is the meal's sale price ?

. . . . . .

. . . . . .

. . . . . .

(3) Mariam wants to buy a vacuum for 6,800 LE , if the discount rate is 12 % . What Mariam will pay to buy ?

. . . . . .

. . . . . .

. . . . . .

(4) Sandy received a 15 % discount rate on a pair of sneakers from the a store .She paid 340 LE .What was the original price of the sneakers?

. . . . . .

. . . . . .

. . . . . .

. . . . . .

(5) If the price of a smart phone displayed in a store is 12,750 LE, and the store offered a 8 % discount rate . what is the phone's price after discount ?

. . . . . .

. . . . . .

. . . . . .

(6) Nancy bought an automatic washing machine for 3,600 L.E and the discount was 10 %. Calculate the original price of the washing machine before discount ?

. . . . . .

. . . . . .

. . . . . .

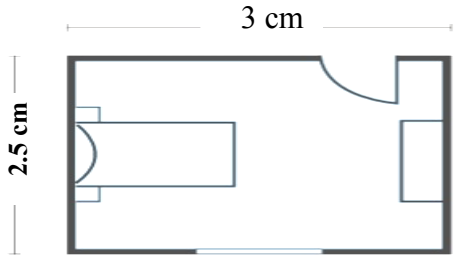
Homework

(1) The corresponding figure represents a model for a room where every 1 cm in the model represents 1.2 meter In fact, what is the real area of the room?

.....

.....

.....



(2) Drawing scale in maps can be expressed as follows:

What is the real distance between two cities is 3 cm ?

.....

(3) The ratio between two numbers is 2 : 5 , if the smallest number is 48 , find the big number ?

(4) A man passed away leaving 150,000 LE for his wife , two sons and one daughter . If his wife's share is $\frac{1}{8}$ of the total amount.

The remaining sum is to be distributed among his children. What is the share of each son and the daughter .considering the ratio of son to a daughter is 2:1?

(5) The ratio among the side lengths of triangle is 1: 2 : 3 , if the perimeter of a triangle is 60 cm , find the length of longest side ?

(6) Three persons invested in a factory the first paid 9,000 L.E , the second paid 6,000 L.E , the third paid 7,500 L.E. At the end of year the profit was 1,500 L.E Calculate the share of each of them ?

(7) If A : B = 4 : 5 , and A + B = 72 , find A and B ?

(8) Two persons started a commercial business , the first paid 5,000 LE the second paid 8,000 LE , at the end of the year , the net profit was 3,900 LE. Calculate the share of each of the profit ?

(9) Tamer bought a mobile for L.E 2,500 with a discount 10% . Find the original price of the mobile before discount ?

(10) Choose the correct answer :

A) If an item price was reduced from 1,500 LE to 1,200 LE Then the deduction rate is .....

(a) 3 % (b) 15 % (c) 20 % (d) 30 %

B) A man deposited 50,000 LE in a bank with an interest 18% The total amount after one year is ..... L E

(a) 9,000 (b) 41,000 (c) 59,000 (d) 68,000

C) The ratio between two numbers 2 : 3 if the greater number is 30 LE , then the first number is ..... LE

(a) 10 (b) 20 (c) 30 (d) 40

Lesson (3) Sets and their Operations

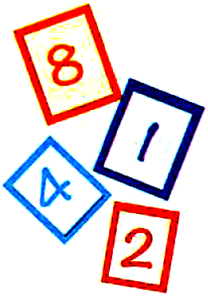
Definition of a set: The set is a collection of known objects that are clearly defined

For Example : Each of the following **can be** considered as **a set**

The colours of traffic lights which are

The name of the months of the year which are

The factors of 8 which are



Remark

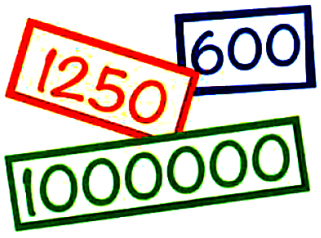
The collection in each of the previous examples is will defined, it means that we can determine exactly all objects of this collection.
So, according to the definition, it is considered as a set.

For Example : Each of the following **can not be** considered as **a set**

The beautiful flowers.

The small animals.

The big numbers.



Remark

The collection in each of the previous examples is not well defined, it means that we can not determine exactly the objects of this collection.
So, according to the definition, it can not be considered as a set.

Example1: Put "set" or "not set" in the blank.

[a] The digits of the number 2010 ——— set

[b] High buildings. ———

[c] Nice fruits. ———

[d] The Arabic alphabet. ———

[e] The letters in the name "Marwan". ———

[f] Beautiful songs.

[g] The prime numbers less than 15 ———

[h] The small trees in the street. ———

Elements of a set

The objects which a set contains are called 'The elements of the set".

For Example :

- The elements of the set of digits in the number 531 are : 5 , 3 and 1
- The elements of the set of days of the week are :

Sunday , Monday , Tuesday , Wednesday , Thursday , Friday and Saturday.

Exercises

1 **State which of the following is a set or not and why ?**

a The colours of the Egyptian flag.----------

b 🕮 The beautiful stories.----------

c Rainbow colours.----------

d Digits of the number 1982 ----------

e 🕮 Months in the Hejira calendar.----------

f 🕮 The letters of the word "Egypt".----------

g Arabic countries.----------

h Big numbers.----------

2 **State which of the following is a set or not and if it is a set, write its elements :**

[a] Prime numbers between 2 and 20 ----------

[b] Digits of the number 30043 ----------

[c] The name of the clever students in your class. ----------

[d] Letters in the word "Egypt". ----------

3 **Write all the elements of each of the following sets :**

a 🕮 The set of the digits in the number 3072 ----------

b The set of letters of the word "element". ----------

c 🕮 The set of the colours in the Egyptian flag ----------

d The set of the main directions. ----------

e The set of odd numbers less than 10 ----------

f The set of whole numbers between 5 and 15 ----------

g 🕮 The set of the days of the week. ----------

h 🕮 The set of the year's months that have less than 30 days.

Mathematical expression of a set

Mathematical expression of a set

There are **two ways** of expressing a set

1. The listing method.
2. The description method.

FIRST The listing method

To express a set using the listing method , write all the elements of the set between **two braces** as { } , then place a **comma** "," between every two elements.

For Example :

- The set of **letters in the word "boy"** is {b , o , y}

Remarks

1) Sets are denoted with capital letter as; X, Y, Z, A,

For Example:

If A is the set of even numbers between 3 and 10 , then : A = {4,6,8}

2) We do not repeat elements when we list the elements of a set.

For Example:

The set of digits of the number 2010 is {2,0,1} not {2,0,1,0}

3) The order of elements is not important in the set , so the elements of a set may be written in any order.

For Example:

The set of digits of the number 135 is {1, 3,5} or {3,5,1} Or {5,3,1}

Example 1

Express each of the following sets by using the listing method :

[a] A = the set of colours of the Egyptian flag. .....

[b] B = the set of 1-digit odd numbers. .....

[c] X = the set of letters in the word "mathematics". .....

[d] Y = the set of digits in the number 7 .....

[e] Z = the set of even numbers. .....

SECOND The description method

To express a set using the description method , we define the property which distinguishes and determines the elements of this set.

For Example :

If X = {r , a , t} , then we can express the set X as one of the following :

- X = the set of letters of the word "rat".
- X = the set of letters of the word "art".

Remark

The previous set X can be written in the following form :

X = {x : x is one of the letters of the word "art"}

It is read as :

X is a set of each x where x is one of the letters of the word "art".

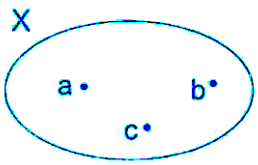
Representing sets by Venn diagrams

We represent every element of a set by putting a point or a mark as "X" , then we surround them by a suitable geometric shape as a triangle , a square , a circle or any closed curve

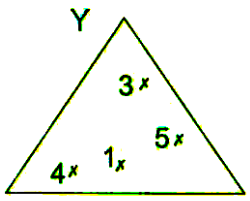
Example :

The following gives some figures of **Venn diagrams** for some sets :

X = {a , b , c}



Y = the set of digits of the number 4315



Z = the set of days of the week

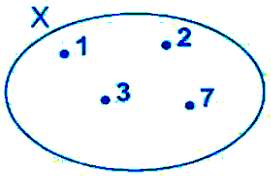
Z

- Saturday
- Sunday
- Monday
- Tuesday
- Wednesday
- Thursday
- Friday

Example 4

Complete the following :

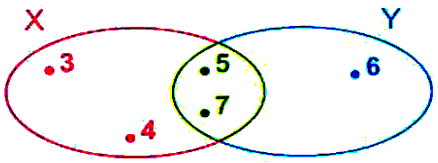
1 In the opposite figure :



[a] X = ... "by listing method"

[b] X = ... "by description method"

2 In the opposite figure :



[a] X = {.....}

[b] Y = {.....}

[c] The set of elements found in X and Y is {.....}

Belonging of an element to a set

If a is an element of a set X , we can write $a \in X$, the symbol $\in$ means "belong to".

For Example: If X is the set of colors of the Egyptian flag, then: X = {red , white , black} and we symbolize that as : red $\in$ X

$\in$ denotes "the **belonging** of an element to a set".

Remark:

$\notin$ denotes "the **not belonging** of an element to a set".

Example 5

Put the suitable sign "∈ or ∉":

[a] 2 ..... {2} , while 2 ..... {22}

[b] 6 ..... {6 , 36} , while 6 ..... {16 , 36}

[c] b ..... the set of letters of the word "book" , while b ..... {book}

[d] 25 ..... the set of digits of the number 2525

[e] 15 ..... {1 , 5}

[f] 7 ..... the set of days of the week.

Types of sets There are **three types of sets**

1. Finite set — 2. Infinite set — 3. Null set

(1) Finite set: A finite set is a set has a limited number of elements.

I.e. The number of its elements can be listed

Examples of finite sets:

• The set {3 ? 5} is a finite set because the number of its elements is 2

(2) Infinite set: An infinite set is a set has an unlimited number of elements,

Examples of Infinite sets:

The set of even numbers = {0,2 ,4,6 ,8 , }

(3) The null (empty) set: The null set is the set that has no elements. It is denoted by symbol { } or **ϕ** which is read as "phi"

Examples of a null set: The set of the whole numbers between 7 and 8

Example:

Put (✓) in front of each set to get the correct choice :

The set	Finite	Infinite	Empty
[a] {5 , 2 , 4 , 0}	.....	.....	.....
[b] The set of odd numbers	.....	.....	.....
[c] The set of even numbers between 4 and 6	.....	.....	.....

Inclusion and subsets

If all the elements of set A are also elements of set B , then A is subset of B

$(A \subset B)$

For Example :

If **A** = {**1** , **2** , **3**} and

B = {**1** , **2** , **3** , **4**}

Then "**A** is a subset of **B**" or

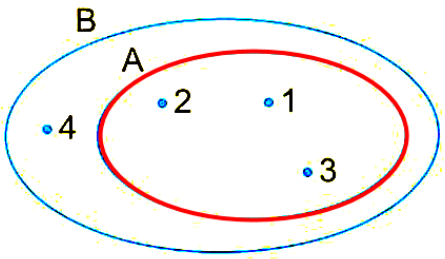
"**A** is included in B" and we write

it as "**A** $\subset$ **B**" where the

symbol "$\subset$" denotes the inclusion

of one set in another set.

We represent that by the following Venn diagram.



The set A lies completely inside the set B

$A \subset B$

Remark

"C is not a subset of B" if there is at least one element in C but not an element in B and we write "$C \not\subset B$"
where the symbol "$\not\subset$" denotes the non inclusion of a set in another set.

For Example :

If B = {2 , 5 , 7 , 8} , A = {2 , 7} , C = {2 , 5 , 3} and

D = {2 , 5 , 7 , 8 , 9} , then :

- $A \subset B$ because each element in A is also an element in B
- $C \not\subset B$ because $3 \in C$ but $3 \notin B$
- $D \not\subset B$ because $9 \in D$ but $9 \notin B$

Remarks

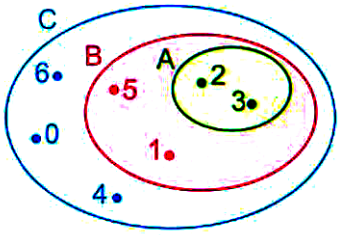
1. The empty set $\varnothing$ is a subset of any set
   For Example : $\varnothing \subset \{a , b , c\}$, $\varnothing \subset \{1 , 2 , 3 , \cdots\}$, $\varnothing \subset \{0\}$
2. Any set is a subset of itself "$X \subset X$"
   For Example : $\{1 , 2\} \subset \{2 , 1\}$

Example 6

In the opposite Venn diagram :

[a] List the elements of the three sets A , B and C

[b] Put the suitable sign (⊂ or ⊄) :



(1) A C	(2) A ..... B	(3) C ..... B
(4) B C	(5) B ..... A	(6) C ..... A

Remarks

1. The two symbols ∈ and ∉ refer to the **relation between an element and a set.**
2. The two symbols ⊂ and ⊄ refer to the **relation between two sets.**

Example 7

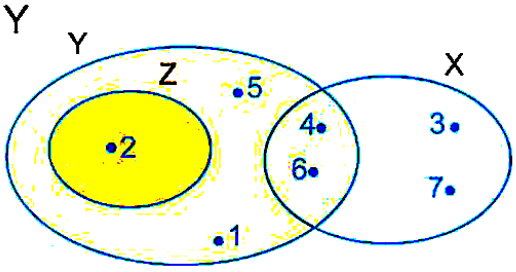
Put the suitable symbol " ∈ , ∉ , ⊂ or ⊄ " :

[a] 7 ..... {5 , 7 , 8 , 9}	**[b]** {5} ..... {55}
[c] 15 ..... {1 , 6 , 3 , 2}	**[d]** {6 , 2} ..... {16 , 62 , 12}
[e] {2 , 0} ..... {0 , 4 , 2 , 8}	**[f]** {5 , 2 , 0} ..... ∅
[g] ∅ ..... {0 , 2 , 3 , 4}	**[h]** {2 , 1 , 3} ..... {1 , 3 , 2}

Example 8

By using the opposite Venn diagram, complete by using the suitable sign " ⊂ , ⊄ , ∈ or ∉ " :

a 3 X	**b** {1, 4} ..... Y
c {1, 6} X	**d** 5 ..... Z
e Z ..... Y	**f** X ..... Y



Equality of Two Sets

If every element of set A belongs to set B , and every element of set B belongs to set A

Therefore : $A = B$

For example : If $A = \{5 , 2 , 4\}$ and $B = \{2 , 4 , 5\}$, then $A = B$

Example 6 **Find the value of each x and y in each of the following :**

1 $\{4 , 3\} = \{3 , x\}$ .....

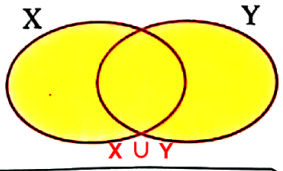
2 $\{3 , x\} = \{5 , y\}$ .....

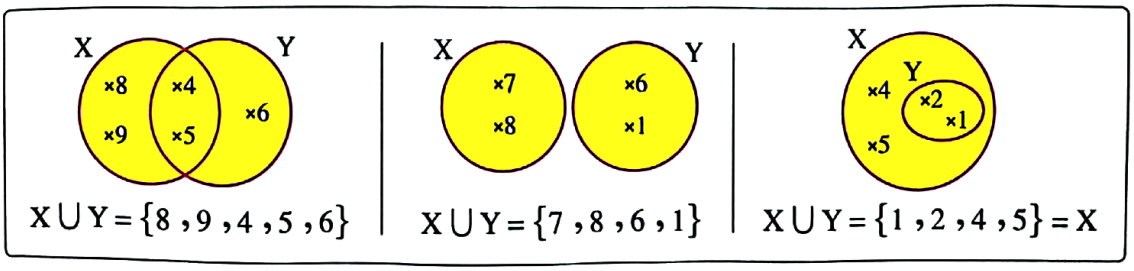
3 $\{2 , 3 , y - 5\} = \{7 , 3 , x + 1\}$ .....

Operations on Sets (Union - Intersection)

First The Union of Two Sets

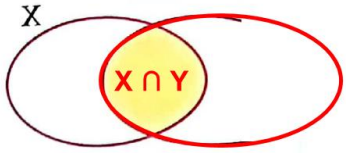
$X \cup Y$ is the set of all elements that belong to either set X or set Y

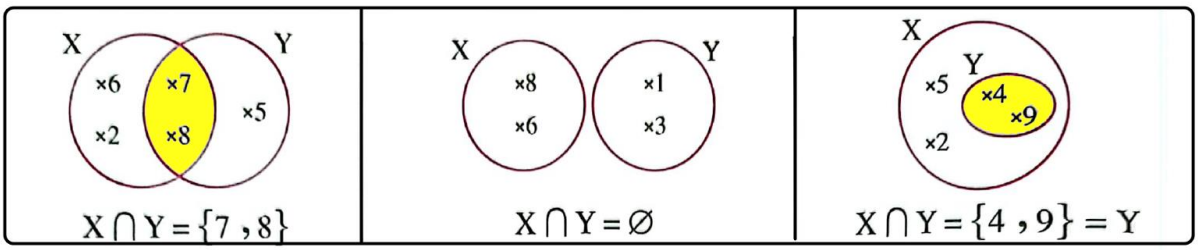




Second The Intersection of Two Sets

$X \cap Y$ is the set of all elements that are common to both sets X and Y

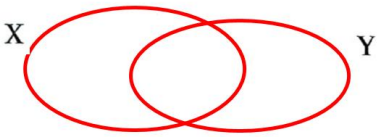




Example 7

If $X = \{7 , 8\}$ and $Y = \{1 , 2 , 7\}$, then represent X and Y by a Venn diagram

, **and find :** $X \cap Y$ and $X \cup Y$



$X \cap Y$

$X \cup Y$

Remarks:

- For any two sets A and B :

(1) If $A \subset B$, then $A \cap B = A$ and $A \cup B = B$

(2) $A \cup \varnothing = A$ and $A \cap \varnothing = \varnothing$

(3) If $A = B$, then $A \cup B = A$ or B

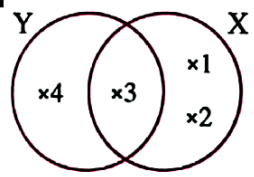
, and $A \cap B = A$ or B

- If A and B are two non-empty sets and $A \cap B = \varnothing$, then the two sets A and B are disjoint (non - intersecting)

Exercise :(1)

Complete the following :

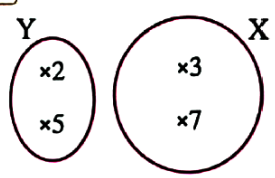
1



$X \cap Y =$ .....

$X \cup Y =$ .....

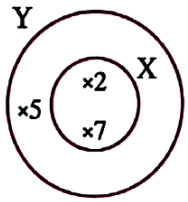
2



$X \cap Y =$ .....

$X \cup Y =$ .....

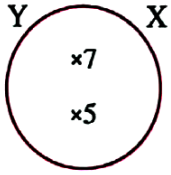
3



$X \cap Y =$ .....

$X \cup Y =$ .....

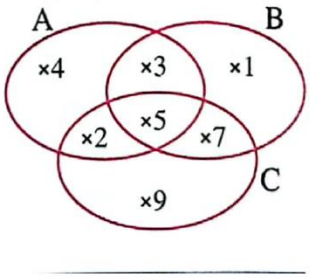
4



$X \cap Y =$ .....

$X \cup Y =$ .....

From the opposite Venn diagram , find :



A= { ..... }

B= { ..... } C= { ..... }

$A \cap B =$ .....

$A \cup C =$ .....

$A \cap (B \cup C) =$ .....

$A \cap B \cap C =$ .....

Homework

1 **Put the suitable symbol of : $\subset, \not\subset, \in, \notin$ in the blank :** (where $\mathbb{N}$ is the set of natural numbers , $\mathbb{Z}$ is the set of integers and $\mathbb{Q}$ is the set of rational numbers)

1. $\{2, 3\}$ ..... $\{1, 2, 3\}$
2. $\{1, 2\}$ ..... $\{2, 3, 4\}$
3. b ..... $\{b, c\}$
4. 0 ..... $\varnothing$
5. $\varnothing$ ..... $\{7, 9\}$
6. $\{0\}$ ..... $\varnothing$
7. 11 ..... $\{211\}$
8. 99 ..... $\{99\}$
9. $\{3, 5, 6\}$ ..... $\{3, 5\}$
10. $\{-2, 5\}$ ..... $\mathbb{N}$
11. $\frac{2}{5}$ ..... $\{2, 5\}$
12. $\{4, 7, 0\}$ ..... $\mathbb{Z}$
13. $\{\frac{1}{2}, 9, -1\}$ ..... $\mathbb{Q}$
14. $\{\frac{3}{4}, 5, -1\}$ ..... $\mathbb{Z}$
15. $\{5, -2\}$ ..... $\{-2, 5\}$
16. $\mathbb{N}$ ..... $\mathbb{Q}$
17. $\mathbb{Q}$ ..... $\mathbb{Z}$
18. $\mathbb{Z}$ ..... $\mathbb{Z}$

2 If $A = \{5, 2, 3, 7\}$, $B = \{4, 5, 7\}$, and $C = \{7, 2\}$, **find :**

1. $A \cap B \cap C$ .....
2. $A \cup B \cup C$ .....
3. $(A \cup B) \cap C$ .....

Choose the correct answer from the given:

(1) **If A={2,5,8} , which of the following is correct?**

(a) $\{2\} \in A$ (b) $\{3\} \notin A$ (c) $\{5\} \subset A$ (d) $\{5,8\} \not\subset A$

(2) **If A={2,4,7} and B⊂A , which of the following could be set B?**

(a) {6,4} (b) {5,3,1} (c) {7,3,4} (d) {2,7}

(3) **If A={2,6,7} and B={8,9,6} , then A∪B =....**

(3) {6} (3) {2,7} (3) {8,9} (3) {9,8,7,6,2}

(4) **If A = { 5 , 2 , 3 , 7 } and B = { 7 , 2 } , then $A \cap B$ =.....**

(a) {2 } (b) {2,7} (c) {2,5} (d) {2,3,5,7}

Lesson (4) + Operations on Integers +

The set of

1) Counting numbers $= \{1, 2, 3, \dots \dots \dots\}$

2) Natural numbers $(\mathbb{N}) = \{0, 1, 2, 3, \dots \dots \dots\}$

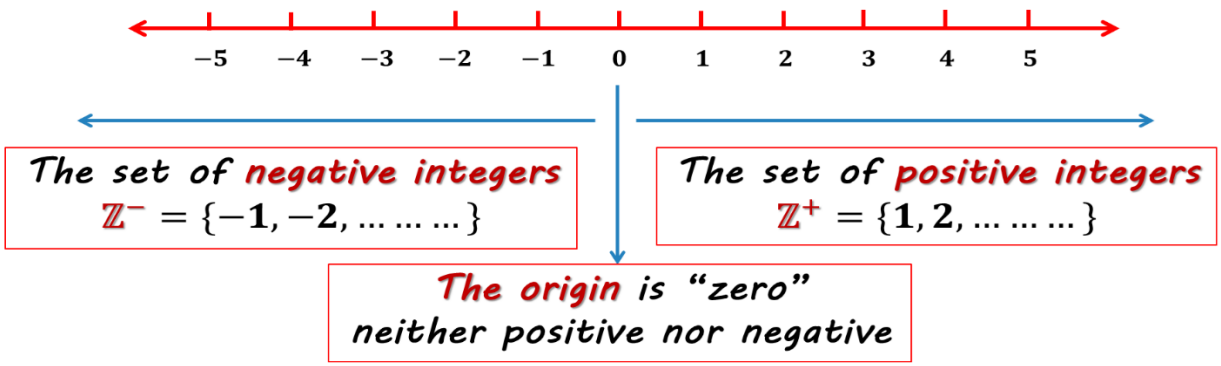
3) Integers $(\mathbb{Z}) = \{\dots \dots \dots, -3, -2, -1, 0, 1, 2, 3, \dots \dots \dots\}$

The set of integers $(\mathbb{Z})$:

The set of positive integers $(\mathbb{Z}^+) = \{1, 2, 3, \dots \dots \dots\}$ (more than zero)

Zero is neither positive nor negative.

The set of negative integers $(\mathbb{Z}^-) = \{-1, -2, -3, \dots \dots \dots\}$ (less than zero)



Notes:

- **The set of non-negative integers** $= \{0, 1, 2, 3, \dots \dots \dots\}$
- **The set of non-positive integers** $= \{0, -1, -2, -3, \dots \dots \dots\}$
- **Fractions** & **decimals** which are not whole numbers **are not integers**.

$$3.4 \notin \mathbb{Z} \quad \& \quad \frac{17}{6} \notin \mathbb{Z}$$

- You can represent integers on **a horizontal** or **a vertical** number line.
- Every integer can be represented by **one point on the number line**.
- The point that represents **zero** is called **"the origin"**.

Addition

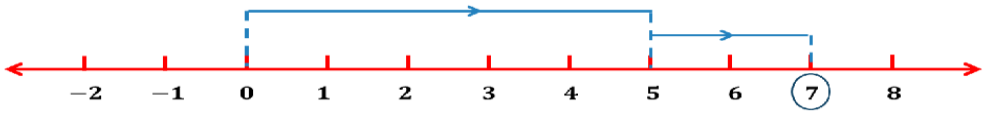
Number line strategy:

- Draw a number line.
- Find the location of the first integer on the number line.
- If the second integer is:

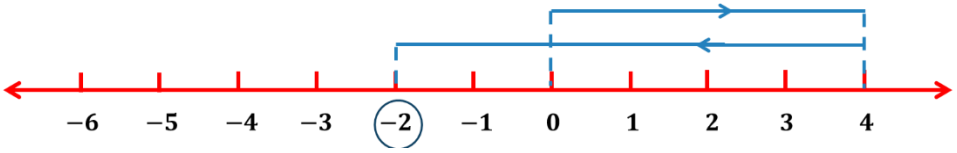
  Positive moves many units to the **right** from the location of the first integer.

  Negative moves many units to the **left** from the location of the first integer.

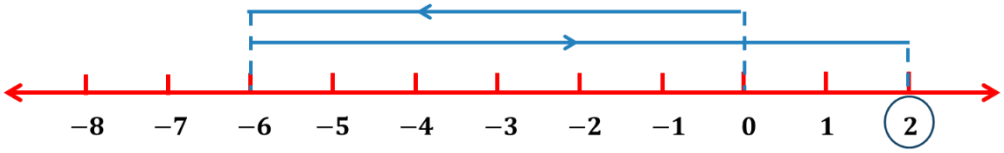
a) $5 + 2 = 7$



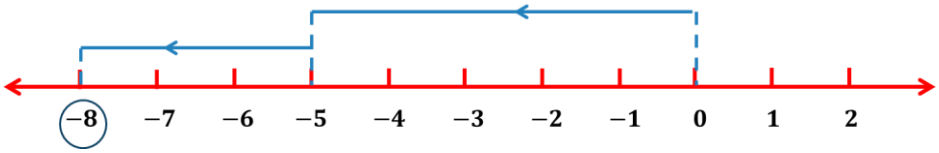
b) $4 + (-6) = -2$



c) $-6 + 8 = 2$



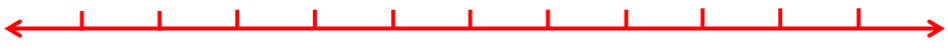
d) $-5 + (-3) = -8$



Ex.1 Using the number line to find the result of:

e) $3 + 5 =$ .....

f) $-4 + (-2) =$ .....



g) $-10 + 3 =$ .....

Absolute value strategy:

- If two integers have **the same sign**, **add** the absolute value of two integers & the result **has the same sign** of two integers.

Example: $6 + 8 = 14$, $-3 + (-4) = -7$

- If two integers have **different signs**, subtract the absolute value of two integers & the result has **the sign of the number with the greater absolute value**

. **Example:** $-4 + 10 = 6$, $3 + (-8) = -5$

Ex.2 Using the absolute value to find the result of:

a) $-20 + 8 =$	b) $5 + (-3) =$ .....
c) $-15 + (-3) =$	d) $-17 + 17 =$ .....
e) $0 + (-9) =$	f) $-5 + 0 =$ .....
g) $7 + 5 =$	h) $-9 + \|-5\| =$ .....
i) $-13 + 4 =$	j) $6 + (-13) =$ .....

Subtraction

Subtracting "b" from "a" means: $a - b = a + (-b)$

Subtracting "a" from "b" means: $b - a = b + (-a)$

Number line strategy:

- Draw a number line.
- Find the location of the first integer on the number line.
- If the second integer is:

  Positive moves many units to the **right** from the location of the first integer.

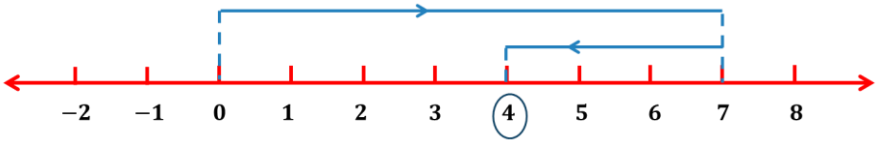
  Negative moves many units to the **left** from the location of the first integer.

- When we subtract two negative integers then the operation turns into addition.

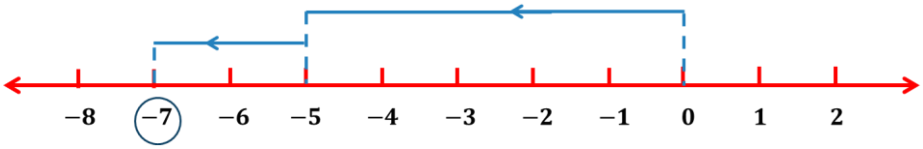
$-8 - (-2) = -8 + 2 = -6$

Example:

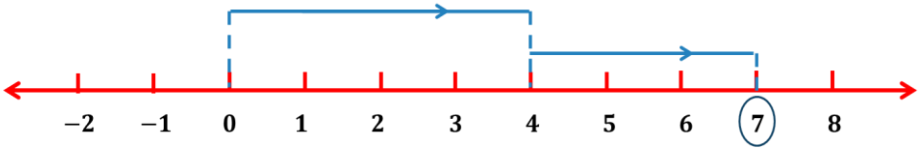
a) $7 - 3 = 4$



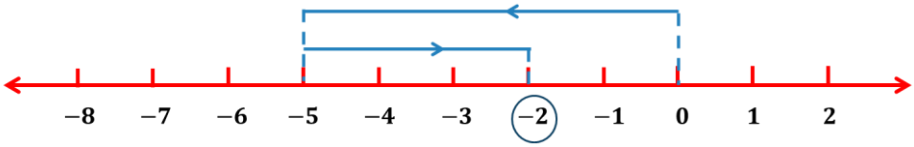
b) $-5 - 2 = -7$



c) $4 - (-3) = 7$



d) $-5 - (-3) = -2$



Ex.3 Using the number line to find the result of:

e) $10 - 7 =$ .....

f) $-12 - 3 =$ .....

g) $-2 - (-1) =$ .....

h) $13 - (-7) =$ .....

Absolute value strategy:

- If two integers have **the same sign**, **add** the absolute value of two integers & the result **has the same sign** of two integers.

Example: $-6 - 8 = -14$, $4 - (-6) = 10$

- If two integers have **different signs**, subtract the absolute value of two integers & the result has **the sign of the number with the greater absolute value**.

Example: $4 - 10 = -6$, $16 - 9 = 7$

30

Ex.4 Using the absolute value to find the result of:

a) $-20-8=$ .....

b) $5-(-3)=$ .....

c) $-15-(-2)=$ .....

d) $-17-17=$ .....

e) $0-(-9)=$ .....

f) $-5-0=$ .....

g) $7-5=$ .....

h) $-9-(-5)=$ .....

i) $-13-|-4|=$ .....

j) $6-(-13)=$ .....

Properties of addition & subtraction

Operations / Properties	Addition	Subtraction
	If $a, b, c \in \mathbb{Z}$ (are integers)	
Closed	$a+b \in \mathbb{Z}$	$a-b \in \mathbb{Z}$
Commutative	$a+b=b+a$	$a-b \neq b-a$
Associative	$(a+b)+c=a+(b+c)$	$(a-b)-c \neq a-(b-c)$
Additive identity	is zero $a+0=0+a=a$	
Additive inverse	$a+(-a)=(-a)+a=0$	

Note: The additive inverse of zero is itself.

Ex.5 Using the properties of addition to find the result of:

a) $-3+6+3=-2+2+(-8)$ (Commutative)

$=(-2+2)+(-8)$ (Associative)

$=0+(-8)$ (Additive inverse)

$=-8$ (Additive identity)

b) $24 + (-19) + (-24) + 9 = 24 + (-24) + (-19) + 9$ **(Commutative)**

$= (24 + (-24)) + (-19 + 9)$ **(Associative)**

$= 0 + (-10)$ **(Additive inverse)**

$= -10$ **(Additive identity)**

c) $-3 + 6 + 3 =$

.....

.....

.....

d) $25 + (-8) + (-25) + 7 =$

.....

.....

.....

e) $-32 + 40 + 32 + (-40) =$

.....

.....

.....

f) $-7 + 2 + (-13) =$

.....

.....

.....

32

g) $5 + (-3) + 7 + (-9) =$

.....

.....

.....

Ex.6 Complete:

a) $4 + (-3) = -3 +$ .....

b) $-15 +$ ..... $= 0$

c) ..... $+(-7) = -7$

d) $0 +$ ..... $= |-9|$

e) $-3 + ($ ..... $+10) = -3$

f) $(-6 + 4) + 9 = -6 + ($ ..... $+9)$

g) $-7 + (-5) = -5 -$ .....

h) $(-3 + 5) + (-2) = -3 + (5 -$ ..... $)$

i) Subtract -7 from -3 is .....

j) Subtract -8 from 2 is .....

k) The additive inverse of 10 is ..... & the additive inverse of $(-2)^3$ is .....

l) The additive inverse of zero is .....

m) The additive inverse of $-|-3|$ is .....

n) The additive identity in $\mathbb{Z}$ is .....

Ex.7 State the name of the property of the addition of integers used in each statement:

a) $-4 + 2 = 2 + (-4)$ (.....)

b) $-8 + (-2 + 3) = \big(-8 + (-2)\big) + 3$ (.....)

c) $0 + (-7) = -7$ (.....)

d) $b + (-b) = 0$ (.....)

e) $4 + (-12) = -8 \in \mathbb{Z}$ (.....)

Ex.8 If $a, b \in \mathbb{Z}$ such that $a - b = 5$ & $3 - (-b) = -7$ Find the value of $a + b$

.....

.....

.....

Ex.9 In Saint Catherine (South Sinai), the maximum temperature recorded on one of the winter days was $11℃$ then during the night, it dropped by $15℃$ becoming the lowest temperature for this city. Find this temperature using integer addition.

.....

Ex.10 A submarine is at a depth of 90 meters below sea level. It rises 60 meters upwards. Find the new depth of the submarine.

.....

Homework

1) Using the number line to find the result of:

a) $3 - 9 =$ .....

b) $-8 + (-3) =$ .....

c) $-4 + 3 =$ .....

2) Using the absolute value to find the result of:

a) $-6 + 4 =$ .....

b) $7 - (-1) =$ .....

c) $-3 - (-8) =$ .....

d) $|-17| + 17 =$ .....

e) $5 + (-5) =$ .....

f) $0 + (-5) =$ .....

g) $-13 + |-2| =$ .....

h) $-9 + (-2) =$ .....

i) $|-14| - |-28| =$ .....

j) $33 - |-11| =$ .....

3) Using the properties of addition to find the result of:

a) $10 + (-13) + (-10) + 13 =$

.....

.....

.....

b) $2025 + 13 + (-2025) =$

.....

.....

.....

c) $-8 + 3 + (-10) =$

.....

.....

.....

d) $5 + (-4) + 6 + (-7) =$

.....

.....

.....

4) State the name of the property of the addition of integers used in each statement:

a) $-7 + 8 = 8 + (-7)$ (.....)

b) $-10 + (-4 + 2) = \big(-10 + (-4)\big) + 2$ (.....)

c) $(-3) + 0 = -3$ (.....)

d) $x + (-x) = 0$ (.....)

e) $-3 - 4 = -7 \in \mathbb{Z}$ (.....)

5) Complete:

a) $-11 + 2 = 2 +$ .....

b) $0 +$ ..... $= |-9|$

c) $-5 + (-4 +$ ..... $) = -5$

d) $(-6 + 4) + 9 = -6 + ($ ..... $+ 9)$

e) $-9 - (-5) =$ ..... $- 9$

f) $|-10| +$ ..... $= 0$

g) $3 +$ ..... $= -3$

h) Subtract -15 from -4 is .....

i) Subtract 4 from -5 is .....

j) $(-3 + 8) + (-4) = -3 + (8 -$ ..... $)$

k) The additive inverse of -4 is ..... & the additive inverse of $(-3)^2$ is .....

l) The additive inverse of zero is .....

m) The additive inverse of $-|7|$ is .....

6) If $a = 3, b = -4, c = -2$. Find the value of $a - b - c$

.....

.....

.....

Multiplication

Multiplication:

is a repeated addition

$$3 \times 4 = 3 + 3 + 3 + 3$$

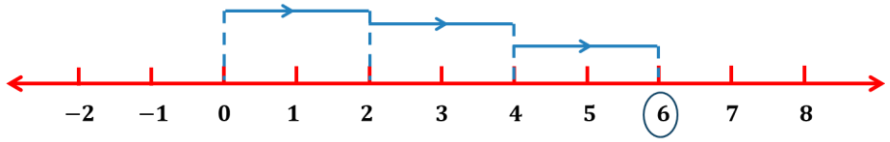
$$4 \times 3 = 4 + 4 + 4$$

Number line strategy:

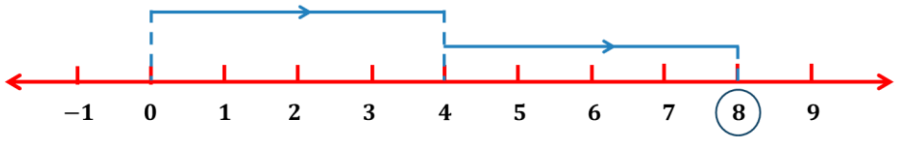
- Draw a number line.
- Find the location of the first integer on the number line.
- We will move **right** on the number line if both integers have the same sign but we will move **left** on the number line if both integers have different signs.
- If both integers are negative then the result is a positive integer.

Ex.1 Using the number line to find the product of:

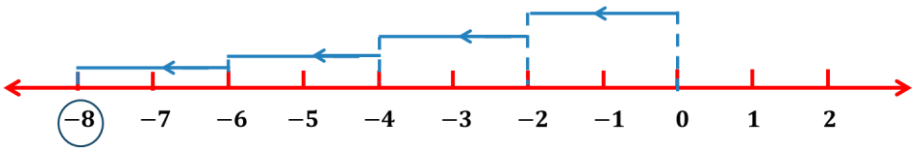
a) $2 \times 3 = 6$



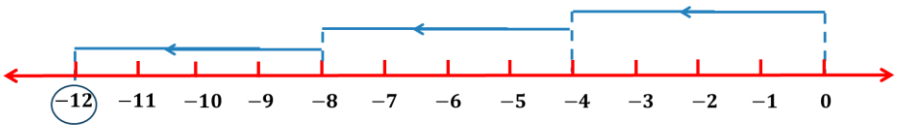
b) $-4 \times (-2) = 8$



c) $2 \times -4 = -8$



d) $-4 \times 3 = -12$



e) $3 \times 4 =$ .....

f) $-4 \times 3 =$ .....

g) $5 \times -2 =$ .....

h) $-2 \times (-4) =$ .....

Absolute value strategy:

- The product of two integers has **the same sign** as a **positive** integer.

Example: $6 \times 8 = 48$, $-3 \times (-4) = 12$

- The product of two integers has **different signs** as a **negative** integer.

Example: $-4 \times 10 = -40$, $3 \times (-8) = -24$

- The product of any integer by **zero** is **zero**.

Ex.2 Using the absolute value to find the product of:

a) $5 \times -3 =$ .....

b) $-15 \times (-2) =$ .....

c) $|-17| \times -1 =$ .....

d) $0 \times -9 =$ .....

e) $-(-2) \times (-8) =$ .....

f) $-1 \times -18 =$ .....

g) $-4 \times 2 =$ .....

h) $-10 \times |-2| =$ .....

i) $-(-12) \times -4 =$ .....

J) $6 \times (-5) =$ .....

Division

Absolute value strategy:

- The quotient of two integers has **the same sign** as a **positive** integer.

Example: $64 \div 8 = 8$, $-12 \div (-4) = 3$

- The quotient of two integers has **different signs** as a **negative** integer.

Example: $-40 \div 5 = -8$, $63 \div (-9) = -7$

- The quotient of any integer by **zero** is **meaningless**.

Ex.3 Using the absolute value to find the product of:

a) $54 \div -6 =$ .....

b) $-16 \div 4 =$ .....

c) $-33 \div (-11) =$ .....

d) $0 \div (-2) =$ .....

e) $-(-6) \div 6 =$ .....

f) $-8 \div 0 =$ .....

g) $|-4| \div 2 =$ .....

h) $-10 \div |-2| =$ .....

i) $-(-20) \div -4 =$ .....

j) $40 \div (-5) =$ .....

Properties of multiplication & division

Operations / Properties	Multiplication	Division
	If $a, b, c \in \mathbb{Z}$ (are integers)	
Closed	$a \times b \in \mathbb{Z}$	$a \div b \notin \mathbb{Z}$
Commutative	$a \times b = b \times a$	$a \div b \neq b \div a$
Associative	$(a \times b) \times c = a \times (b \times c)$	$(a \div b) \div c \neq a \div (b \div c)$
Multiplicative identity	is one $a \times 1 = 1 \times a = a$	
Distribution on addition & subtraction	$a \times (b + c) = (a \times b) + (a \times c)$ $a \times (b - c) = (a \times b) - (a \times c)$	

Ex.4 Using the properties of multiplication to find the product of:

a) $-4 \times -19 \times 25 = -4 \times 25 \times (-19)$ (Commutative)

$= (-4 \times 25) \times (-19)$ (Associative)

$= -100 \times (-19) = 1900$ (Closure)

$2 \times 5 = 10$

$4 \times 25 = 100$

$8 \times 125 = 1000$

b) $8 \times 2 \times 125 \times (-50) = 8 \times 125 \times 2 \times (-50)$ (Commutative)

$= (8 \times 125) + \big(2 \times (-50)\big)$ (Associative)

$= 1000 \times (-100) = -100000$ (Closure)

c) $20 \times 16 \times -5 =$

.....

.....

d) $-8 \times -3 \times -125 =$

.....

.....

.....

e) $-2 \times (-25) \times (-50) \times 40 =$

.....

.....

.....

f) $-2 \times 80 \times 5 \times -125 =$

.....

.....

.....

Ex.5 Using the distributive property of multiplication to find the product of:

a) $4 \times 2 + 4 \times 8 = (4 \times 2) + (4 \times 8) = 4 \times (2 + 8) = 4 \times 10 = 40$

b) $3 \times 4 - 3 + 3 \times 7 = (3 \times 4) - (3 \times 1) + (3 \times 7) = 3 \times (4 - 1 + 7) = 30$

c) $12 \times 103 = 12 \times (100 + 3) = (12 \times 100) + (12 \times 3) = 12200 + 36 = 1236$

d) $-13 \times 98 = -13 \times (100 - 2) = (-13 \times 100) — 13 \times 2 = 1300 - 26 = 1274$

e) $(-3) + 3 \times 5 =$

.....

.....

f) $45 \times (-6) + (-47) \times (-6) + (-6) =$

.....

.....

.....

g) $-14 \times 1001 =$

.....

.....

.....

h) $-9 \times 102 =$

.....

.....

.....

Ex.6 Complete:

a) $-8 \times \ldots\ldots = 0$

b) $\ldots\ldots \div 7 = -6$

c) $6 \div \ldots\ldots =$ meaningless

d) $\ldots\ldots \times -9 = 27$

e) The multiplicative identity in $\mathbb{Z}$ is .....

f) The product of any two integers with different signs is a ..... integer.

g) If $a = 3, b = -2$ then the value of $3ab =$ .....

h) If $a \times b = a, a \neq 0$ then $b =$ .....

i) If $a \div b = a, ab \neq 0$ then $b =$ .....

Ex.7 State the name of the property of the multiplication of integers used in each statement:

a) $-4 \times 2 = 2 \times (-4)$ (.....)

b) $-2(6 - 9) = (-2 \times 6) - (-2 \times 9)$ (.....)

c) $-8 \times (-2 \times 3) = \big(-8 \times (-2)\big) \times 3$ (.....)

d) $1 \times (-7) = -7$ (.....)

Ex.8 If $x = 2, y = 1$ and $z = -5$ Find the value of:

a) $(3x - y) \div z =$ .....

.....

b) $4x - 2y + z =$ .....

.....

Ex.9 **The submarine moves vertically downwards under the water. When it reached a depth of 84 meters, the submarine started to rise again. Determine its location after 20 minutes if it was moving upwards at a speed of 4 meters/minute.**

.....

Homework

1) Using the number line to find the product of:

a) $3 \times 9 =$ .....

b) $-8 \times (-3) =$

c) $-4 \times 3 =$ .....

d) $2 \times (-7) =$ .....

2) Using the absolute value to find the result of:

a) $-6 \times 4 =$ .....

b) $7 \div (-1) =$ .....

c) $-3 \div (-3) =$ .....

d) $|-17| \times 2 =$ .....

e) $5 \times (-6) =$ .....

f) $0 \div (-5) =$ .....

g) $-12 \div |-2| =$ .....

h) $-9 \times -4 =$ .....

i) $|-4| \times |-20| =$ .....

j) $|-11| \div 0 =$ .....

3) Using the properties of multiplication to find the product of:

a) $2 \times 6 \times (-5) \times 4 =$

.....

.....

b) $-125 \times -3 \times 8 =$

.....

.....

4) Using the distributive property of multiplication to find the product of:

a) $-4 \times (-5) + 2 \times (-5) =$

.....

.....

b) $6 \times 102 =$

.....

.....

c) $4 \times 3 - 13 \times 3 - 3 =$

.....

.....

d) $-2 \times 97 =$

.....

.....

5) Complete:

a) The multiplicative identity in $\mathbb{Z}$ is .....

b) If $x = -4$, $y = -3$ then the value of $7xy =$ .....

c) The product of any two integers with same sign is a ..... integer.

d) ..... $\times -4 = 0$

e) $6 \div$ ..... = meaningless

f) $-35 \div$ ..... $= 5$

g) $-9 \times$ ..... $= 27$

h) ..... $\times (-4) = 32$

i) ..... $\div (-3) = -5$

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6) State the name of the property of the addition of integers used in each statement:

a) $(-15-4)\times -3=(-3\times -15)-(-3\times 4)$ (.....)

b) $3\times(-5)=(-5)\times 3$ (.....)

c) $-10\times 1=-10$ (.....)

d) $(3\times -8)\times 4=3\times\big((-8)\times 4\big)$ (.....)

7) If $a=|-3|$, $b=-4$, $c=2$. Find the value of:

a) $(ab)\div(-c)=$ .....

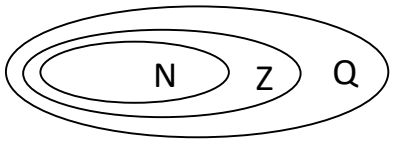
.....

b) $4ac-3b=$ .....

.....

Lesson (5) Operations on Rational Numbers

The set of rational numbers " Q " = { x : x = $\frac{a}{b}$; a, b ∈ Z , b ≠ 0 }



- N ⊂ Z ⊂ Q .
- Q* = Q - { 0 }

Different forms of a rational number:

- $\frac{3}{4}$ = 0.75 (finite decimal)
- $\frac{3}{4}$ = % 75 (percentage)
- $\frac{1}{3}$ = 0.333333..... = $0.\dot{3}$ (infinite decimal)

1) Express each number as a rational number $\frac{a}{b}$:

a) $0.\dot{5}$ = ...

b) $1.\dot{2}\dot{5}$ = ...

c) - $0.31\dot{6}$ = ...

d) $2.3\dot{9}$ = ...

e) $0.\dot{4}0\dot{7}$ = ...

f) 0.75 % = ...

g) |– 3 | = ...

h) - |– 7 | = ...

i) 0 = ...

---

1 Write each of the following numbers as a percentage :

1) $\frac{4}{5}$ 2) $\frac{3}{1000}$ 3) $2\frac{1}{2}$ 4) 6.7

2 Write each of the following numbers as a decimal :

1) $\frac{3}{4}$ 2) $\frac{11}{20}$ 3) $-5\frac{7}{8}$

1 Addition Operation of Rational Numbers

- If $\frac{a}{b}$ and $\frac{c}{b}$ are two rational numbers with equal denominators , then $\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}$
- If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers with different denominators , then $\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$

Add :

1) $-\frac{5}{6}+\frac{1}{6}$ 2) $\frac{3}{8}+\frac{1}{4}$ 3) $3+\frac{2}{5}$

4) $3\frac{1}{4}+\left(-2\frac{1}{5}\right)$ 5) $0.\overline{3}+\left(-\frac{10}{15}\right)$

Add each of the following :

1 $\frac{1}{5} + \frac{2}{5}$ | 2 $\frac{2}{5} + \frac{1}{3}$ | 3 $\frac{1}{2} + \left(-\frac{5}{6}\right)$

4 $-\frac{3}{4} + \frac{1}{5}$ | 5 $\frac{8}{12} + \left(-\frac{15}{18}\right)$

Properties of Addition Operation in $\mathbb{Q}$:

The properties of addition operation in $\mathbb{Q}$ are the same as the properties of addition operation in $\mathbb{Z}$, which are as follows :

1 Closure. 2 Commutative. 3 Associative.

4 Existence of Additive Identity(0). 5 Existence of Additive Inverse.

1 Use the properties of addition in $\mathbb{Q}$ to find the result of : $\frac{4}{5} + \left(-\frac{3}{7}\right) + \frac{1}{5} + \frac{3}{7}$

2 Choose the correct answer from the given ones :

1) The additive inverse of the number $-\frac{3}{4}$ is .....

a. $\left|-\frac{3}{4}\right|$ b. $-\frac{4}{3}$ c. $-\frac{3}{4}$ d. $-\left|-\frac{3}{4}\right|$

2) If $\frac{2}{3} + x = \frac{2}{3}$, then $x =$ .....

a. $-\frac{2}{3}$ b. 0 c. 1 d. $\frac{2}{3}$

2 Subtraction Operation of Rational Numbers

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers , then subtracting $\frac{c}{d}$ from $\frac{a}{b}$ is adding $\frac{a}{b}$ to the additive inverse of $\frac{c}{d}$

i.e. $\frac{a}{b} - \frac{c}{d} = \frac{a}{b} + \left(-\frac{c}{d}\right)$

Find the result of each of the following :

1 $7\frac{2}{5} - 3\frac{1}{4}$ | 2 $\frac{5}{7} - 1$ | 3 $-3\frac{1}{2} - 2\frac{1}{4}$

4 $25\,\% - \left|-\frac{3}{5}\right|$ | 5 $-0.\overline{3} - \frac{1}{4}$

Homework

1 Complete the following :

1. $\frac{7}{20}$ = ... %
2. $\frac{21}{1000}$ = ... %
3. $|-0.4|$ = ... %
4. If $\frac{5}{a}$ is a rational number , then a ≠ .....
5. $0.\overline{2}$ = ... (in the form of $\frac{a}{b}$)
6. $0.\overline{123}$ = ... (in the form of $\frac{a}{b}$)
7. $0.3\overline{28}$ = ... (in the form of $\frac{a}{b}$)
8. $\frac{5}{8}$ = ... (in the decimal form)
9. $\frac{7}{20}$ = ... (in the decimal form)
10. $\frac{1}{3}$ = ... (in the decimal form)
11. The additive identity in $\mathbb{Q}$ is .....
12. The additive inverse of $\frac{3}{7}$ is .....
13. The additive inverse of $-\frac{4}{9}$ is .....
14. The additive inverse of -2.3 is .....
15. $\frac{-6}{-11}$ is the additive inverse of .....
16. The additive inverse of $(-2)^3$ is .....
17. The additive inverse of $|-\frac{4}{5}|$ is.....
18. The additive inverse of zero is.....

2 Find the result of each of the following :

1. $\frac{2}{7} + \frac{3}{7}$
2. $\frac{5}{8} + (\frac{-7}{8})$
3. $\frac{5}{9} + |-\frac{4}{9}|$
4. $-3.2 + (-1.\overline{3})$
5. $\frac{1}{5} - \frac{2}{3}$
6. $-\frac{2}{5} - \frac{3}{15}$
7. $-\frac{9}{12} + \frac{3}{16}$
8. $-\frac{15}{18} + \frac{12}{16}$
9. $\frac{3}{7} - (-\frac{2}{5})$
10. $\frac{1}{4} + 2\frac{3}{8}$
11. $3\frac{1}{6} - 6\frac{2}{3}$
12. $2\frac{3}{5} - \frac{1}{2}$
13. $-2.125 + 2.6 + (-7\frac{21}{24})$
14. $60\,\% + \frac{3}{10} - 0.14$
15. $\frac{2}{3} - 0.\overline{3} + 2$

3) **Using the properties of addition in $\mathbb{Q}$, find the result of each of the following in its simplest form :**

1. $\frac{3}{4} + \frac{1}{2} + \frac{1}{4}$
2. $\frac{2}{7} + \frac{3}{4} + \frac{5}{7} + \frac{1}{4}$
3. $\frac{5}{4} + (-\frac{13}{5}) + (-\frac{25}{4}) + \frac{28}{5}$
4. $\frac{5}{8} + (-\frac{3}{4}) + \frac{3}{8} + \frac{3}{4}$

4) Find $m - n$ if $n = 3\frac{3}{5}$ and $m = -7\frac{2}{3}$

5) Find $a + b$ if $b = 2.2$ and $a = -3\frac{1}{4}$

6) **If $x = \frac{5}{6}$, $y = -\frac{1}{3}$ and $z = \frac{1}{2}$, calculate the value of each of the following :**

1. $x + z$
2. $x + y$
3. $x - y$
4. $(z + y) - x$

Choose the correct answer from the given ones :

1 $\frac{3}{4}$ + 50 % = ...

(a) 75 % (b) 150 %

(c) $\frac{5}{4}$ (d) $\frac{3}{2}$

2 The result of adding : $\frac{1}{5} + \left(-\frac{6}{5}\right)$ equals .....

(a) 1 (b) – 1

(c) $\frac{7}{5}$ (d) $-\frac{7}{5}$

3 The remainder of subtracting : $\frac{1}{7}$ from $\frac{8}{7}$ equals .....

(a) 1 (b) – 1

(c) $-\frac{9}{7}$ (d) $\frac{9}{7}$

4 $\frac{3}{4}$ exceeds $\frac{3}{8}$ by the amount of .....

(a) $-\frac{3}{8}$ (b) $\frac{3}{8}$

(c) $-\frac{9}{8}$ (d) $\frac{9}{8}$

5 The additive inverse of the remainder of subtracting : $-\frac{2}{9}$ from $\frac{5}{9}$ is .....

(a) $-\frac{7}{9}$ (b) $-\frac{3}{9}$

(c) $\frac{3}{9}$ (d) $\frac{7}{9}$

6 If $a + \frac{6}{7} = 0$, then a = ...

(a) zero (b) 1

(c) $\frac{6}{7}$ (d) $-\frac{6}{7}$

7 **Marine Creatures :** A dolphin dived to a depth of $3\frac{1}{4}$ meters underwater , then it dived a further $2\frac{1}{2}$ meters. Which of the following does not express the dolphin's position relative to the water surface ?

(a) $-3\frac{1}{4} + \left(-2\frac{1}{2}\right)$ (b) $-3\frac{1}{4} + \left|-2\frac{1}{2}\right|$

(c) $-3\frac{1}{4} - 2\frac{1}{2}$ (d) $-\left(3\frac{1}{4} + 2\frac{1}{2}\right)$

8 **Economy :** If the stock price of a company listed on the Egyptian exchange increased by 3.25 LE , and then if decreased by 2.75 LE , which of the following expresses the change in the stock price at that moment ?

(a) – 3.25 + 2.75 (b) 3.25 – 2.75

(c) 3.25 + 2.75 (d) – 3.25 – 2.75

Lesson (5)
Follow Rational Numbers

1) Complete each of the following:

a) The number $\frac{2}{a}$ is a rational number if a ≠ .....

b) The number $\frac{3}{x-2}$ is a rational number if x ≠ .....

c) The number $\frac{x-3}{x+4}$ is a rational number if x ≠ ..... and equals zero if x = ...

d) The number $\frac{x+4}{x+7}$ is not a rational number if x = ...

e) The number $\frac{x+6}{x-8}$ is not a rational number if x = ... and is zero if x = ...

Multiplication operation of rational numbers

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers , then : $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$

Example (1) :

Find the result of each of the following in its simplest form :

1) $\frac{3}{6} \times \frac{2}{5}$

2) $75\ \% \times (-0.\overline{2})$

3) $-\frac{2}{4} \times 2$

4) $-4\frac{2}{7} \times \left(-3\frac{1}{6}\right)$

Example (2) :

Find the result of each of the following in its simplest form :

1 $\frac{3}{2} \times \frac{5}{9}$

2 $1.8 \times -0.\overline{4}$

3 $-5 \times 30\ \%$

4 $-4\frac{1}{2} \times \left(-\frac{5}{9}\right)$

The properties of multiplication operation in ℚ are :

1 Closure. 2 Commutative. 3 Associative.

4 Existence of Multiplicative Identity (1). 5 Existence of Multiplicative Inverse.

Note

The multiplicative inverse for $\frac{a}{b}$ is $\frac{b}{a}$ (where : $a \neq 0$)

6 Property of Distributing Multiplication Over Addition and Subtraction :

If a , b and c are three rational numbers , then :

(1) $a \times (b \pm c) = a \times b \pm a \times c$ (2) $(b \pm c) \times a = b \times a \pm c \times a$

Example (3) :

Use the distributive property to find the value of each of the following :

1) $\frac{5}{11} \times \frac{1}{7} + \frac{5}{11} \times \frac{6}{7}$

2) $\frac{9}{17} \times 21 - \frac{9}{17} \times 4$

3) $\frac{22}{25} \times \frac{6}{11} + \frac{5}{11} \times \frac{22}{25} - \frac{22}{25}$

4) $\frac{7}{12} \times 5 + \frac{49}{12} - \frac{7}{12} \times 11$

Example (4) :

1 Complete the following :

1) The multiplicative inverse of 0.3 is .....

2) The multiplicative identity in $\mathbb{Q}$ is .....

2 Use the distributive property to find the value of each of the following :

1) $\frac{5}{7} \times \frac{2}{3} - \frac{5}{7} \times \frac{1}{3}$

2) $7 \times \frac{3}{10} + 4 \times \frac{3}{10} - \frac{3}{10}$

Division operation of rational numbers

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers , then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$ where $\frac{c}{d} \neq 0$

Example (5) :

Find the quotient of each of the following :

1) $-\frac{2}{3} \div \frac{5}{3}$

2) $\frac{3}{7} \div (-8)$

3) $2\frac{1}{5} \div 5\frac{1}{2}$

4) $-0.\overline{54} \div \frac{5}{22}$

Example (6) :

If $x = -\frac{1}{3}$, $y = \frac{3}{4}$ and $z = -3$, find the numerical value of each of the following :

1) $\frac{y}{z}$

2) $\frac{xy}{z}$

3) $\frac{x}{y} - \frac{y}{z}$

Example (7) :

If $x = -\frac{1}{3}$, $y = \frac{3}{4}$ and $z = -3$, find the numerical value of each of the following :

1 xyz

2 $xy + yz$

Notes:

a) 0 is the additive identity element in Q .

b) 1 is the multiplicative identity element in Q.

c) The additive inverse of 0 is itself but it has no additive inverse.

d) Multiplying a rational number by zero, the product equals zero.

e) $\frac{a}{b} - \frac{c}{d} = \frac{a}{b} + (-\frac{c}{d})$.

f) $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$.

Homework

1) Write each of the additive inverse and the multiplicative inverse for each of the following rational number:

[a] $\frac{3}{7}$ [c] zero [e] – 2.3

[b] $-\frac{4}{9}$ [d] – 6 [f] 5.41

2) Complete:

a) The multiplicative inverse of the rational number ($-\frac{2}{3}$) is ..... and its additive inverse is .....

b) To divide $-\frac{7}{12}$ by $-\frac{2}{3}$, you would multiply ..... by .....

c) 0 ÷ (-14) = ...

d) $-\frac{4}{3} \times (-\frac{3}{4})$ = ...

e) The additive identity element in Q is ..... and the multiplicative identity element is .....

f) The additive inverse of $(\frac{5}{3})^{zero}$ is ..... and its multiplicative inverse is .....

g) The additive inverse of zero is ..... and its multiplicative inverse is .....

3) Use the properties of addition and multiplication to calculate:

[a] $\frac{4}{9} \times 11 + \frac{4}{9} \times 16$

[b] $\frac{5}{12} \times 3 + \frac{5}{12} \times 9$

[c] $-\frac{3}{7} \times 8 + 5 \times (-\frac{3}{7}) + (-\frac{3}{7})$

[d] $\frac{18}{5} \times \frac{25}{9} + (-\frac{3}{7}) \times \frac{25}{9}$

4) if $x = \frac{3}{2}$, $y = -\frac{1}{4}$, $Z = -2$, **find in simplest form the numerical value of each**

a) $x - z \div y$

b) $\frac{x}{y} - \frac{z}{y}$

c) $\frac{1}{xyz}$

d) $\frac{y}{xz}$

5) Choose the correct answer:

9 If $\frac{2}{3} \times X = \frac{5}{7} \times \frac{2}{3}$, then $X =$ .....

(a) $\frac{2}{3}$ (b) $\frac{5}{7}$

(c) $\frac{3}{2}$ (d) $\frac{7}{5}$

10 If $\frac{2}{5} \div X = \frac{2}{5} \times -\frac{7}{9}$, then $X =$ .....

(a) $-\frac{9}{7}$ (b) $-\frac{7}{9}$

(c) $\frac{7}{9}$ (d) $\frac{9}{7}$

11 If three times a number is 27 , then $\frac{1}{3}$ of this number equals .....

(a) 3 (b) 1

(c) $\frac{3}{2}$ (d) $\frac{9}{4}$

12 Which of the following expressions has the same product as $\frac{-3}{8} \times \frac{8}{3}$?

(a) $2\frac{1}{4} \times \frac{4}{9}$ (b) $\frac{-2}{5} \times 3\frac{1}{2}$

(c) $-\frac{1}{4} \times (-4)$ (d) $2\frac{1}{2} - 3.5$

13 Which of the following operations has the same result of $2\frac{2}{3} \div \left(-1\frac{3}{7}\right)$?

(a) $2\frac{2}{3} \times \left(-1\frac{7}{3}\right)$ (b) $2\frac{2}{3} \times 1\frac{3}{7}$

(c) $-2\frac{2}{3} \times \frac{7}{10}$ (d) $-1\frac{3}{7} \times 2\frac{2}{3}$

14 What is the multiplicative inverse of $-3\frac{1}{2}$?

(a) $-\frac{7}{2}$ (b) $-2\frac{1}{3}$

(c) $\frac{-2}{7}$ (d) $\frac{2}{7}$

15 If $\frac{X}{y} = \frac{2}{3}$, then $\frac{3X}{2y} =$ .....

(a) $\frac{1}{3}$ (b) 1

(c) $\frac{3}{2}$ (d) $\frac{9}{4}$

16 If $\frac{a}{b} = 70$, then $\frac{a}{2b} =$ .....

(a) 35 (b) 68

(c) 72 (d) 140

17 If $\frac{|X|}{5} = 3$, then $X =$ .....

(a) 5 (b) 10

(c) 15 (d) ± 15

18 If $(X-1)$ is the multiplicative inverse of $\frac{1}{5}$, then $X =$ .....

(a) 4 (b) 5

(c) 6 (d) $1\frac{1}{5}$

UNIT 2

Algebra

Unit Lessons

2-1 Mathematical Expressions and Formulas

2-2 Linear Equations

Digital technology and algebra share a mutual relationship; the progress of one influences the other. Technology facilitates the creation of virtual learning environments, and software can also be used to solve algebraic equations.

▶ Can digital technology apps be further developed to solve more complex algebraic equations?

Issues and Life Skills		Values	
- Mathematical Communication	- Sustainable Development	- Responsibility	- Belonging
- Language & Translation	- Creative Thinking	- Work	- Respecting Others

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Lesson (1)

Mathematical Expressions and Formulas

1) Numerical expression

Contains numbers and operation (+ , - , × , ÷)

Examples : 8 or 20 + 5 , 6 - 3 , 10 ÷ 2 , 9 × 4

2) Algebraic expression

Contains variables , numbers and operation (+ , - , × , ÷)

Examples : $n \times \frac{3}{4}$, 3y , $M \div 5 + 3$

3) The Equation

consists of two mathematical expressions with an equal sign (=) between them

Examples : 3m + 1 = 5

4) The Inequality

consists of two mathematical expressions with one of inequality sign ($\leq , \geq , > , <$) between them

Examples : 3m + 1 > 5

5) The Mathematical Formula

It represents a fact , rule or principle

Example : Area of rectangle = length x width

Express each of the following , state its type:

1) amount that ahmed will pay to buy 4 kg of apples , 3 kg of mango for n LE

2) the area of a triangle (A) with base length (b) and its corresponding height (h).

3) the age of a child (y) must be at least four years to be accepted in a school.

4) the total number of boys (x) and girls (y) in a classroom equals 35.

5) when 32 is subtracted from the temperature in Fahrenheit (F) and the result is multiplied by ___ , you got the temperature in Celsius (C).

6) Ahmed savedK LE , and Sameh saved f LE. and three times what ahmed saved minus 20 LE equals what Sameh saved.

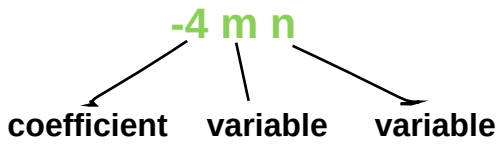
7) Rana is looking for an item online its price is x LE , plus the shipping costs of LE, and the total cost is at most 500 LE.

8) the perimeter of a triangle P , with sides lengths M ,K ,B ,

The algebraic term

An algebraic term is the product (or quotient) of a non-zero number and at least one variable.

Examples :



Complete the table:

Algebraic term	$2ab^2$	$7ab^3c$	$-8x^2b^4$	3	$(-2)^3$	$\frac{1}{2}x^3yz^2$
Coefficient						
Variables						

Like algebraic terms

terms with the same variable raised to the same power, even if their coefficients differ.

Examples : 3x and -5x , 4xy and 9xy

but x^2 and 7x are different

Shade the box of the like terms :

1) **13 $a^2 b^2$** : ○ 31 ab ○ -3 $b^2 a^2$ ○ 5 a b^2 ○ 7 a^2 b

Adding and Subtracting Like algebraic terms (add or subtract coefficients of like terms)

- 8x + 2 - 7x + 7y + 8 -3y

  = (8x - 7x) + (7y - 3y) + (2 + 8)

  = x + 4y + 10

Notes

1) subtracting 5x from -3x written as - 3x - 5x
2) the increase of 2a than - 7a written as 2a - (- 7a)
3) the decrease of 2y than 8y written as 8y - 2y
4) the term that if it is added to 3k , the result is - 4k written as - 4x - 3k
5) the term that if it is subtracted from 5b , the result is 7b written as 5b - 7b

The algebraic expression

consists of two or more algebraic terms (monomials)

Examples of Algebraic Expressions

$2x$

$3x - 4y + 5$

$8a^2 + 4ab - 3$

Complete the following table :

Algebraic expression	Number of terms	Coefficient of terms	Like terms	Constant terms
• $-5xy - 3y + 5x + 5$	.....	.....	.....	.....
• $7ab - 3 + 4ba$	.....	.....	.....	.....
• $12 + 28k + 7$	.....	.....	.....	.....
• $-3xyz^2$	.....	.....	.....	.....

1 - Put (✓) for the correct statement and (×) for the incorrect one:

(1) The two algebraic terms x^2 and $2x$ are like terms. ( )

(2) The two algebraic terms $3\,a\,b^2$ and $-\,a\,b^2$ are like terms. ( )

(3) The two algebraic terms $7x^2$ and $2x^7$ are like terms. ( )

(4) The two algebraic terms $3\,a^2\,b^3$ and $-2\,b^3\,a^2$ are like terms. ( )

2 - Find the result :

$9a - 5a$ = ……. $4a + 6a - 9a$ = ……

$-\,4b + 7b$ = ……. $2y - 7y + 5y$ = …..

$2c - 6c$ = ……. $3x - x - 4x$ = …….

3 - Find the result of each of the following:

(1) $3x + x$ = …… (2) $7y - y$ = ……

(3) $3x + 2x$ = …… (4) $5y - 3y$ = ……

(5) $4z - 11z$ = …… (6) $-7x - 2x$ = ……

(7) $2a + 3a - 4a$ = …… (8) $-3a^2 + 5a^2$ = ……

(9) $\frac{5x}{4} + \frac{3x}{4}$ = …… (10) = ……

4 - Simplify each of the following algebraic expressions :

1 $-2x + y - 5 + 3y + 3$

2 $4(-2a + 3b) - 2(8b - 3) + a$

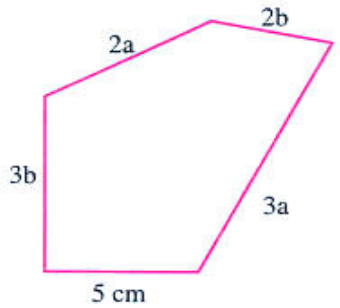
5- Simplify each of the following expressions :

1 $7b + 3a - 5b + a + 7$

2 $-5(2x - 3y) + 2(-3x + y)$

5- Write a mathematical expression in its simplest form that represents the perimeter of the opposite shape , then find the numerical value of the perimeter when

$a = 3$ cm and $b = 2$ cm



Homework

1 Complete each the following

1. The constant term in the algebraic expression : $2x y + 3x - 5$ is .....
2. The number of terms in the algebraic expression : 35 a b c is .....
3. The like term to the term $a^2 b$ in the algebraic expression : $b^2 a - 3 a b + 7 b a^2$ is .....
4. The coefficient of the term like to the term $5 x y$ from the terms of the algebraic expression : $3 y x - 5 x^2 y + 2 x y^2$ is .....
5. $- 3 y + 5 y - 6$ in its simplest form equals .....
6. The result of subtracting 2 b from 5 b is .....
7. The algebraic term that , if added to $- 7$ m , the result will be $- 9$ m is .....
8. The algebraic term that , if subtracted from $8x$, the result will be $- 4x$ is .....

2 Express each of the following mathematically , stating its type (algebraic expression , equation , inequality , or mathematical formula) :

1. Double the number x added to 5 is equal to 1
2. The grade x that a student must obtain to pass the test is at least 18.
3. The area of a square (A) with side length (ℓ).
4. Samer saved x banknotes of 10 LE and y banknotes of 20 LE , then spent 8 LE.
5. The speed at which a car can travel on the (Cairo-Alexandria) desert road must not exceed 120 km/h.
6. Sally's age 5 years ago , if she is now x years old.
7. The arithmetic mean of the two numbers x and y is not less than 18.
8. A rectangle with dimensions x and y , whose area is 36 square centimeters.

3 The algebraic expression $4x + 3 y - 8 x y - 6$ consist of four terms , write :

1. The terms of the expression.
2. The constant term.
3. The coefficient of $x y$
4. The variable whose coefficient is 3

4 **Complete the following table :**

Algebraic expression	Number of terms	Coefficients of the terms	Like terms	Constant terms
1 $4x+2-7-x$	.....	.....	.....	.....
2 $6n-9n-4+n$	.....	.....	.....	.....
3 $-7xy-3x+4y+5$	.....	.....	.....	.....
4 $13abc^2$	.....	.....	.....	.....
5 $6-3x^2y^2+5xy-x^2y^2$	.....	.....	.....	.....

5 **Find the value of each of the following expressions given that a = 8 , b = 3 , d = 4 , f = – 1 :**

1 $\frac{-a}{4}$

2 b^2-2f

3 $af+3d$

4 $2f-ad$

5 $ab-3fd$

6 $2f-3a+4b-d$

6 **Write the following algebraic expressions in their simplest forms :**

1 $7m-2n-7m+1$

2 $-x+2y-8y+5x+7$

3 $5k-3(2k-4)$

4 $-2n+3(n-1)$

Choose the correct answer from the given options :

1 If $2\mathcal{X} = 2$, then $(3\mathcal{X} - 1) =$ .....

(a) 1 (b) 2

(c) 3 (d) 4

2 If $4(\mathcal{X} + 2) = 12$, and the substitution set is $\{2, 3, 4\}$, then the solution set = ...

(a) $\{1\}$ (b) $\{2\}$

(c) $\{3\}$ (d) $\varnothing$

3 Two consecutive numbers whose sum is 29. Which of the following equations expresses that ?

(a) $\mathcal{X} + \mathcal{X} + 2 = 29$ (b) $\mathcal{X} + \mathcal{X} + 1 = 29$

(c) $\mathcal{X} + \mathcal{X} - 1 = 28$ (d) $\mathcal{X} + \mathcal{X} + 1 = 30$

4 Ziad is $\mathcal{X}$ years old now. Seven years ago , he was 18 years old. Which of the following equations represents this situation ?

(a) $\mathcal{X} + 7 = 25$ (b) $\mathcal{X} - 7 = 11$

(c) $\mathcal{X} + 7 = 18$ (d) $\mathcal{X} - 7 = 18$

5 The solution set in $\mathbb{Z}$ for the equation : $7 - 3\mathcal{X} = \mathcal{X} - 3$ is .....

(a) $\varnothing$ (b) $\{2\}$

(c) $\{1\}$ (d) $\{3\}$

6 The solution set in $\mathbb{Q}$ for the equation : $4(\mathcal{X} - 8) = 2\mathcal{X} + 1$ is .....

(a) $\{17\}$ (b) $\varnothing$

(c) $\{16\frac{1}{2}\}$ (d) $\{20\}$

7 Which of the following represents the solution of the equation : $2(\mathcal{X} - 5) = 0$ in $\mathbb{Q}$?

(a) 0 (b) 5

(c) -5 (d) 10

8 Which of the following equations does not have a solution in $\mathbb{Z}$?

(a) $6\mathcal{X} = 12$ (b) $6\mathcal{X} = 15$

(c) $6\mathcal{X} = 18$ (d) $6\mathcal{X} = 24$

9 Safaa bought three identical suitcases and paid 1500 LE and she got a 300 LE discount. The price of one suitcase before the discount is ..... LE.

(a) 500 (b) 600

(c) 700 (d) 800

10 If $0.\overline{6}\,\mathcal{X} - \frac{1}{3} = 1$, then what is the value of $\mathcal{X}$?

(a) $\frac{20}{9}$ (b) $\frac{2}{9}$

(c) 2 (d) $\frac{1}{2}$

Adding and subtracting Algebraic expressions.

First: Adding

Add : $2x - 5z + y$ and $7x + 4y - 2z$

Solution

1- Using the horizontal method

$= 2x - 5z + y + 7x + 4y - 2z$

$= (2x + 7x) + (-5z - 2z) + (y + 4y)$

$= 9x - 7z + 5y$

2- Using the vertical method

$$\begin{array}{r} 2x - 5z + y \\ +\ 7x - 2z - 4y \\ \hline 9x - 7z + 5y \end{array}$$

[1] Add:

1) $(3x - 2y + 5)$ and $(x + 2y - 2)$

.....

.....

.....

.....

.....

2) $(3x + 3y - 5)$ and $(2x - 2y + 5)$

.....

.....

.....

.....

.....

3) $5x^2 - 3x + 9$, $x^2 + 2x - 5$, $x^2 - 3 - 6x$

.....

.....

.....

.....

4) $3x - 4x^2 + 2$, $x^2 + x - 5$, $3 + 3x^2 - 4x$

.....

.....

.....

.....

Second: Subtracting

Subtract $-a^2 - 5ab + 4b^2$ from $3a^2 - 2ab - 2b^2$ By using the vertical method

Change the signs of the lower expression [comes before word (from)]

then add $3a^2 - 2ab - 2b^2$

$a^2 + 5ab + 4b^2$

The sum (result) = $4a^2 + 3ab - 6b^2$

[1] Answer each of the following:

(1) Subtract y^2 from $-3y^2$ ……

(2) Subtract $-6x^2y$ from $9x^2y$ ……

(3) What is the increase $-2x$ of $-5x$? ……

(4) What is the increase $3a^2b$ of a^2b? ……

(5) What is the decrease $-3ab$ of $2ab$? ……

(6) What is the decrease $6x^2y$ of $-7x^2y$? ……

[2] Subtract:

(1) $x - 2$ from $2x - 5$

……

……

……

……

……

(2) $2x + 6y - 7$ from $2x - 5y + 2$

……

……

……

……

……

[3] What is the increase of:

(1) $5a + 7b$ than $3a - 2b$

……

……

……

(2) $x^2 - 5x - 1$ than $3x^2 + 2x - 3$

……

……

……

[4] What is the decrease of:

(1) $2a + 3b$ than $5b - 3a$

……

……

……

(2) $3y^2-2xy+x^2$ than $3x^2-5xy+y^2$

……

……

……

[5] Subtract $x + x^2 - 5$ from $2x^2 + x - 3$, then find the numerical value of the result when $x = 6$

……

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Lesson (2)

Linear Equations

The concept of the equation

It is a two expression with equality relation contains variable or (symbol) like x.

for example:

$$2x = 6$$

An equation can be named according to the highest exponent of its variables

for example:

- $x + 9 = 11$ it is a linear equation
- $x^2 + 4x = 12$ it is a quadratic equation
- $x^3 + x = 2$ it is a cubic equation

Express each of the following situations with a suitable equation :

1 A number added to 5 gives 7

2 Two consecutive integers whose sum is 21

3 If 2 is added to three times the result of subtracting a number from 6 and the result is 11

The substitution set:

it is the set that contains the probable values of the unknown.

The solution set:

it is the set whose elements satisfy the equation and it is a subset of the substitution set.

Equivalent equations:

linear equation in one variable have the same solution

for example:

$$3x-1=5,\quad 3x=6,\quad x=2$$

properties of equality

if A = B , therefore A + C = B + C

if A = B , therefore A - C = B - C

if A = B , therefore A x C = B x C

if A = B , therefore $\frac{A}{C} = \frac{B}{C}$

for example:

Find the solution set in Z for each of the following:

$$3x + 1 = -4$$

by using the properties of equality

3x + 1 = - 4

(subtract 1 from both sides)

3x + 1 - 1 = - 4 - 1

3x = - 5

$$\frac{3x}{3} = \frac{-5}{3}$$

$$x = \frac{-5}{3}$$

the solution set = $\emptyset$ as its value not belong to Z

by using the inverse operation

3x (+ 1) = - 4

(move with opposite sign)

3x = - 4 - 1

(3) x = - 5

(reverse the multiplication)

$$x = \frac{-5}{3}$$

the solution set = $\emptyset$

Solve

Find the solution set in $\mathbb{Z}$ for each of the following equations :

1. $2x - 3 = 5$

2. $3x - 3 = x + 6$

.....

.....

.....

.....

Solve

Adel bought a TV for 18,000 LE , and he has a deal with the shop owner to pay 2,000 LE down payment and to pay the remaining amount in 5 equal monthly installments

1 Write an equation representing this situation.

2 What is the value of the monthly installment?

.....

.....

Homework

1 Complete each of the following :

1 If $x + 3 = 5$, then x = ...

2 If $5 - x = 3$, then x = ...

3 If $7x = 21$, then $3x$ = ...

4 If $3(x - 1) = 9$, then x = ...

5 The solution set of the equation : $2x + 1 = 5$ in $\mathbb{Q}$ is.....

6 The solution set of the equation : $3x - 1 = 4$ in $\mathbb{N}$ is .....

7 If x is an even integer , then the next even integer is .....

8 The sum of two numbers is 15 , if one of them is y , then the other is .....

9 A rectangle with a perimeter of 24 cm and length ℓ cm , then its width is ..... cm

2 Represent each of the following situations in the form of a suitable equation :

1 When 5 is added to a number , the result is (– 3)

2 When 3 is added to a number , the result is 0

3 When 15 is subtracted from twice a number , the result is 12

4 When (– 8) is subtracted from three times a number , the result is (– 5)

3 Find the solution set for each of the following equations using the given substitution set :

1 The equation : $y + 1 = -3$, the substitution set : $\{-2, -3, -4\}$

2 The equation : $7 - k = 2$, the substitution set : $\{3, 4, 5, 6\}$

7 **Find the solution set for each of the following equations in $\mathbb{Z}$:**

1. $2X - 5 = -17$
2. $4 - 3X = 19$
3. $5(X - 1) = 20$
4. $-2(X + 1) = 3$
5. $4(X - 1) = 3(X + 1)$
6. $2(X + 1) = 3(5 - X)$

8 **Find the solution set for each of the following equations in $\mathbb{Q}$:**

1. $3(X - 5) = -18$
2. $2(X - 3) = 8$
3. $2(X + 3) = 3(1 - X)$
4. $\frac{1}{3}X + 3 = 12$
5. $2X + 5 = 12 + 3X$
6. $7 = 2(X + 3)$
7. $3(X - 1) + 4 = 3$
8. $4(X - 3) = 2(X + 4)$

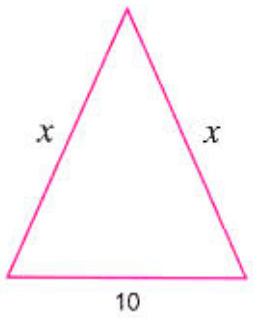
14 The length of a rectangle exceeds its width by 7 meters. If its perimeter is 66 meters , find its dimensions.

---

15 Hazem has a wooden strip of length 180 centimeters. He intends to create a rectangular picture frame where one of the dimensions exceeds the other by 15 centimeters.
Write the equation expressing this situation and solve it to determine the frame's dimensions.

12 **Geometry :**

If the perimeter of the opposite triangle equals 34 , then what is the value of X ?



UNIT 3

Statistics

Unit Lessons

Machine Learning is used in analyzing data to extract the information it contains, which helps in making decisions and improving performance in various areas.

▶ Can Machine Learning algorithms be developed to solve statistical problems in our daily lives?

Issues and Life Skills

- Information Technology
- Creative Thinking
- Predicting
- Decision Making

Values

- Responsibility
- Work
- Justice
- Persistence

Lesson (1) Organizing Data

Mode

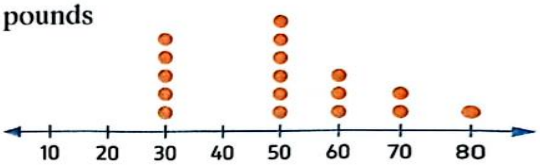
It is the most common value (the most frequent value).

1 Mode for a set of values :

- From the data set : 5 , 3 , 8 , 9 , 8 , 12
  The mode is 8
- From the data set 3 , 7 , 2 , 7 , 3 , 8
  The mode is 7 , 3 (bimodal)
- From the data set 15 , 16 , 9 , 18 , 11
  There is no mode as all values are different .

2 Mode from a dot plot:

Savings of some students last month in pounds



The mode is 50 pounds because it has the highest frequency.

Median

It is the value that lies in the middle of the values after arranging.

To find the median of a set of values , follow these steps :

Arrange the values

If the number of values is odd	If the number of values is even
The median is the value exactly in the middle.	The median = $\frac{\text{sum of the two middle values}}{2}$
• For Example : 42 , 23 , 17 , 30 , 20 • Arrange the values in ascending order : 17 , 20 , 23 , 30 , 42 Median = 23	• For Example : 27 , 13 , 23 , 24 , 13 , 21 • Arrange the values in ascending order : 13 , 13 , 21 , 23 , 24 , 27 Median = $\frac{21+23}{2}=22$

Lower Quartile (First Quartile)
It is the median of the lower half of the values.

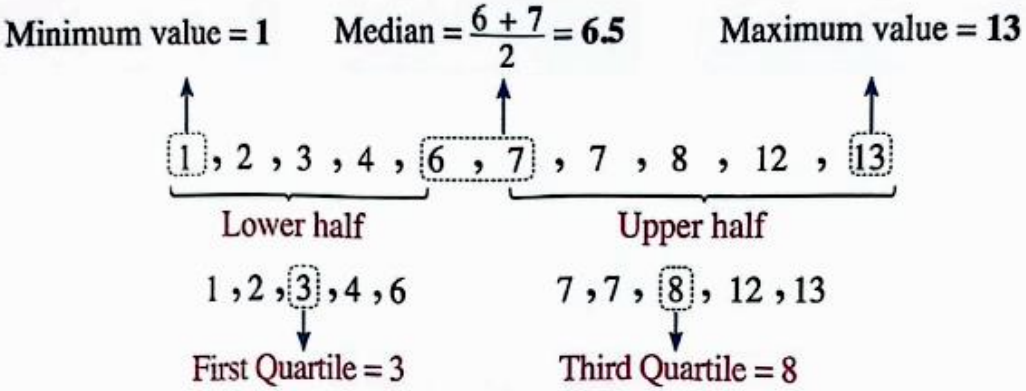
Upper Quartile (Third Quartile)
It is the median of the upper half of the values.

How to find the first and third quartiles

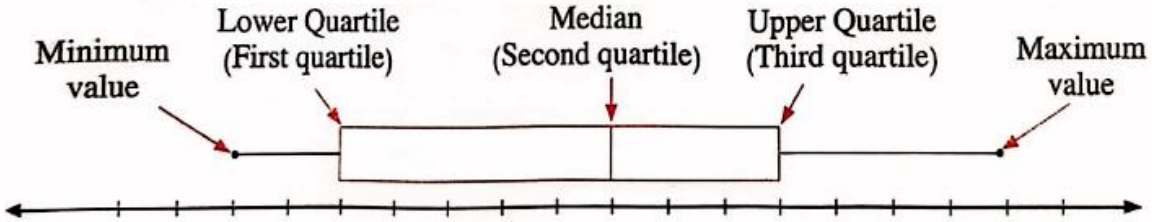
- To find the first and third quartiles for values: 2 , 13 , 7 , 7 , 6 , 4 , 1 , 12 , 8 , 3

Arrange the values in ascending order and find the median.

Then find the median of the lower half of the values (first quartile) and the median of the upper half of the values (third quartile).



Box plot using the five-number summary



Statistics

Statistics is a branch of mathematics that involves collecting , organizing , presenting , analyzing , and interpreting data to make a decision.

It has applications in various fields such as medicine , economics , social sciences , etc.

The Scientific Steps of Statistical Research :

1 **Identify the problem or the question :**

Formulate it as a statistical question.

2 **Collect data :**

This can be done using various methods such as observation , survey or interview.

3 **Organize and present data :**

Use one of the appropriate charts
(dot plot , bar graph , histogram , etc.).

4 **Analyze and interpret data.**

5 **Make prediction and decisions.**

Frequency Table with Intervals and its Representation Using Histogram

When the number of data points is large and the values vary over a wide range , when it is organized , it is divided into a suitable number of groups or equal intervals and the number of items belong to each group are determined. Then , a table containing these groups and their corresponding frequencies is created. This is called a **frequency table with intervals**.

Bar graph

Example 1

A teacher asked his students inside the class : "What is your favorite hobby?"
The responses were as follows:

Drawing	Singing	Acting	Drawing	Reading	Reading	Acting
Reading	Drawing	Singing	Music	Reading	Singing	Music
Reading	Music	Drawing	Music	Reading	Acting	Music.

Represent this data with a bar graph and answer the following:

1 What is the hobby which is the largest number of students preferred ?
2 What is the hobby which is the least number of students preferred ?
3 How many more students prefer music than drawing ?

Solution

Arrange the data in a frequency table , and to make it easier use tally marks as follows :

Hobby	Tally Marks	Frequency
Drawing	𝍸	4
Acting	𝍸	3
Reading	𝍸 𝍸	6
Singing	𝍸	3
Music	𝍸	5

The frequency table can be simplified by removing the tally marks , to get the following frequency table

Hobby	Drawing	Acting	Reading	Singing	Music
Frequency	4	3	6	3	5

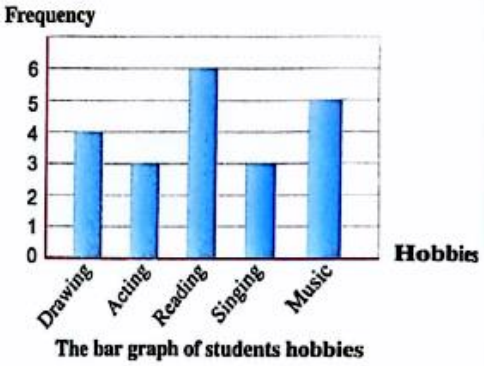
Note that

The Hobbies can be written in the table in any order.

1 Reading 2 Singing , Acting 3 $5-4=1$

Notes on Bar graph

- Bar graph is used for qualitative or quantitative data.
- There are equal gaps between the bars.
- The bars can be reordered.
- The graph shows the actual data values.



The bar graph of students hobbies

Dot Plot

Example ❷

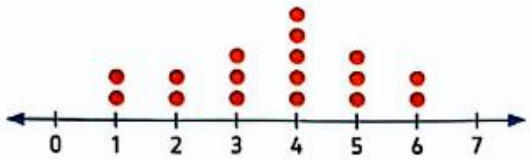
A teacher asked his students : "How many books did you read last year?" The responses were as follows :

1 5 4 4 2 4 3 5 4 4 3 5 1 6 2 3 6

1 Represent this data with a dot plot.
2 Find the mode.
3 Find the median , first quartile and third quartile.
4 Draw the box plot.

Solution

1



2 **The mode** is 4 (as it is the most frequent).

3 • **The median** is the value in the middle after arranging the data = 4

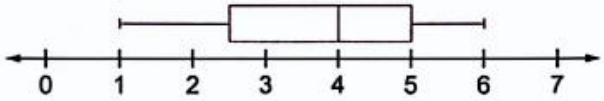
• **The first quartile** (is the median of values less than 4) $= \frac{2+3}{2} = 2.5$

• **The Third quartile** (is the median of values greater than 4) $= \frac{5+5}{2} = 5$

4 To draw the box plot , create a summary table of the five values.

Minimum Value	First Quartile	Median	Third Quartile	Maximum value
1	2.5	4	5	6

And from it , we draw the box plot



Notes on Dot Plot

- Dot plot is used to represent quantitative data only.
- Each data point is represented by a dot on a number line.
- The order of the number line is followed.
- The plot shows the actual data values.

Stem-and-Leaf Plot

A stem-and-leaf diagram is a way of displaying quantitative data based on place value for every digit in the data values , separating the last digit on the right to form the leaves , and the rest of the digits forming the stem. For example :

If a data point is (143) the units (3) represents the leaf , and the rest of the number (14) represents the stem. The plot is formed by drawing a vertical line , with leaf on the right and the stem on the left , and a key indicating how to read the data.

Examples of Stem-and-Leaf Plots:

Stems	Leaves
5	1 1
6	0 3 5
7	4 6
8	1 9 9

Key	7 | 4 represents 74

Stems	Leaves
12	3 4
13	2 2 6
16	1 8
17	0 4

Key	17 | 4 means 174

Stems	Leaves
15	0 1 1
16	2 3
17	1 2 2
18	5

Key	16 | 2 represents 16.2

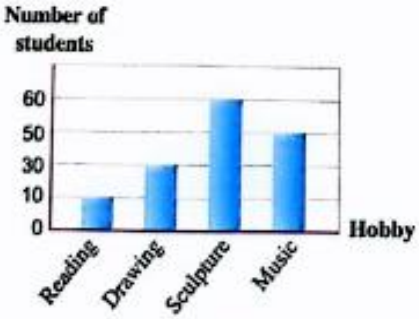
Note

If the data includes decimal point like 17.1 , 16.2 , the decimal part (1) , (2) represent the leaves , and the whole number (17) , (16) represent the stems.

The Misleading Graphs

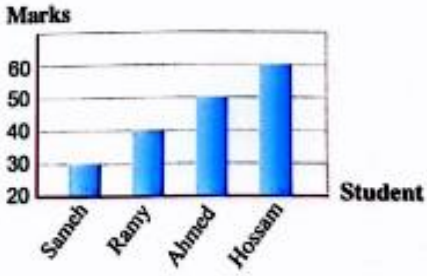
- Since graphs convey information and provide us with insights into the original data , any error in these graphs can lead to misleading information.
- Some common ways that make a graph misleading :
  1. If the vertical axis does not begin with zero.
  2. If an uneven scale is used on the vertical axis.

Examples of Misleading Graphs



"Uneven scale on the vertical axis"

Misleading because it shows that the number of those who chose drawing is twice that of those who chose reading , while it is actually three times.

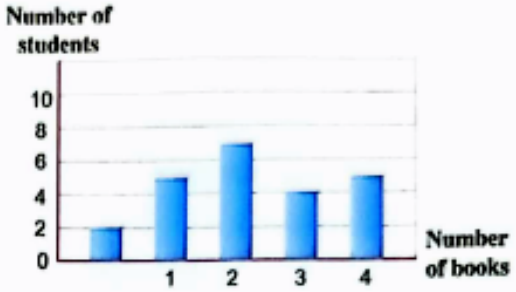


"The vertical axis does not begin at zero"

Misleading because it shows that Ramy marks as twice that of Sameh's , which is not true.

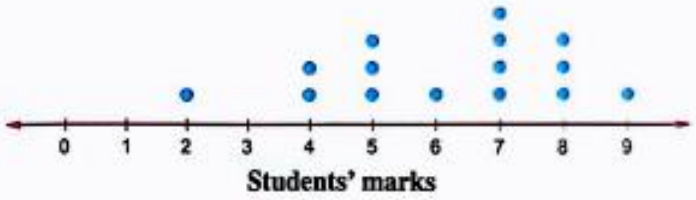
1 Complete each of the following :

1. The opposite bar graph shows the number of books read by your classmates last month :



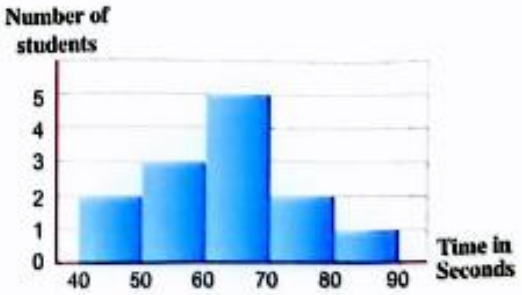
- The number of students who read three books = ...
- The number of students who read fewer than three books = ...
- The total number of students in the class = ...

2 The teacher recorded the students scores in the math test and represented them with the opposite dot plot :



- The number of students who scored 5 points or more = ...
- The score that most students received = ...
- The median = ...
- The first quartile = ...
- The third quartile = ...

3 In a running race , the physical education teacher measured the time each student took to finish the race and divided these measurements into intervals in the opposite histogram :



- The number of students who finished the race in less than 60 seconds = ...
- The number of students who finished the race in 80 seconds or more = ...
- The interval that contains the largest number of students is .....
- The total number of students who participated in the race = ...

4 The opposite stem-and-leaf plot represents the total scores of some students :

Stems	Leaves
25	3 5
26	0 1 1 3
27	2 3 5
28	8 9

Key | 26 | 0 means 260

- The number of students who scored more than 270 points = ...
- The number of students who scored less than 260 points = ...
- The mode = ...
- The median = ...
- The first quartile = ...
- The third quartile = ...

5 🕮 **Business Administration :** The opposite table demonstrates the sales of a computer shop last year.

Represent this data using a suitable graphical chart.

Product	Frequency
PC	60
Laptop	100
Tablet	80
Electronic components	240

🕮 **A teacher recorded the marks of 25 students in a quiz as follows :**

6 5 6 3 4 6 7 5 6 3 8 6 6 7 7 6 7 8 4 6 8 7 5 6 5

1 Represent this data using a dot plot and a bar graph.

2 How many students got at least 7 marks ?

3 What is the mark that most students got ?

🕮 **The following data represents the heights of 32 students in centimeters :**

134	152	140	134	130	142	131	144
144	132	147	143	135	135	145	137
148	151	133	142	136	138	132	146
140	139	141	148	130	144	149	139

1 Form a frequency table with intervals (130 – , 135 – , ...) , then represent it using a histogram.

2 How many students have heights less than 145 cm ?

The opposite stem-and-leaf plot represents the ages of mathematics teachers at a school.

Answer the following questions :

1 How many mathematics teachers are there at this school ?

2 What is the age of the youngest teacher ?

3 What is the age of the oldest teacher ?

4 How many teachers are younger than 45 years old ?

5 What is the median age ?

Stems	Leaves
3	5 7 9
4	1 4 5 8
5	0 3 6

Key	315 means 35 years

If the number of people who went to a restaurant over 11 days is as follows :

104	131	120	115	109	124
128	118	116	120	125	

Stems	Leaves
.....	.....
.....	.....
.....	.....
.....	.....

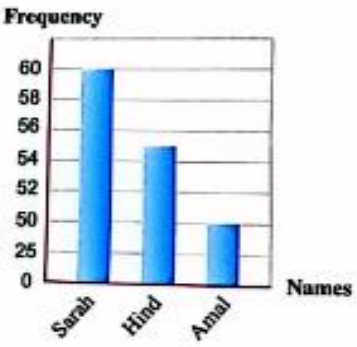
Key	13 | 1 means 131

1 Complete the adjacent plot.

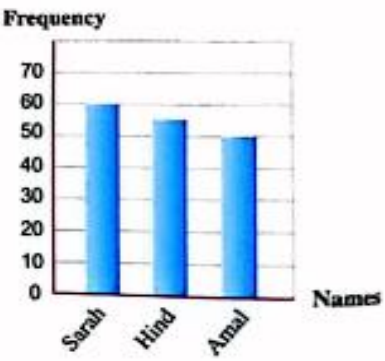
2 **Find :** the mode , median , and range.

Both of the following graphs represent the masses of Sarah , Hind , and Amal in kilograms. Which graph could be misleading and why ?

1



2

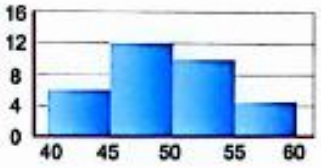


Choose the correct answer from the given ones :

1 Which of the following is suitable for representing categorical data .....
(a) Bar graph.
(b) Histogram.
(c) Stem-and-leaf plot.
(d) Both a and b

2 Which of the following charts does not display the actual data ?
(a) Dot plot.
(b) Histogram.
(c) Stem-and-leaf plot.
(d) Bar graph.

3 From the following histogram :
The frequency in the 50 – interval is .....
(a) 12
(b) 8
(c) 10
(d) 9



4 From the opposite stem-and-leaf plot , what is the median ?
(a) 16
(b) 17
(c) 18
(d) 20

Stems	Leaves
0	9
1	0 2 2 2 3 4 5 6 6
2	0 1 1 5 7 8 9
3	1 2 3

Key	3 | 1 means 31

Lesson (2) The arithmetic mean

Measures of Central Tendency :

are values that describe the center of data set such that the arithmetic mean, median, mode.

1) The arithmetic mean of set of values = $\frac{The\ sum\ of\ values}{their\ number}$.

Example (1): The arithmetic mean of the values 5 , 7 , 3 , 1 = $\frac{5+7+3+1}{4}$ = 4

2) The median of set of values

Arrange the values ascending (or descending) order then take the middle value.

- If (n **number of values, where n is odd number**) , then
  the order of the median = $\frac{n+1}{2}$
- If (n **number of values, where n is even number**) , then
  the order of the median = $\frac{n}{2}$ and $\frac{n}{2}+1$
- If we have two middle values then we take their mean

Ex: If the median of six consecutive even numbers is 39 , find the arithmetic mean of the smallest two numbers.

solution : since the median is 39 , we have six even value ,
so the values are 34 , 36 , 38 before 39 and 40 , 42 , 44 after it
the arithmetic mean for 34 & 36 = $(\frac{34+36}{2})$ = 35

3) The mode of set of values : is the most repeated value .

Choose the correct answer :

1) The order of the median of the set of values : 4 , 5 , 6 , 7 and 8 is .....
(a) third. (b) fourth. (c) fifth. (d) sixth.

2) The order of the median of the set of values : 8 , 4 , 7 , 6 , 5 is .....
(a) 7 (b) 6 (c) 3 (d) 5

3) If the order of the median of a set of values is the fourth , then the number of these values is .....
(a) 3 (b) 5 (c) 7 (d) 9

Ex: **If the ages of five players at a sports club are 16 , 21 ,15 , 23 , 15 years , and the age of a sixth player is added as 13 years , which of the following statements is true**

(a) the mean decreases. **(b) the mean does not change.**

(c) the median increases. **(d) the mode increases.**

solution:

1) the arithmetic mean of five values = $\frac{15+15+16+21+23}{5}$ = 18 , when we add 13 and get the mean = $\frac{15+15+16+21+23+13}{6}$ = 17.16 , so **the value of mean decreases**

2) 15 , 15 , 16 , 21 , 23 median is 16 , if we add 13 to the values , the values are 13 , 15 , 15 , 16 , 21 , 23 median will be = $\frac{(15+16)}{2}$ = 15.5 , so **the median decreases**

3) **the mode does not change** = 15 in the two cases.

so, (a) is true & (b) is false & (c) is false & (d) is false

EX: **If the arithmetic mean of the numbers 16 , n+3 , 14 , 2n - 2 is 15.25 find the value of n , then find the median of these numbers.**

solution:

mean = $\frac{16+n+3+14+2n-2}{4} = \frac{3n+31}{4} = \frac{15.25}{1}$

so , 3n +31 = 61

3n = 30

n = 10

then the values are 16 , 10+3 , 14 , 2(10) - 2

16 , 13 , 14 , 18

arrange the values , 13 14 16 18 , the median = $\frac{14+16}{2}$ = 15

Think

The daily pocket money for five students are 70 , 60 , 90 , 60 and 50. If a sixth student's pocket money of 100 is added , then which of the following statements is true ?

(a) The median decreases. (b) The mean increases.

(c) The mode decreases. (d) The mean does not change.

The Arithmetic Mean of a Frequency Distribution $(\bar{x})$

$$\text{The arithmetic mean } (\bar{x}) = \frac{\Sigma (f . x)}{\Sigma f}$$

Where : Σf is the sum of the frequencies
and $\Sigma (f . x)$ is the sum of the products of f and x

Ex:

The following table shows the masses of 30 students

The masses (x)	35	38	42	44	52
Number of students (f)	4	10	8	6	2

Find the arithmetic mean of the mass of the students

Solution :

x	f	f . x
35	4	140
38	10	380
42	8	336
44	6	264
52	2	104
total	30	1224

$$\text{The arithmetic mean } (\bar{x}) = \frac{\Sigma (f . x)}{\Sigma f}$$

$$= \frac{1224}{30}$$

$$= 40.8 \text{ kg}$$

Ex: The given table shows the number of minutes a group of students spend on phone calls. find the number of students who spend 4 minutes so that the arithmetic mean of the call duration is 5.2 minutes.

N.of minutes (x)	Frequency (f)
3	6
4	a
5	14
6	11
7	10

Solution :

$$\bar{x} = \frac{\Sigma (f . x)}{\Sigma f}$$

$$5.2 = \frac{4a + 224}{a + 41}$$

$$4a + 224 = 5.2 (a + 41)$$

$$4a + 224 = 5.2a + 213.2$$

$$224 - 213.2 = 5.2a - 4a$$

$$10.8 = 1.2a$$

$$a = \frac{10.8}{1.2} = 9$$

x	f	f . x
3	6	18
4	a	4a
5	14	70
6	11	66
7	10	70
total	a+41	a4+224

Exercises

1) Complete each of the following :

1. The arithmetic mean of the numbers 3 , 0 , 9 , 8 , 5 is .....
2. The arithmetic mean of the numbers x , 1 , 7 , 8 , 9 , 8 , 7 , 2 is 6 , then x equals .....
3. If the arithmetic mean of the values $x+1$, $x-3$, 9 , $4x+6$ is 7.25 , then x equals .....
4. The median of the numbers 25 , 15 , 7 , 18 , 17 , 30 is .....

2) Sarah took five tests , each with full marks of 100. If her marks in three tests are 94 , 88 , 81 , what is the least mark she could get in one of the other two tests to achieve an average of 83 marks in the five tests ?

3) The following frequency table shows a student's daily pocket money over two weeks : Find the average daily pocket money for this student

Daily pocket money in LE (x)	25	29	34	39	55
Number of days (f)	3	5	3	2	1

4) The opposite table shows the number of minutes a group of people spend on phone calls. Calculate the average number of minutes a person spends on one phone call

No. of minutes	2	3	4	5	6
Frequency	12	20	36	20	12

5) The following table shows the number of working hours for a group of workers :

No. of hours (x)	5	6	7	8	9	10
No. of workers (f)	12	n	30	39	16	9

Find the number of workers who work 6 hours such that the arithmetic mean of the number of working hours is 7.5

The Misleading Statistical Data

Using the arithmetic mean regardless of the overall distribution of the data can be misleading. If there are some extreme values (very large value or very small value) , this can significantly affect the arithmetic mean. Therefore , in such cases , it is better to use the median or the mode.

Ex:

A car dealership declares the price list of cars in thousands of LE and announces that the average car price is 740,000 LE.
Explain why the average price of the car is misleading.

Type of Car	Price in Thousands of LE
Megane	960
Tipo	940
Nissan	890
MG	780
Motorcycle	130

solution:

The arithmetic mean of the prices in the list (in thousands of LE) is :

$$\frac{960 + 940 + 890 + 780 + 130}{5} = 740$$

The dealership used the arithmetic mean to report the average car price , but this measure is misleading because there is an extreme value (130) that does not represent the price of any car. but the dealership used this to claim that the average car price is 740,000 LE , which is not real.

So , a more accurate measure in case of extreme values is needed , in this case it is the median , which is 890,000 LE , that shows more accurately and truthfully about the car prices.

Homework

1) The following table shows the internet usage in GB by a family during a month. Calculate the arithmetic mean of the daily usage for this family.

The daily consumption in GB	5	7	10	12	15	18
Number of days	4	6	7	8	3	2

2) The following table shows the daily additional wages in LE for a group of workers :

The addition wage	50	60	70	80	90	95
Number of workers	7	5	k	10	8	6

Find the number of workers who receive an additional wage of 70 LE so that the arithmetic mean of the daily wage is 75.5 LE.

3) If the masses of the school's Karate team players in kilograms are given in the following table :

Mass (kg)	Frequency
71	1
72	2
73	4
74	3
75	5
76	3
77	2

Calculate the arithmetic mean of the masses of these players

4) The following table shows the lengths of some infants in centimeters :

Length (x)	43	45	47	48	50	53	60
Frequency (f)	4	6	11	9	5	k	2

Find the number of infants who are 53 cm tall so that the arithmetic mean of the infant's length is 48 cm

Choose the correct answer from the given ones :

1 If the median of eight consecutive odd numbers is 22, then what is the arithmetic mean of the largest two numbers ?

(a) 25 (b) 26

(c) 28 (d) 29

2 If the arithmetic mean of six consecutive even numbers is 35, then what is the smallest of these numbers ?

(a) 36 (b) 34

(c) 32 (d) 30

3 For a set of values :

If $\Sigma f = 10$ and $\Sigma (f \cdot x) = 40$,

then what is the value of $\overline{x}$?

(a) 4 (b) 30

(c) 50 (d) 400

4 If some data is organized in the following stem-and-leaf diagram , then what is the arithmetic mean of this data ?

(a) 22

(b) 23

(c) 24

(d) 25

Stems	Leaves
1	0 2 9
2	4 5 5 6 7
3	4 8
Key	3 | 4 means 34

5 If the arithmetic mean of Magdy's marks in 4 tests is 16 , then what mark should Magdy have in the fifth test to have average of 18 across all tests?

(a) 24 (b) 25

(c) 26 (d) 27

6 55 , 75 , 55 , 42 , and 47 represent the numbers of children who took the infantile paralysis vaccine over five days. If the number of children who took the vaccine on the sixth day is 95 , then which of the following statements is true ?

(a) The median decreases

(b) The mode increases

7 For a set of values :

If $\overline{x} = 20$ and $\Sigma (f \cdot x) = 1{,}500$

What is the value of Σf ?

(a) 75 (b) 150

(c) 3,000 (d) 30,000

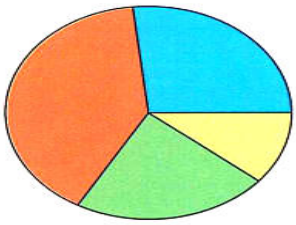
8 Four numbers , the average of the first two numbers is 25 and the average of the last two numbers is 34. What is the average of all four numbers?

(a) 27.5 (b) 28

(c) 29.5 (d) 30

Lesson (3) pie chart

A pie chart is a circle divided into sectors based on the percentage of frequencies in each group of the distribution. It is a simplified graphical method to show parts-to-whole relationship and visually comparing the areas of the sectors.



Ex: **The following table shows the contributions of four persons (in thousand LE) in one of the projects :**

Name	Ibrahim	Mahmoud	Islam	Wael
Amount	10	11	6	9

Represent the share of each person in the project using a pie charts.

Solution

- The total amount = 10 + 11 + 6 + 9 = 36 thousand LE.
- Find the measure of the central angle of each sector :

Measure of the central angle

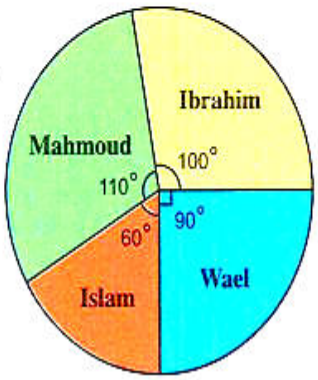
$$= \frac{\text{amount of each person}}{\text{Total amount}} \times 360°$$

«Ibrahim» $\frac{10}{36} \times 360° = 100°$

«Mahmoud» $\frac{11}{36} \times 360° = 110°$

«Islam» $\frac{6}{36} \times 360° = 60°$

«Wael» $\frac{9}{36} \times 360° = 90°$



Steps for Drawing

- Draw an appropriate circle and draw its radius.
- Use a protractor to draw a central angle of measure 100°; this sector represents "Ibrahim".
- Similarly , draw a central angle of measure 110° to represent "Mahmoud".
- Also , draw a central angle of measure 60° to represent "Islam".
- The remaining part is the sector that represents "Wael".

Ex:

The following table shows the percentages of weekly study hours for each school subject for one of the students :

Subject	Arabic	Mathematics	English	Science	Social studies
Percentage	25 %	35 %	15 %	.....	5 %

Represent this data using a pie chart.

Solution

- The percentage for Science = 100 % – (25 % + 35 % + 15 % + 5 %) = 20 %
- The measure of the central angle representing each sector :

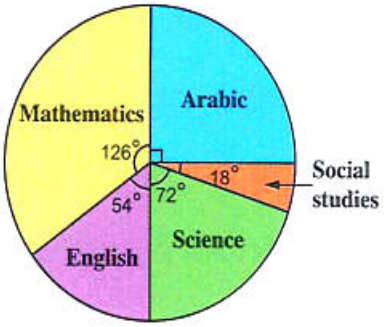
«Arabic» $\frac{25}{100} \times 360° = 90°$

«Mathematics» $\frac{35}{100} \times 360° = 126°$

«English» $\frac{15}{100} \times 360° = 54°$

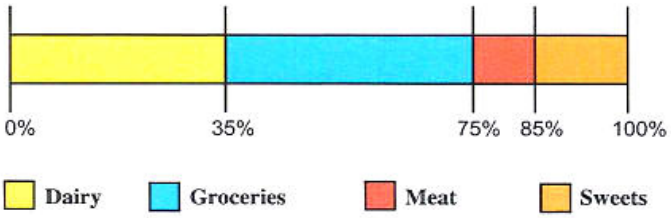
«Science» $\frac{20}{100} \times 360° = 72°$

«Social Studies» $\frac{5}{100} \times 360° = 18°$



Ex:

The opposite chart shows the amount of products sold in a commercial market
Represent this data using a pie chart.



Solution

From the above chart , form the following table :

Product	Dairy	Groceries	Meat	Sweets
Percentage	35 %	40 %	10 %	15 %

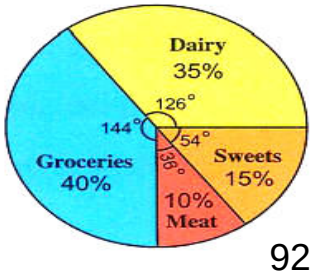
- The measure of the central angle representing each sector :

«Dairy» $\frac{35}{100} \times 360° = 126°$

«Groceries» $\frac{40}{100} \times 360° = 144°$

«Meat» $\frac{10}{100} \times 360° = 36°$

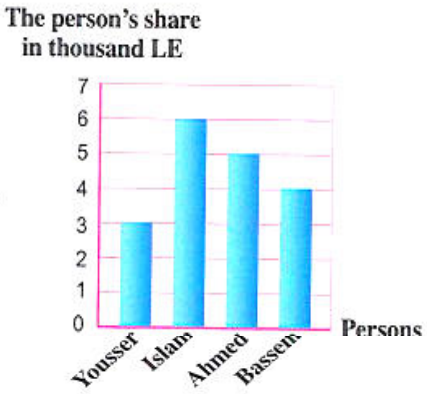
«Sweets» $\frac{15}{100} \times 360° = 54°$



Ex:

The bar graph shows the contributions of four friends (in thousand LE) to a project.

Represent the share of each in the project using a pie chart.



Solution

From the bar graph , from the following table :

Person	Youssef	Islam	Ahmed	Bassem
The person's share in thousand LE	3	6	5	4

Total capital = 3 + 6 + 5 + 4 = 18

• The measure of the central angle representing each sector :

«Youssef» $\frac{3}{18} \times 360° = 60°$

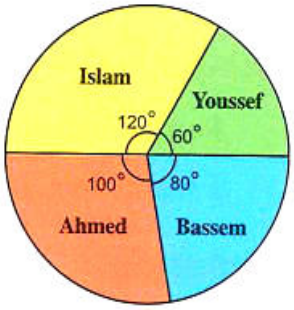
«Islam» $\frac{6}{18} \times 360° = 120°$

«Ahmed» $\frac{5}{18} \times 360° = 100°$

«Bassem» $\frac{4}{18} \times 360° = 80°$

Note

When you have a dot plot , bar graph , or histogram , first create the table , then represent the data using a pie charts.



Ex:

The following table shows the savings of a group of children in a school :

Interval	0 –	100 –	200 –	300 –
Frequency	22	8	6	4

Represent this data using a pie charts.

Solution

Total frequency = 22 + 8 + 6 + 4 = 40

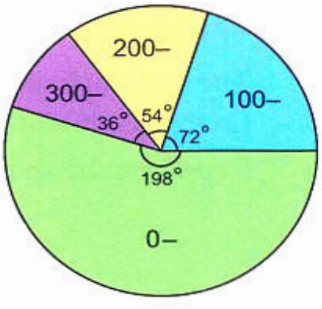
• The measure of the central angle representing each sector :

«interval 0–» $\frac{22}{40} \times 360° = 198°$

«interval 100–» $\frac{8}{40} \times 360° = 72°$

«interval 200–» $\frac{6}{40} \times 360° = 54°$

«interval 300–» $\frac{4}{40} \times 360° = 36°$



Exercises

1) The following table shows the favorite TV programs for a number of students:

TV Programs	Sports	News	Series	Movies
Number of Students	15	5	10	30

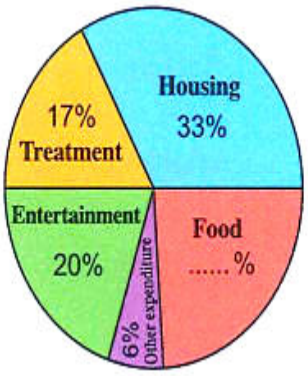
Represent this data using a pie chart.

2) The following table shows the percentages of production for a factory :

Product	Fridge	Washing Machine	Cooker	Television
Percentage	30 %	15 %	.... %	45 %

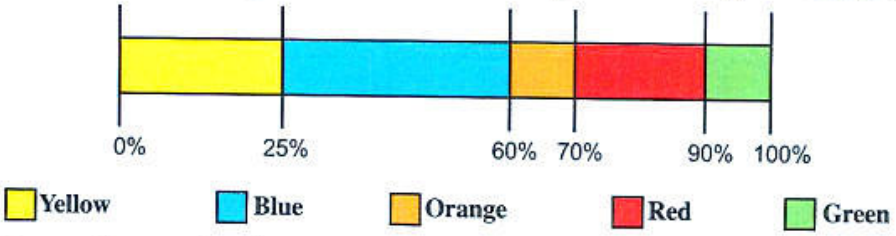
Represent this data using a pie chart.

3) The opposite diagram represents the pie chart for the monthly expenses of a family with a monthly income of 14,000 LE :



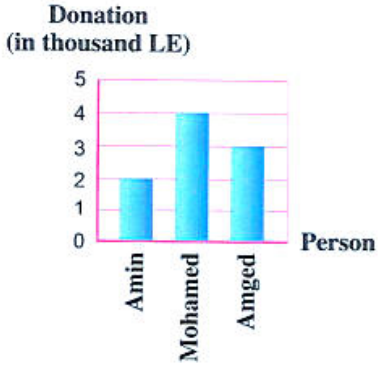
1. Find the monthly expenses on housing.
2. Find the monthly expenses on food.
3. Calculate the central angle for the entertainment sector.

4) If the favorite colors of a group of students are represented by the following chart :



Represent this data using a pie chart.

5) The bar graph shows the donations of three friends (in thousand LE) to a public hospital.
Represent each person's share in the contribution using a pie charts.



6) **The following table shows the scores of some students in a class :**

Interval	5 –	10 –	15 –	20 –
Frequency	15	20	60	5

Represent this data using a pie chart.

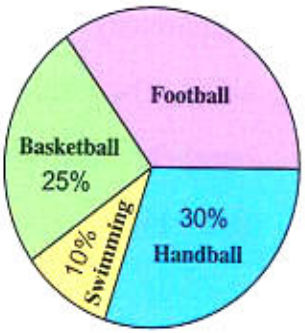
Homework

1 **Complete each the following :**

1 The sum of the measure of the accumulative angles around the center of a circle = ... °

2 The measure of the central angle of the sector whose area is $\frac{1}{4}$ of the area of the circle = ...°

3 The opposite pie chart represents the distribution of sports activities for students in a school. If the number of students in the school is 1,200 students , then



First : The percentage of students participating in football = ...

Second : The measure of the central angle of the sector representing the students participating in handball = ...

Third : The number of students participating in swimming = ...

2 🕮 **Sports :** In a tennis match , a player hit 15 smashes , distributed as shown in the table opposite. Draw a pie chart for this distribution.

Set	First	Second	Third
No. of hits	7	3	5

3 If the favorite sport of 160 students is shown in the opposite table. Represent this data using a pie charts.

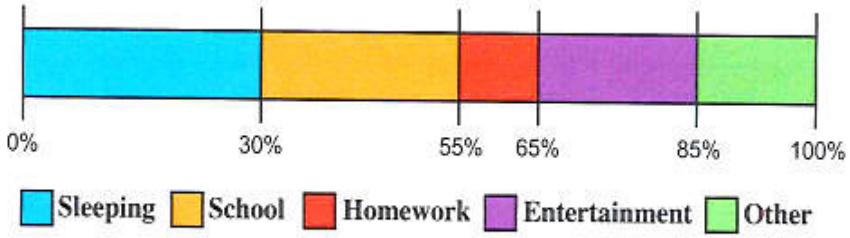
Sports	Football	Swimming	Volleyball	Karate
Frequency	96	16	16	32

4 **The following table shows the weekly study hours for a student :**

Subject	Arabic	Math	Science	English	Social Studies
Number of hours	9	10	6	7	4

Represent this table using a pie chart.

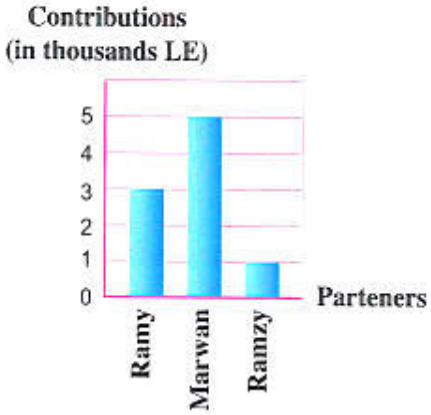
5 🕮 **Daily Life :** The figure below illustrates how Bassem spends his entire day :



Draw a pie chart expressing this routine.

6 The bar graph shows the contributions of three partners (in thousand LE) to a project

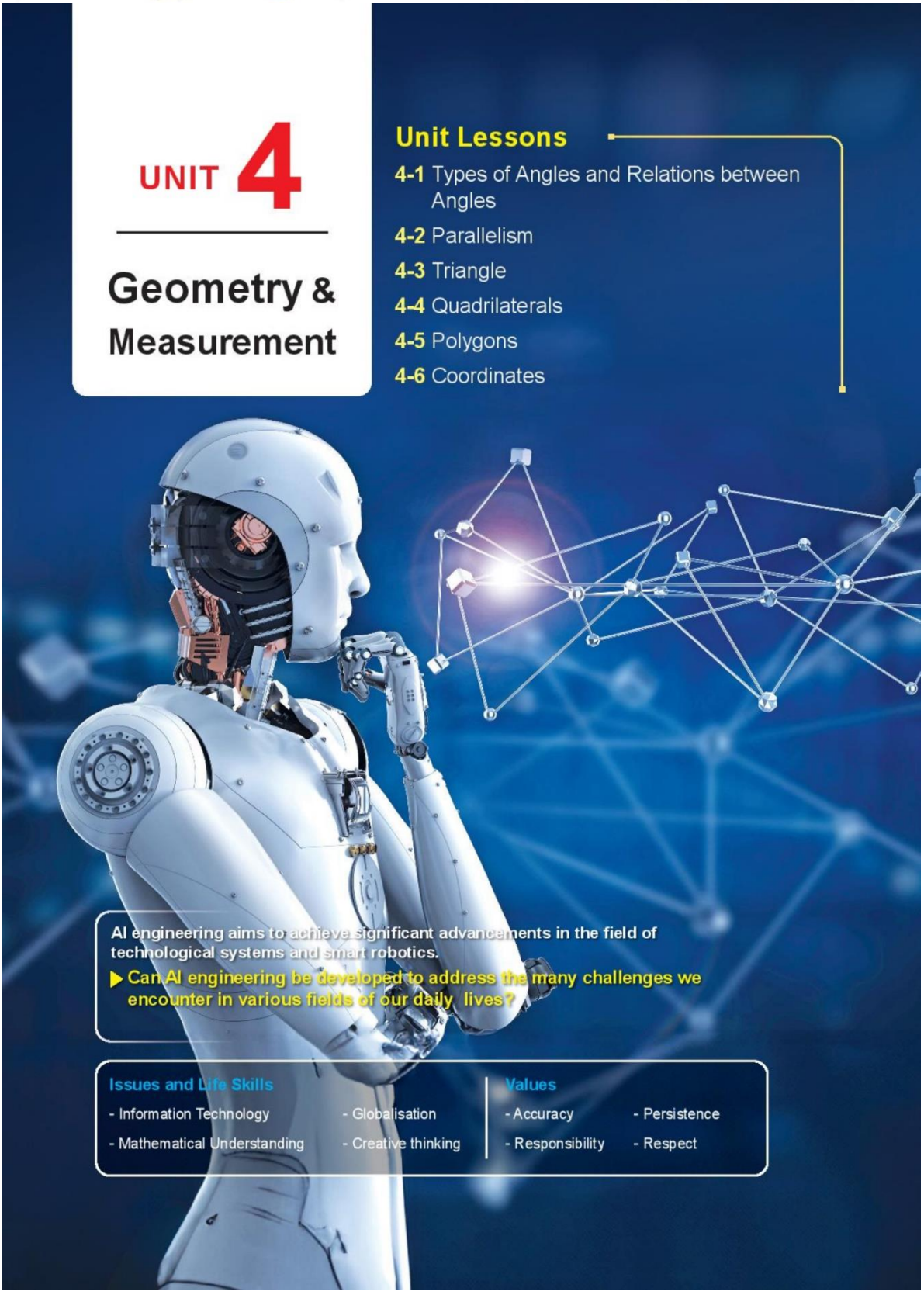
Represent the share of each partner in the project using a pie chart.



UNIT 4

Geometry & Measurement

Unit Lessons



AI engineering aims to achieve significant advancements in the field of technological systems and smart robotics.

▶ Can AI engineering be developed to address the many challenges we encounter in various fields of our daily lives?

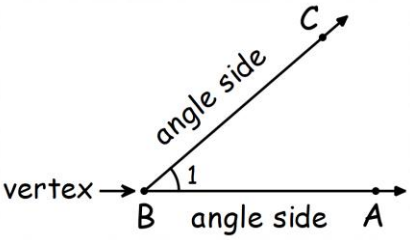
Issues and Life Skills		Values	
- Information Technology	- Globalisation	- Accuracy	- Persistence
- Mathematical Understanding	- Creative thinking	- Responsibility	- Respect

Lesson (1) Types of Angles and Relations between Angles

Concept of angle

Is the union of two rays with the same initial point.

* The initial point of the two rays is called vertex of the angle.
* Each one of the two rays is called the side of the angle.



Ex: In the opposite figure:

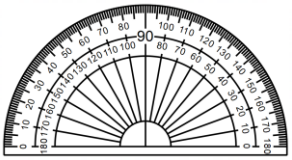
$\overrightarrow{BA} \cup \overrightarrow{BC} = \angle ABC$

$\angle ABC$ can be written also as: $\angle CBA$, $\angle B$ or $\angle 1$

Units of measurement for the angles

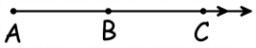
Degree , minute , second ,

* We use the protractor to measure an angle.
* A degree is equal to 60 minutes. ($1° = 60'$)
* A minute is equal to 60 seconds. ($1' = 60''$)



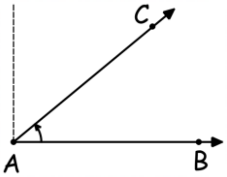
Types of Angles

1 Zero angle:



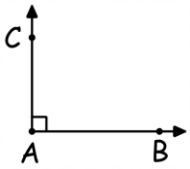
Its measure = 0°
Sides are coincide

2 Acute angle:



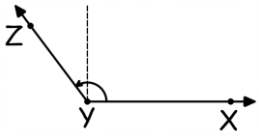
Its measure more than 0° and less than 90°

3 Right angle:



Its measure = 90°

4 Obtuse angle:

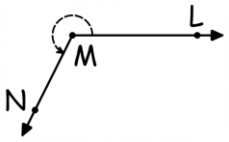


Its measure more than 90° and less than 180°

5 Straight angle:

Its measure = 180°
Sides are on opposite directions

6 Reflex angle:



Its measure more than 180° and less than 360°

Remark

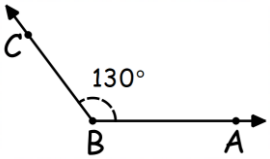
The measure of the circle is 360° , **then**:

* The measure of $\angle M$ + the reflex of $\angle M = 360°$

Ex: In the opposite figure:

$M(\angle ABC) = 130°$

Find: the measure of the reflex $\angle ABC$



Solution:

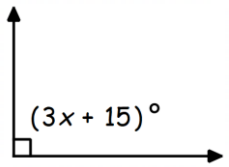
$m(\angle ABC) + m(\angle ABC \text{ reflex}) = 360°$

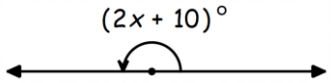
$m(\angle ABC \text{ reflex}) = 360° - 130° = 230°$

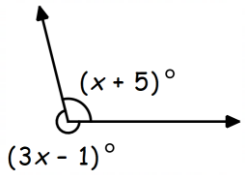
Ex: Complete:

m(∠ ABC)	85°	100°	135°	52.5°
m(∠ ABC) reflex	.....	.....	.....	.....

Ex: Find the value of x in each of the following:

1) 

2) 

3) 

Relation between angles

① Adjacent angles

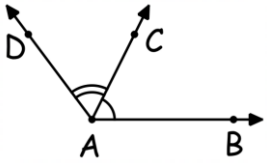
They are two angles thar lie in the same plane, share the same vertex,
have a common side, and their outer sides lie on different sides of the common side.

Ex: ∠ BAC and ∠ CAD are adjacent angles, **because**:

they share the same vertex "A" , and have a common side $\overrightarrow{AC}$

the outer sides $\overrightarrow{AD}$ and $\overrightarrow{AB}$ lie on different sides

of the common side $\overrightarrow{AC}$



Remarks

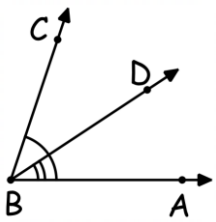
1) **In the opposite figure:**

∠ ABC , ∠ ABD are not adjacent angles,

because:

the sides $\overrightarrow{BD}$, $\overrightarrow{BC}$ are on the same side

of the common side $\overrightarrow{BA}$

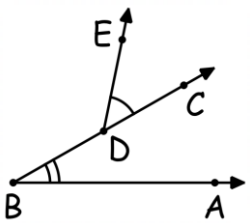


2) **In the opposite figure:**

∠ ABC , ∠ CDE are not adjacent angles,

because:

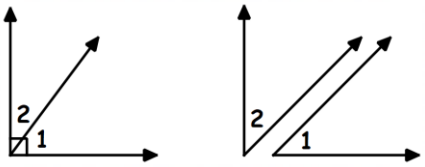
they do not share the same vertex,
and do not have a common side.



② Complementary angles

When the sum of two angles equal to 90° , they are called **complementary angles**.

Ex: If m(∠ 1) = 60° and m(∠ 2) = 30°
Then: ∠ 1 and ∠ 2 are complementary angles, because:
m(∠ 1) + m(∠ 2) = 60° + 30° = 90°



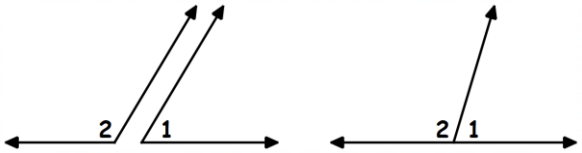
Remarks

* The two complementary angles may be acute angles or one zero angle and the other is right angle.
* Complementary angles of the same angle (or equal angles) equal in measure,
if ∠ A complements ∠ B , ∠ B complements ∠ C ⇝ then m(∠ A) = m(∠ C)

③ Supplementary angles

When the sum of two angles equal to 180° , they are called **supplementary angles**.

Ex: If m(∠ 1) = 60° and m(∠ 2) = 120°
Then: ∠ 1 and ∠ 2 are supplementary angles,
because:
m(∠ 1) + m(∠ 2) = 60° + 120° = 180°



Remarks

* The two supplementary angles may be acute angle and obtuse angle or one zero angle and the other is straight angle or two right angles.
* Supplementary angles of the same angle (or equal angles) equal in measure,
if ∠ A supplements ∠ B , ∠ B supplements ∠ C ⇝ then m(∠ A) = m(∠ C)

Ex: Choose the correct answer:

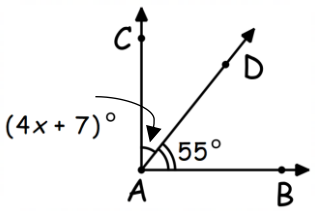
① The angle 68° complements angle of measure ……
(a) 22° (b) 125° (c) 110° (d) 305°

② the angle 23° supplements angle of measure ……
(a) 23° (b) 67° (c) 157° (d) 337°

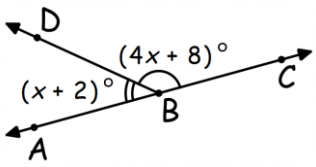
③ If m(∠ X) = m(∠ Y) , ∠ X complements ∠ Y, then ∠ Y = ……
(a) 90° (b) 45° (c) 180° (d) 360°

Ex: Find the value of x in each of the following:

♺ If $\overrightarrow{AB}$ is perpendicular to $\overrightarrow{AC}$

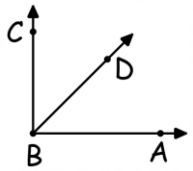


♺ If A, B, C are on a straight line



Complementary adjacent angles

The two adjacent angles in which their outer sides are perpendicular, are complementary.

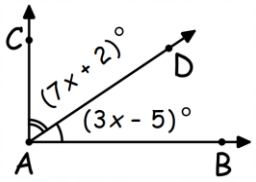


* If $\overrightarrow{BA} \perp \overrightarrow{BC}$, then:

$m(\angle ABD) + m(\angle DBC) = 90°$

Ex: In the opposite figure, find the value of x:

♺ If $\overrightarrow{AC} \perp \overrightarrow{AB}$



Solution: the 2 angles are complementary

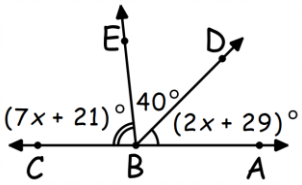
$7x + 2 + 3x - 5 = 90°$

$10x - 3 = 90°$

$10x = 90° + 3$

$x = \frac{93°}{10} = 9.3°$

♺ A, B, C are on a straight line



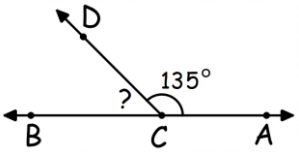
Supplementary adjacent angles

The two adjacent angles which formed from a straight line and a ray (its endpoint on this line) , are supplementary.

Ex: In the opposite figure:

If $\overline{AB} \cap \overline{CD} = \{ C \}$, $m(\angle ACD) = 135°$

then: $m(\angle BCD) = \ldots\ldots\ldots°$



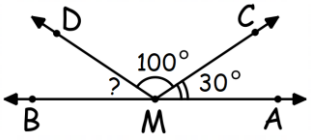
solution:

$m(\angle BCD) = 180 - 135 = 45°$

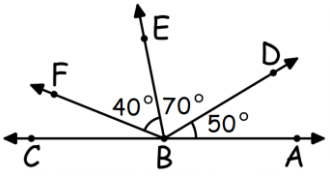
Ex: In the opposite figure:

♺ $M \in \overleftrightarrow{AB}$, $m(\angle AMC) = 30°$

$m(\angle CMD) = 100°$, **find:** $m(\angle DMB)$



♺ $B \in \overleftrightarrow{AC}$, **find:** $m(\angle FBC)$

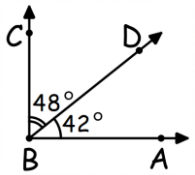


The two outer sides of the two adjacent angles

① If the two adjacent angles are complementary, then:
their outer sides are perpendicular.

$m(\angle ABD) + m(\angle DBC) = 42° + 48° = 90°$

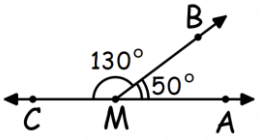
then: $\overrightarrow{BC}$ and $\overrightarrow{BA}$ are perpendicular.



② If the two adjacent angles are supplementary, then:
their outer sides are on the same straight line.

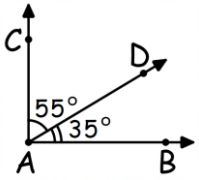
$m(\angle AMB) + m(\angle BMC) = 50° + 130° = 180°$

then: $\overrightarrow{BC}$ and $\overrightarrow{BA}$ are on the same straight line.

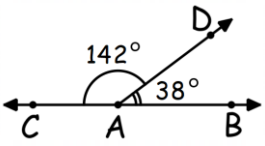


Ex: In the opposite figure:

① Is $\overrightarrow{AB} \perp \overrightarrow{AC}$? Give a reason.



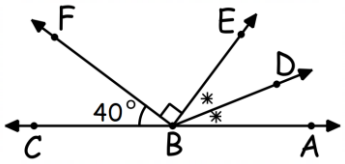
② Do $\overrightarrow{AB}$ and $\overrightarrow{AC}$ form a straight line ?
Give a reason.



Ex: In the opposite figure:

$m(\angle FBE) = 90°$, $B \in \overleftrightarrow{AC}$, $m(\angle CBF) = 40°$, $\overrightarrow{BD}$ bisects $\angle ABE$

Find: $m(\angle ABD)$



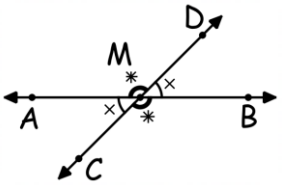
The **angle bisector** is a ray divides the angle into two equal parts in measure

④ Vertically opposite angles

The two vertically opposite angles are non-adjacent angles formed by two intersecting straight lines.

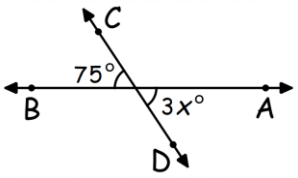
$\angle AMC$ and $\angle BMD$ are (V.O.A) ⇝ $m(\angle AMC) = m(\angle BMD)$
$\angle AMD$ and $\angle BMC$ are (V.O.A) ⇝ $m(\angle AMD) = m(\angle BMC)$

The two vertically opposite angles are **congruent** (equal in measure)

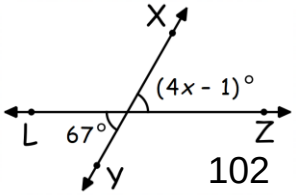


Ex: Find the value of *x*:

① $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{ M \}$



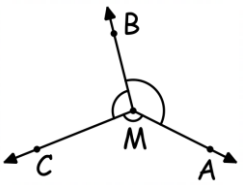
② $\overleftrightarrow{XY} \cap \overleftrightarrow{LZ} = \{ M \}$



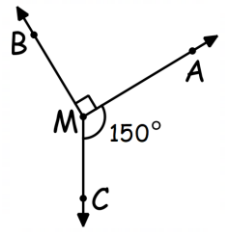
⑤ Accumulative angles at a point

The sum of measures of accumulative angles at a point = 360°

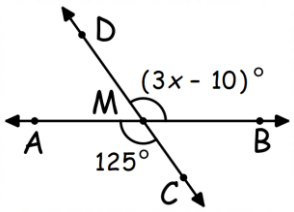
m(∠ AMC) + m(∠ AMB) + m(∠ BMC) = 360°



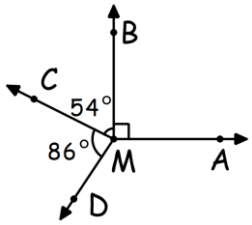
Ex: In the opposite figures:

① 

Find: m(∠ BMC)

② 

Find: the value of x

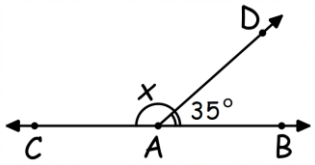
③ 

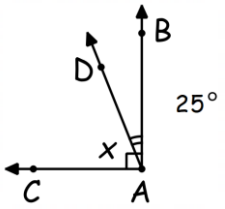
Find: m(∠ AMD)

Homework

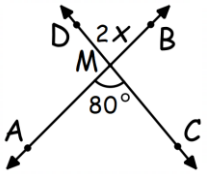
① calculate the value of x:

① $A \in \overleftrightarrow{BC}$



② 

③ $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{ M \}$



② Choose the correct answer:

① What is the type of the angle supplements an acute angle?

(a) Acute (b) Obtuse (c) Straight (d) Reflex

② What is the type of the angle complements a right angle?

(a) Acute (b) Obtuse (c) Zero (d) Straight

③ If the angles A and B are complementary, and m(∠ A) = 40°, then m(∠ B) = ……

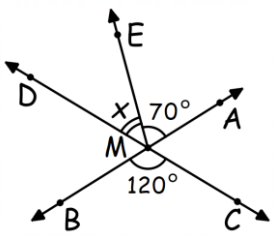
(a) 40° (b) 50° (c) 90° (d) 140°

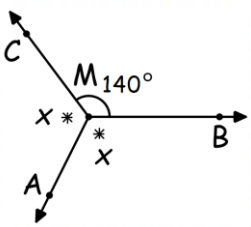
④ What is the measure of the angle that supplements an angle of measure 34° 60'' ?

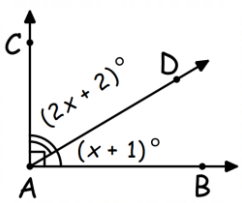
(a) 55° (b) 56° (c) 145° (d) 146°

③ Find the value of x in each of the following:

① $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{ M \}$

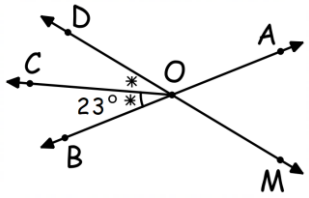


② 

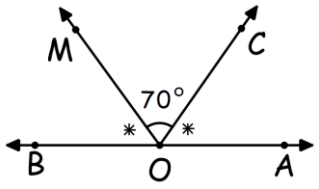
③ 

④ In each of the following, calculate m(∠ AOM):

① $\overleftrightarrow{AB} \cap \overleftrightarrow{DM} = \{ O \}$

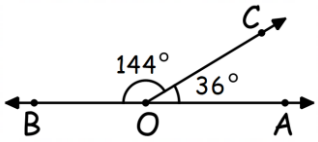


② $O \in \overleftrightarrow{AB}$

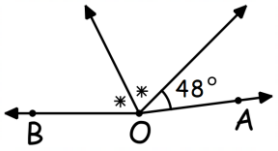


⑤ In each of the following, do $\overrightarrow{OA}$ and $\overrightarrow{OB}$ form a straight line or not ? why ?

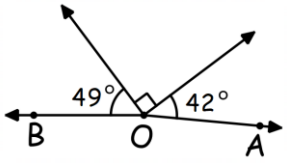
①



②



③



⑥ Complete:

① The angle 50° complements angle ……° and supplements angle ……°

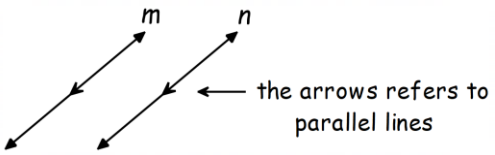
② The obtuse angle supplements …… angle.

③ If m(∠ A) = 70°, then m(∠ A reflex) = ……°

④ The sum of the accumulative angle at a point equals ……°

Lesson (2) Parallelism

The relation between two straight lines

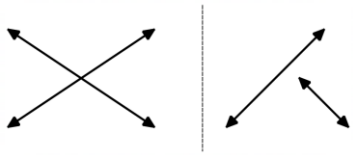


1 Two parallel straight lines:

* Are two straight lines that never intersect.
  m // n

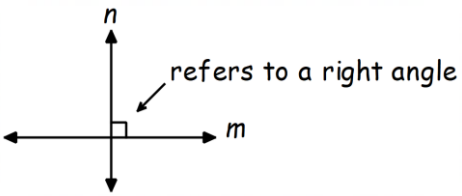
2 Two intersecting straight lines:

Are two straight lines that intersect at a point.



3 Two perpendicular straight lines:

* Are two straight lines intersect and form 4 right angles.
  m ⊥ n



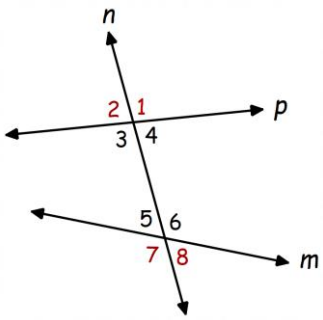
Transversal

* Is a straight line (n) that intersects two or more straight lines (p , m).

* if a straight line that intersects two straight lines, then: (8) angles are formed:

  1 (4) angles are called **interior angles** ∠ **3** , ∠ **4** , ∠ **5** , ∠ **6**

  2 (4) angles are called **exterior angles** ∠ **1** , ∠ **2** , ∠ **7** , ∠ **8**



The relation between pairs of angles formed when a straight line intersects two parallel straight lines

Any pair of angles formed by the intersection are **equal in measure** or **supplementary.**

Ex: If m // n and the straight line (q) is a transversal, then:

1 Each two **corresponding** angles are equal in measure:

m(∠ 2) = m(∠ 6) m(∠ 1) = m(∠ 5)
m(∠ 3) = m(∠ 7) m(∠ 4) = m(∠ 8)

2 Each two **alternating** interior angles are equal in measure:

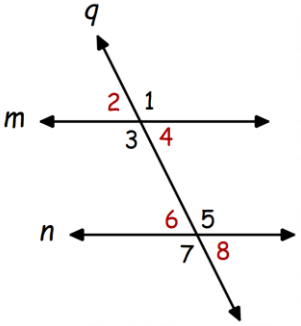
m(∠ 3) = m(∠ 5) m(∠ 4) = m(∠ 6)

3 Each two **alternating** exterior angles are equal in measure:

m(∠ 2) = m(∠ 8) m(∠ 1) = m(∠ 7)

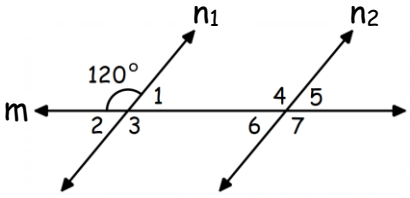
4 Each two **interior** angles are supplementary:

m(∠ 4) + m(∠ 5) = 180° m(∠ 3) + m(∠ 6) = 180°



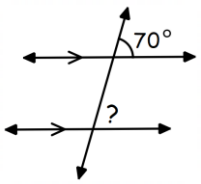
Ex: In the opposite figure:

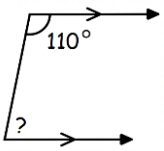
There are three angles, the measure of each is 120°
Identify these angles and state the reason.
Given that n_1 // n_2 and m is a transversal.

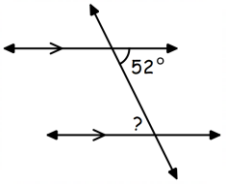


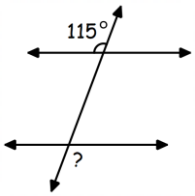
Ex: In the opposite figure:

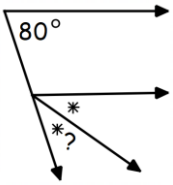
Find the measure of the angle indicated by (?) , state the reason.

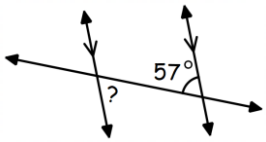
1) 

2) 

3) 

4) 

5) 

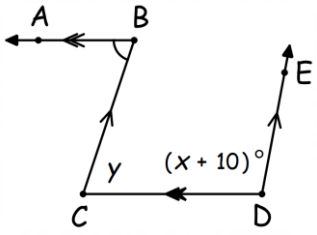
6) 

How to write a proof in geometry

1) Read the problem carefully.
2) Write the given information in form of points.
3) Write the required.
4) Think of a plan for the proof, which includes the steps needed to reach the required.
5) Write the proof by writing mathematical statements and its reason.

Ex: In the opposite figure:

Find with proof the value of x , y



Solution:

Given: $\overrightarrow{CD}$ // $\overrightarrow{BA}$, $\overrightarrow{BC}$ // $\overrightarrow{DE}$, m(∠ ABC) = 70°

Required: Find the value of x , y

Proof:

∵ $\overrightarrow{CD}$ // $\overrightarrow{BA}$, $\overline{BC}$ is a transversal.

∴ m(∠ ABC) = m(∠ BCD) = 70° (alternate angles)

∴ y = 70°

∵ $\overrightarrow{BC}$ // $\overrightarrow{DE}$, $\overline{CD}$ is a transversal.

∴ m(∠ CDE) = m(∠ BCD) = 180° (interior angles)

∴ m(∠ CDE) = 180° − 70° = 110°

∴ x + 10° = 110°

∴ x = 110° − 70° = 100°

* The symbol (∵) read as **since** and is placed before a given fact or theorem.

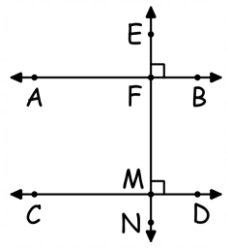
* the symbol (∴) read as **then** and is placed before a derived mathematical statement.

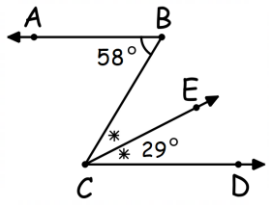
How to prove that two straight lines are parallel

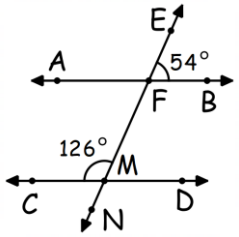
Two straight lines are parallel if a transversal intersects them and one of the following conditions has occurred:

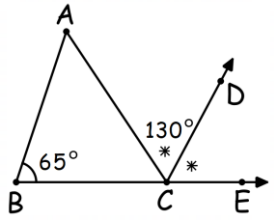
1. Two corresponding are equal in measure.
2. Two alternate angles (interior or exterior) are equal in measure.
3. Two interior angles on the same side of the transversal are supplementary.

Ex: In each of the following, prove that $\overleftrightarrow{AB}$ // $\overleftrightarrow{CD}$:

1) 

2) 

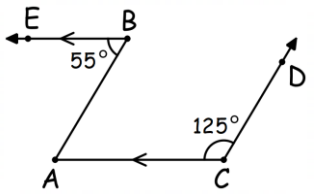
3) 

4) 

Ex: In the opposite figure:

$\overline{CA}$ // $\overrightarrow{BE}$, m(∠ B) = 55° , m(∠ C) = 125°

Prove that: $\overline{AB}$ // $\overrightarrow{CD}$



Given	Conclusion	Conclusion	Required
$\overline{CA}$ // $\overrightarrow{BE}$ AB is a transversal	m(∠ A) = m(∠ B) = 55° (alternating interior angles)	m(∠ A) + m(∠ C) = 55° + 125° = 180° (interior angles)	$\overline{AB}$ // $\overrightarrow{CD}$

Solution:

Given: $\overline{CA}$ // $\overrightarrow{BE}$, $\overleftrightarrow{AB}$ is a transversal.

Required to prove: $\overline{AB}$ // $\overrightarrow{CD}$

Proof:

∵ CA // BE , AB is a transversal

∴ m(∠ A) = m(∠ B) = 55° (alternating interior angles)

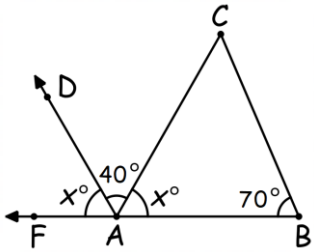
∵ m(∠) + m(∠) = 55° + 125° = 180° (interior angles on the same side of the transversal)

∴ $\overline{AB}$ // $\overrightarrow{CD}$

Ex: In the opposite figure:

F ∈ $\overleftrightarrow{BA}$, m(∠ CAD) = 40° , m(∠ B) = 70°

Prove that: $\overrightarrow{AD}$ // $\overline{BC}$

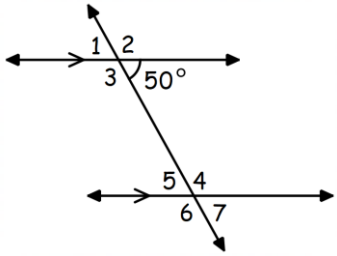


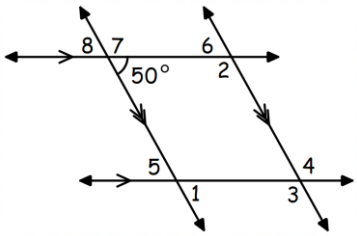
Homework

① Complete each of the following:

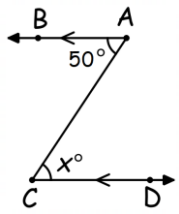
1. If a straight line intersects two parallel lines, then any two alternating angles are ……
2. If a straight line intersects two parallel straight lines, then any two corresponding angles are ……
3. If a straight line intersects two parallel straight lines, then any two interior angles on the same side of the transversal are ……
4. If a straight line intersects two straight lines and produces two corresponding angles that are equal in measure, then the straight lines are ……
5. If a straight line intersects two straight lines and the interior angles on the same side of the transversal are supplementary, then the two straight lines are ……

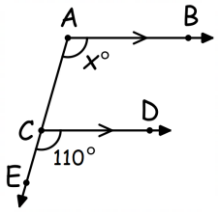
② Find the measure of the numbered angles in each figure, state the reason for each:

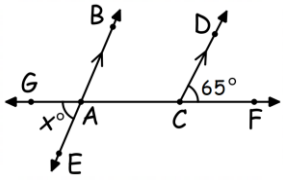
1. 

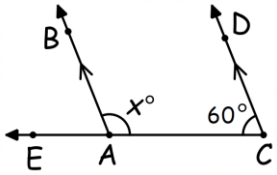
2. 

③ Find the value of x in each of the following:

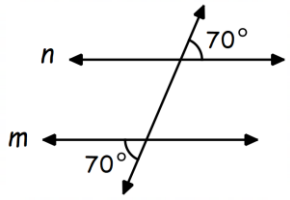
1. 

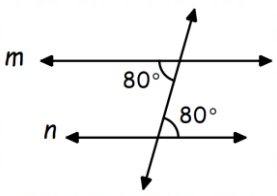
2. 

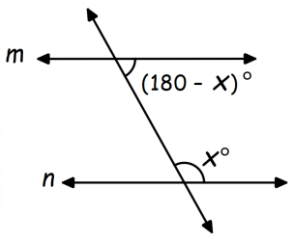
3. 

4. 

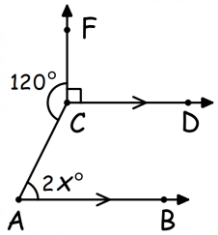
④ In each of the following, prove that: m // n:

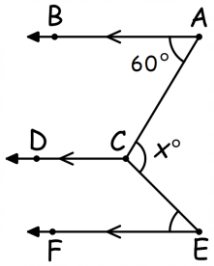
1. 

2. 

3. 

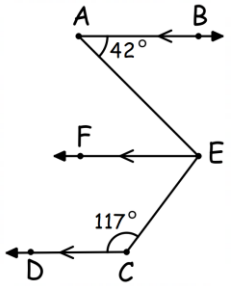
⑤ Calculate with proof the value of x :

1. 

2. 

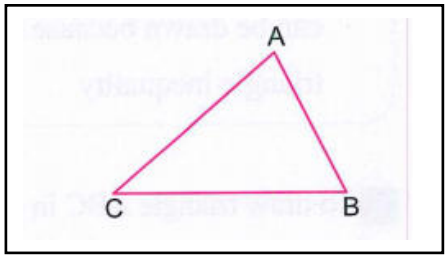
⑥ In the opposite figure:

AB // CD , EF // CD ,
m(∠ A) = 42° ,
m(∠ C) = 117°
Find by proof: m(∠ AEC)



Lesson (3) **Triangle**

Any triangle has 3 angles, 3 sides and 3 vertices



Rule

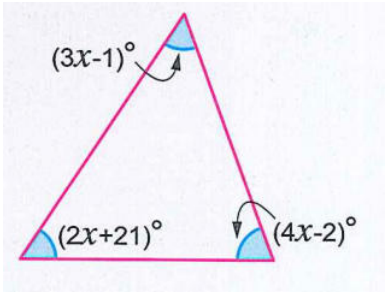
The sum of the interior angles in any triangle is 180

$$m(A) + m(B) + m(C) = 180$$

Example 1

In the opposite figure :

Find the value of x



Solution

$\because$ The sum of the measures of the interior angles of a triangle is 180°

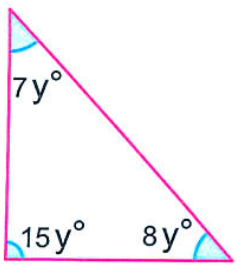
$\therefore 3x - 1° + 4x - 2° + 2x + 21° = 180°$

$\therefore 9x + 18° = 180°$ $\quad\quad$ $\therefore 9x = 180° - 18° = 162°$

$\therefore x = \frac{162°}{9} = 18°$

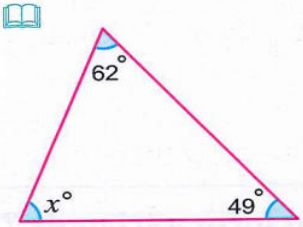
Try by your self :

Find the value of Y



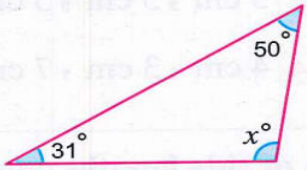
1) Find the value of the variable in each of the following :

1



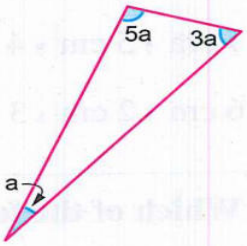
X = ...

2



X = ...

3



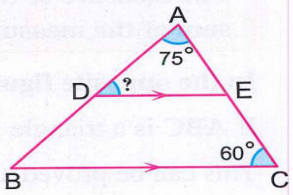
a = ...

Example ❷

$\overline{DE} \parallel \overline{BC}$

$m(\angle A) = 75°$, $m(\angle C) = 60°$

Find by proof : $m(\angle ADE)$



Solution

$\because \overline{DE} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore m(\angle AED) = m(\angle ACB) = 60°$ (Corresponding angles)

In triangle ADE :

$\because$ The sum of the measures of the interior angles of a triangle is 180°

$\therefore m(\angle ADE) = 180° - [75° + 60°] = 45°$

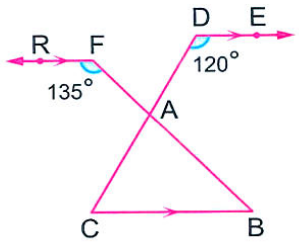
Try by your self :

In the opposite figure :

$\overrightarrow{DE} \parallel \overrightarrow{CB} \parallel \overrightarrow{FR}$

$m(\angle EDC) = 120°$, $m(\angle RFB) = 135°$

Find the measures of the interior angles of the triangle ABC

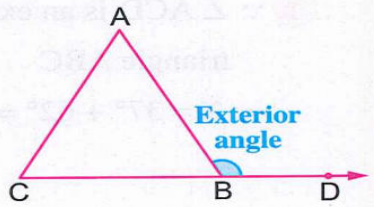


The Exterior Angle of a Triangle

In the opposite figure :

If ABC is a triangle , $D \in \overrightarrow{CB}$, $D \notin \overline{CB}$

then $\angle ABD$ is called an exterior angle of the triangle ABC



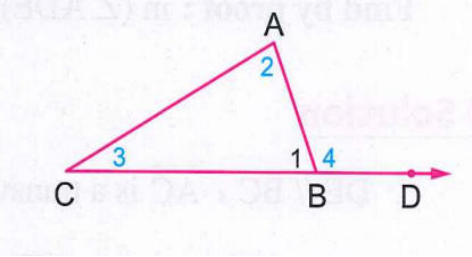
Rule :

The measure of the exterior angle of any triangle is equal to the sum of the measures of the two non-adjacent interior angles

$m(\angle 4) = m(\angle 2) + m(\angle 3)$

m(3) = m(4) - m(2)

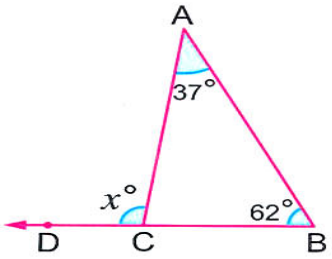
m(2) = m(4) - m(3)



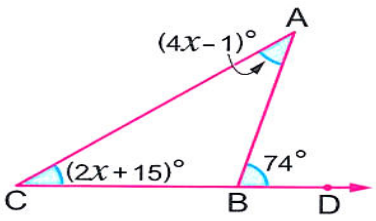
Example ❸

Find the value of the variable in each of the following :

1



2



Solution

1 ∵ $\angle ACD$ is an exterior angle of triangle ABC

$\therefore \mathcal{X} = 37° + 62° = 99°$

2 ∵ $\angle ABD$ is an exterior angle of triangle ABC

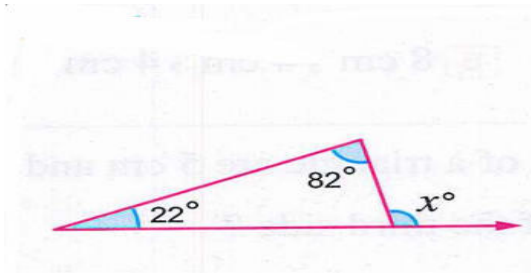
$\therefore 4\mathcal{X} - 1° + 2\mathcal{X} + 15° = 74°$

$\therefore 6\mathcal{X} + 14° = 74°$

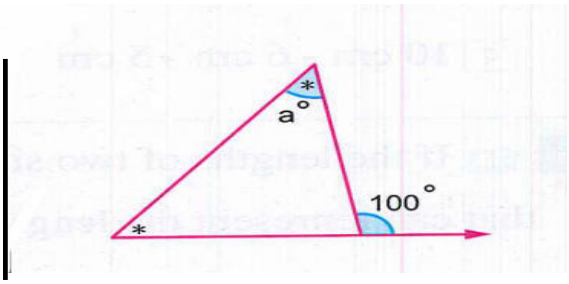
$\therefore 6\mathcal{X} = 74° - 14° = 60$

$\therefore \mathcal{X} = \frac{60°}{6} = 10°$

2) Find the value of the variable in each of the following :



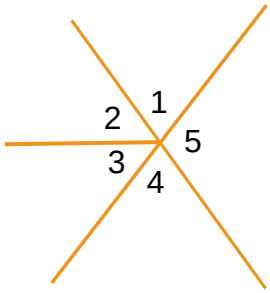
X = ...



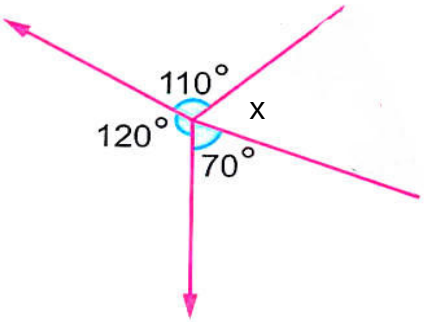
a = ...

Accumulative Angles at a point:

The sum of measures of accumulative angles at a point is 360



m(1) + m(2) + m(3) + m(4) + m(5) = 360



X =.....

Triangle Inequality

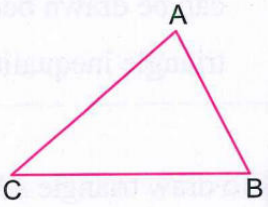
The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

For example : In triangle ABC :

AB + BC > AC

AB + AC > BC

AC + BC > AB



Example (4)

Can a triangle be drawn with side lengths of :

1) **4cm , 6cm , 11cm**

$$4 + 6 = 10$$

$$4 + 6 < 11$$

4 , 6 and11 can't be a sides of triangle as the sum of the shortest less than the greatest one

2) **14cm , 9cm ,7cm**

$$7 + 9 = 16$$

$$7 + 9 > 14$$

7 , 9 and 14 can't be a sides of triangle as the sum of the shortest less than the greatest one

Try by your self :

Can a triangle be drawn with side lengths of :

1. 2 cm , 3 cm , 4 cm
2. 3 cm , 6 cm , 2 cm
3. 10 cm , 3 cm , 7 cm

Note:

The length of any side of a triangle is greater than the difference between the lengths of the other two sides and smaller than their sum.

Example 5

If the lengths of two sides of a triangle are 7 cm and 3 cm , what is the largest integer that can represent the length of the third side ?

Solution

$$7 - 3 = 4$$

$$7 + 3 = 10$$

The length of the third side is greater than the difference between 7 cm and 3 cm and smaller than the sum of 7 cm and 3 cm.

This means the length of the third side is greater than 4 cm and smaller than 10 cm. Since the length of the third side must be an integer , the possible lengths for the third side could be 5 cm or 6 cm or 7 cm or 8 cm or 9 cm. Therefore , the largest integer length for the third side is 9 cm.

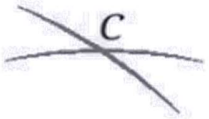
Triangle Inequality

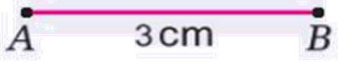
Practical Activity

Draw a triangle with side lengths 3 cm, 4 cm, and 5 cm, using a ruler and a compass.

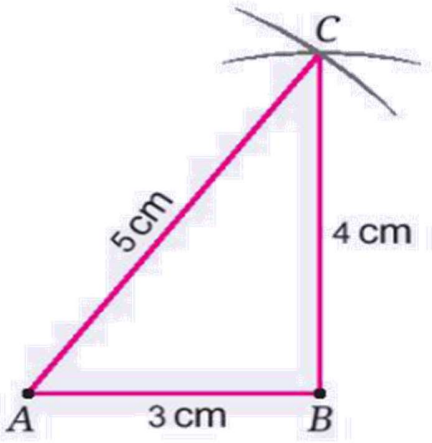
1. Use the ruler to draw a line segment $\overline{AB}$ of length 3 cm.

2. Set the compass to a length of 4 cm, place the needle at point B, and draw an arc. Then, set the compass to a length of 5 cm, place the needle at point A, and draw another arc that intersects the first arc at point C.





3. Draw $\overline{AC}$ and $\overline{BC}$ to form triangle ABC with side lengths of 3 cm, 4 cm, and 5 cm.



Note

You can apply the triangle inequality without drawing it as follows :

Add the lengths of the shorter two sides and compare their sum to the length of the third side.

- If the sum of the lengths of the two shortest sides > the length of the third side , then the lengths can be side lengths of a triangle.
- If the sum of the lengths of the two shortest sides ≤ the length of the third side , then the lengths cannot be side lengths of a triangle.

Try by your self :

Can you draw a triangle with each of the following sets of side lengths:

1. 10 cm, 6 cm, 5 cm.
2. 8 cm, 4 cm, 4 cm.
3. 12 cm, 3 cm, 6 cm.

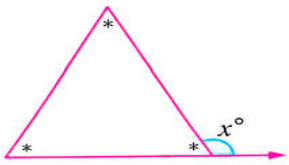
Exercises

1 Complete each of the following :

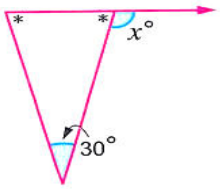
1. The sum of the measures of the interior angles of a triangle = ...°
2. The measure of the exterior angle of any triangle equals the sum of .....
3. In triangle ABC , if m (∠ A) = 70° and m (∠ C) = 50° , then m (∠ B) = ...°
4. The measure of each interior angle of a triangle can be equal to .....
5. If the measures of the three angles in a triangle are equal , then the measure of each angle is .....°

2 Find the value of the variable in each of the following :

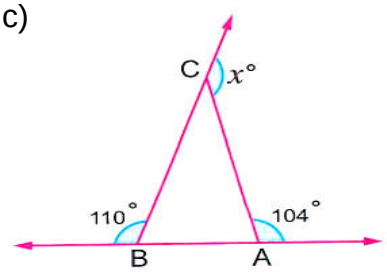
a)



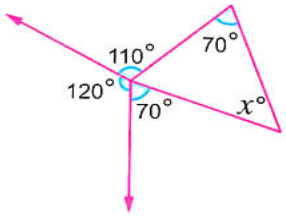
b)



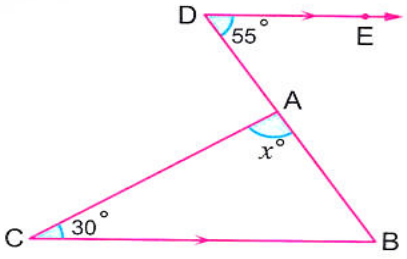
c)



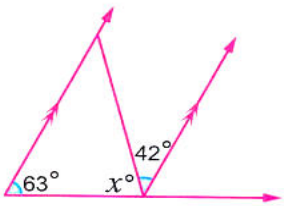
d)



e)



f)



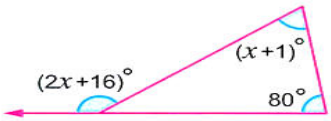
3 Which of the following sets of side lengths can be used to form a triangle ? Why ?

1. 9 cm , 5 cm , 4 cm
2. 7 cm , 6 cm , 5 cm
3. 10 m , 5 m , 2 m

4 If the lengths of two sides of a triangle are 5 cm and 2 cm , what is the largest integer that can represent the length of the third side ?

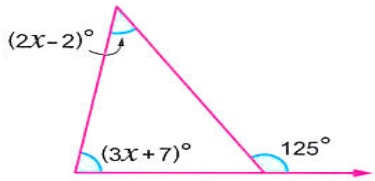
5 **Find with proof the value of the required angles in each of the following figures :**

1



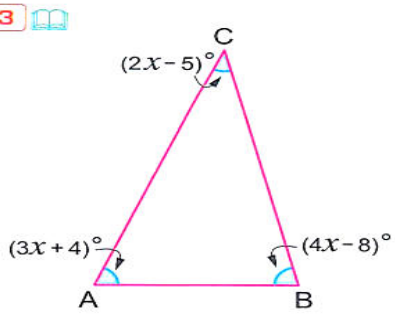
Find the value of x

2



Find the value of x

3



Find m (∠ B)

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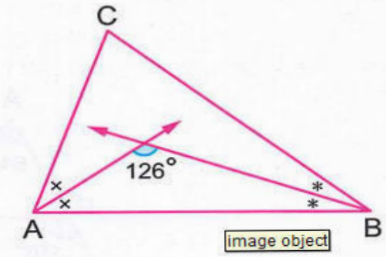
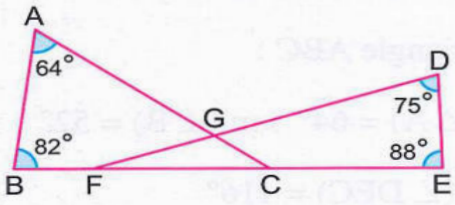
4)



Find m (∠ C)

5)



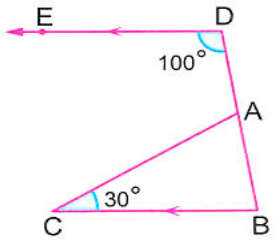
Find m (∠ FGC)

6) In the opposite figure :

$\overline{ED} // \overline{CB}$, $m(\angle D) = 100°$

, $m(\angle C) = 30°$, $A \in \overline{BD}$

Find : $m(\angle BAC)$

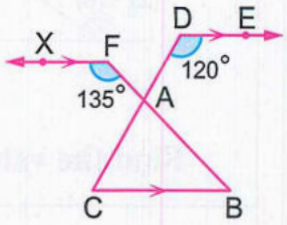


7) In the opposite figure :

$\overrightarrow{DE} // \overrightarrow{FX} // \overline{CB}$

، $m(\angle D) = 120°$ ، $m(\angle F) = 135°$

Calculate : the measures of the interior angles of the triangle ABC



8) In the opposite figure :

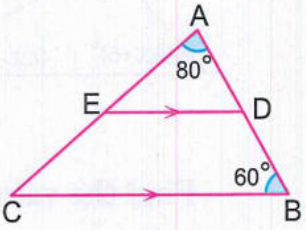
In triangle ABC :

$m(\angle A) = 80°$, $m(\angle B) = 60°$

$D \in \overline{AB}$, $\overline{ED} // \overline{CB}$

, $E \in \overline{AC}$

Find : $m(\angle AED)$ and $m(\angle DEC)$



Home Work

Choose the correct answer from the given ones :

1 If the sum of the measures of two angles in a triangle is 130°, then what is the measure of the third angle ?

(a)20° (b) 30° (c) 50° (d) 60°

2 In triangle XYZ , if m (∠ X) = 50° and m (∠ Y) = 100° , then what is the measure of ∠ Z ?

(a) 30° (b)50° (c) 80° (d) 100°

3 In triangle ABC ,
if m (∠ A) + m (∠ B) = 110° , then what is the measure of angle C ?

(a)110° (b) 90° (c) 70° (d) 55°

4 The sum of the measures of the interior angles of a triangle equals the measure of .....

(a) a right angle. (b) a straight angle.
(c) an acute angle. (d) a reflex angle.

5 If the measures of two angles in a triangle are 30° and 70° , which of the following cannot be the measure of an exterior angle of the triangle ?

(a)150° (b) 130° (c) 110° (d) 100°

6 The triangle consists of two ..... angles at least

(a) acute (b) obtuse
(c) right (d) reflex

7 Which of the following sets of numbers cannot be used as side lengths of a triangle ?

(a) 4 cm , 7 cm , 7 cm
(b) 3 cm , 4 cm , 7 cm
(c) 7 cm , 7 cm , 7 cm
(d) 9 cm , 7 cm , 5 cm

8 The lengths of two sides of an isosceles triangle are 3 cm and 7 cm. What is the length of the third side ?

(a) 3 cm (b) 4 cm
(c) 5 cm (d) 7 cm

9 The sum of the lengths of any two sides in a triangle ..... the length of the third side.

(a) smaller than (b) greater than
(c) equal (d) half

10 If ABC is a scalene triangle in which the length of $\overline{AC}$ is 3 cm , and the length of $\overline{BC}$ is 5 cm. How many integers could be the length of $\overline{AB}$?

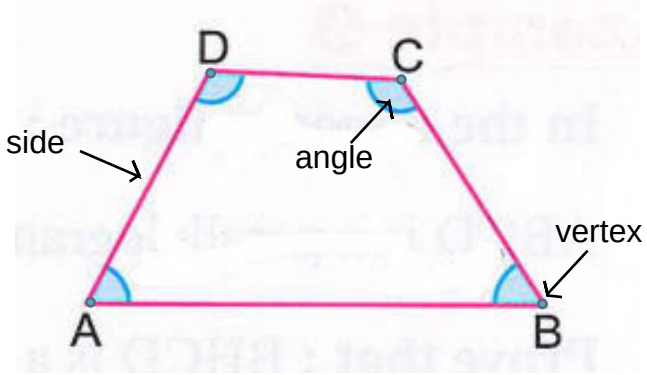
(a) 2 (b) 3 (c)4 (d) 5

11 In the triangle ABC , $\frac{AB + BC}{AC}$ 1

(a) < (b) > (c) = (d) ≥

Lesson (4) Quadrilaterals

Any quadrilateral has 4 sides , 4 angles and 4 vertices

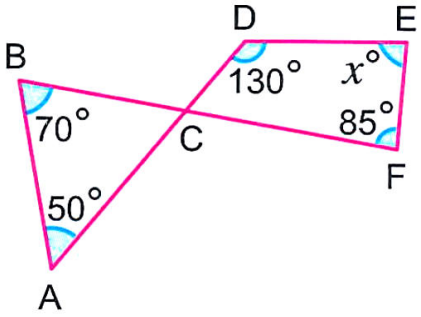


Rule:

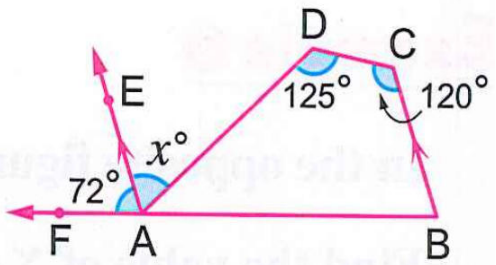
The sum of the measures of the interior angles of any quadrilateral is 360°.

1) Find with proof the value of x

1)



2)



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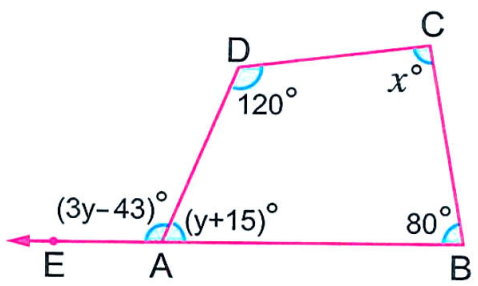

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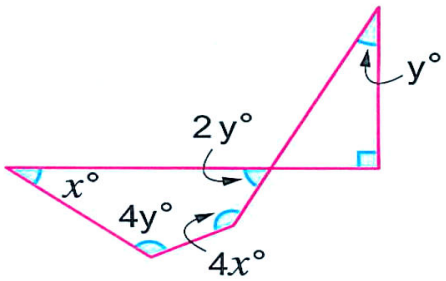
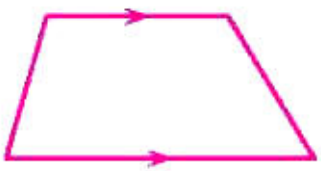


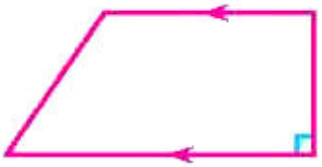
Special cases quadrilateral

1 Trapezium

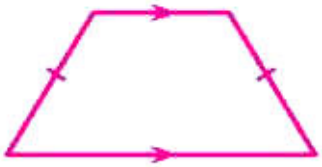
It is a quadrilateral that has only two parallel sides.



Trapezium

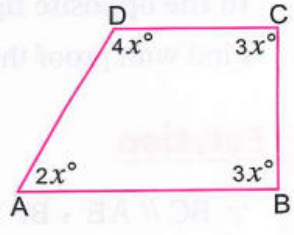


Right Trapezium

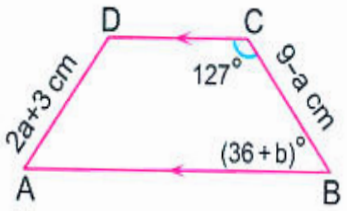


Isosceles Trapezium

2) **Find the value of x and determine if the quadrilateral ABCD is a right trapezium or not ?**

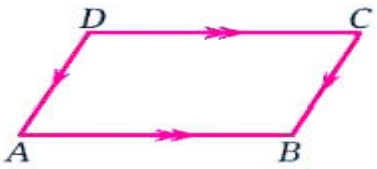


3) If the opposite figure ABCD is an isosceles trapezium , find the values of a and b



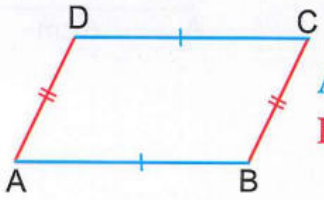
2 Parallelogram

A parallelogram is a quadrilateral in which each two opposite sides are parallel.



Properties of a Parallelogram

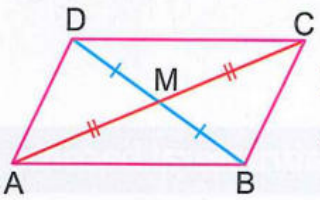
1 Each two opposite sides are equal in length.



$AB = DC$

$BC = AD$

2 The diagonals bisect each other.



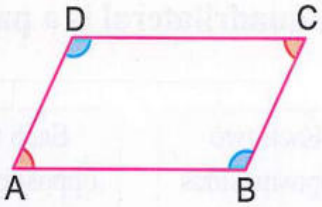
$BM = DM$

$AM = CM$

3 Each two opposite angles are equal in measure.

$m(\angle A) = m(\angle C)$

$m(\angle B) = m(\angle D)$



4 Each two consecutive angles are supplementary.

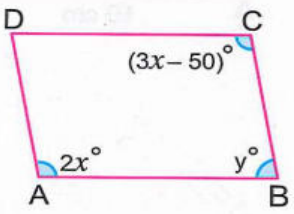
$m(\angle A) + m(\angle B) = 180°$, $m(\angle B) + m(\angle C) = 180°$

$m(\angle C) + m(\angle D) = 180°$, $m(\angle D) + m(\angle A) = 180°$

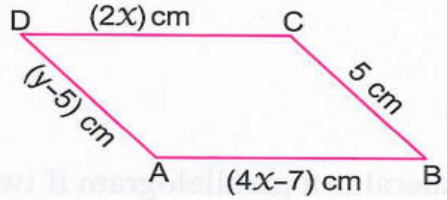
4) **In each of the following figures find the value of :**

x and y where ABCD is a parallelogram.

1



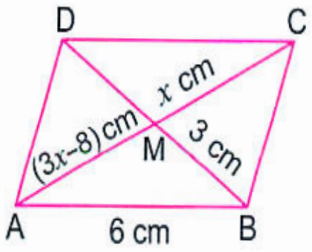
2



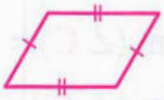
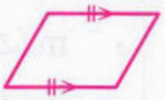
5) **In the opposite figure :**

ABCD is a parallelogram ,

Calculate : the perimeter of the triangle CMD

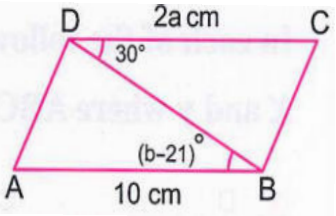


A quadrilateral is a parallelogram if any of the following conditions has occured

Each two opposite sides are parallel.	Each two opposite sides are equal in length.	Two opposite sides are both parallel and equal in length.	Diagonals bisect each other.	Each two opposite angles are equal in measure.
				

6) **In the opposite figure :**

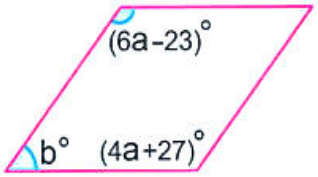
Find the values of a and b that make the quadrilateral ABCD a parallelogram.



7) **Find the values of a and b that make the quadrilateral a parallelogram :**

1

2

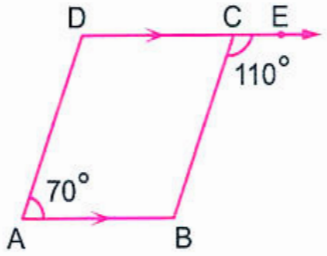


8) **In the opposite figure :**

ABCD is a quadrilateral , $\overline{AB} \parallel \overline{DC}$

$m(\angle DAB) = 70°$, $m(\angle ECB) = 110°$, $E \in \overrightarrow{DC}$

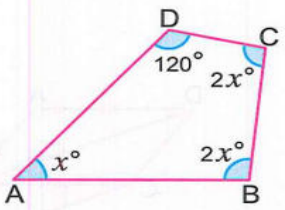
Prove that : ABCD is a parallelogram.



Exercises

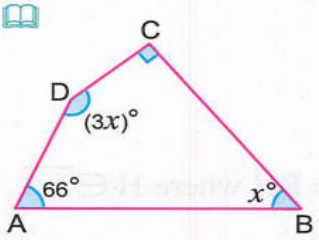
1 **Find the value of x in each of the following figures :**

1



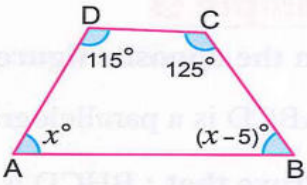
X =.....

2



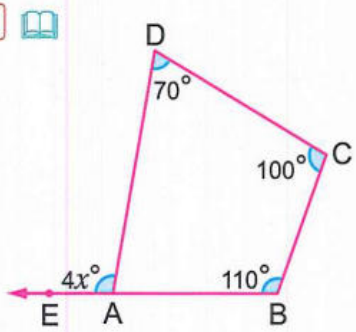
X = ...

3



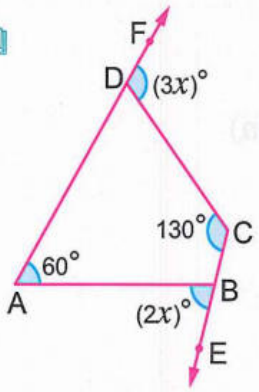
X = ...

4



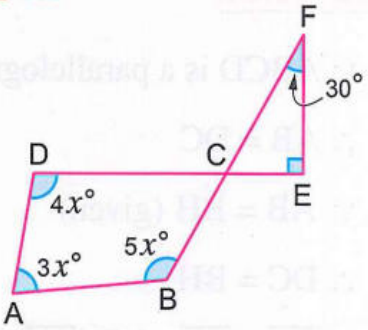
X =.....

5



X = ...

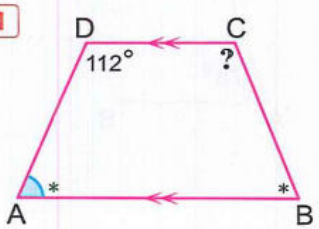
6



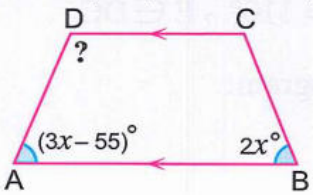
X = ...

2 **In each of the following shapes , if ABCD is a trapezium , find the measure of the angle indicated by the symbol (?) :**

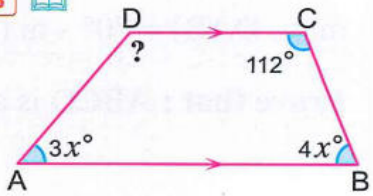
1



2 If m (∠ A) = m (∠ B)



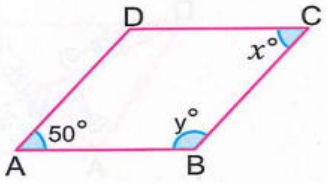
3



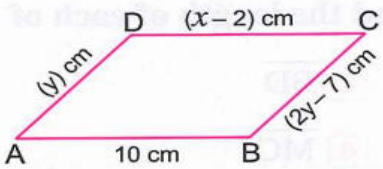
3) **In each of the following shapes , if ABCD is parallelogram , find the value of :**

x and y

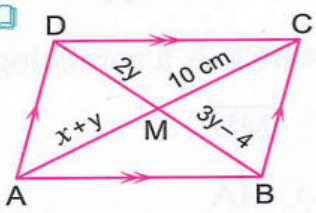
1



2

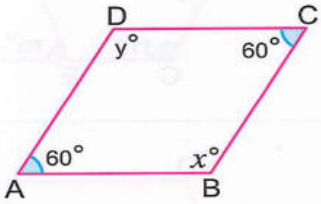


3

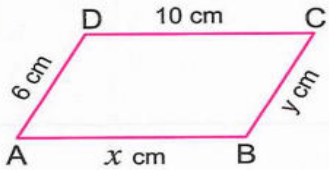


4) **Find the value of x and y that make ABCD a parallelogram in each of the following cases :**

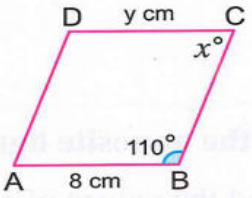
1



2



3



5) **In the opposite figure :**

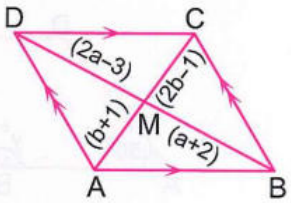
ABCD is a parallelogram. **Find the length of each of :**

1 $\overline{MB}$

2 $\overline{BD}$

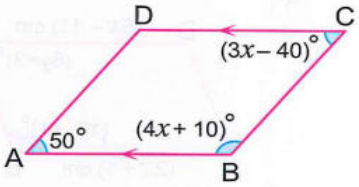
3 $\overline{MA}$

4 $\overline{MC}$



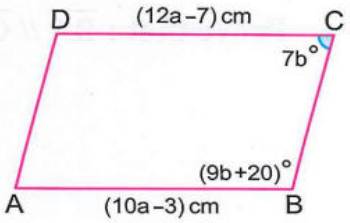
6) **In the opposite figure :**

Prove that : ABCD is a parallelogram.



7) In the opposite figure :

Prove that : ABCD is a parallelogram when $a = 2$, $b = 10°$



Home Work

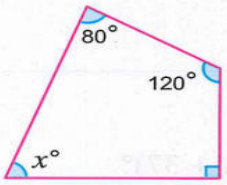
Choose the correct answer from the given ones :

1 **In the opposite figure :**

What is the value of X ?

(a) 70° (b) 80°

(c) 90° (d) 120°

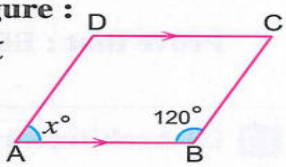


2 **In the opposite figure :**

What is the value of X that makes ABCD a parallelogram ?

(a) 109° (b) 120°

(c) 80° (d) 60°

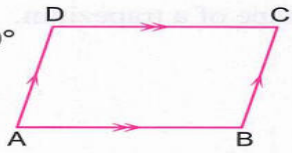


3 **In the opposite figure :**

If m (∠ A) + m (∠ C) = 140° , then what is the measure of ∠ B ?

(a) 70° (b) 40°

(c) 110° (d) 220°



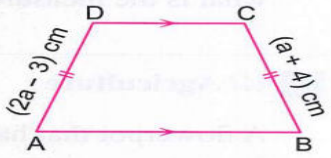
4 **In the opposite figure :**

ABCD is an isosceles trapezium.

What is the length of $\overline{AD}$ in centimeters ?

(a) 4 (b) 6

(c) 7 (d) 11

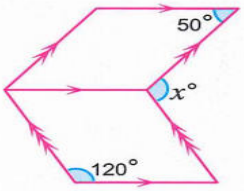


5 **In the opposite figure :**

What is the value of X ?

(a) 110° (b) 120°

(c) 130° (d) 140°



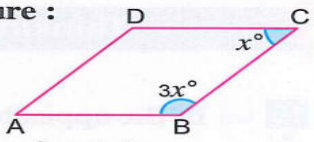
6 **In the opposite figure :**

ABCD is a parallelogram.

What is the measure of ∠ A ?

(a) 30° (b) 45°

(c) 60° (d) 135°



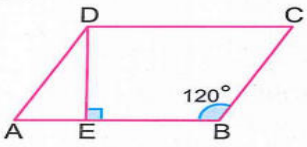
7 **In the opposite figure :**

ABCD is a parallelogram.

What is the measure of ∠ EDA ?

(a) 30° (b) 60°

(c) 45° (d) 120°



8 **In the opposite figure :**

ABCD is a parallelogram.

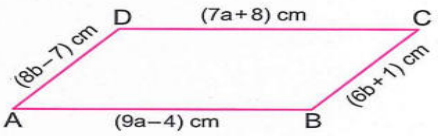
What is the value of a + b ?

(a) 4

(b) 7

(c) 9

(d) 10



Lesson (4) Follow Quadrilaterals

(Special Cases of Parallelograms)

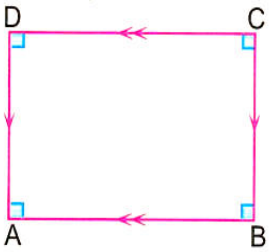
1 The Rectangle

The rectangle is a parallelogram that has one of its angles is a right angle

ABCD is a parallelogram ,

If m (∠ A) = 90°

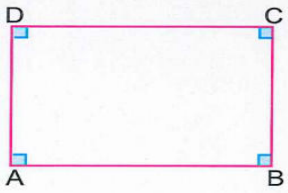
then ABCD is a rectangle.



Properties of the Rectangle

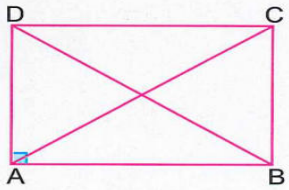
The rectangle is a parallelogram , so it has the same properties of the parallelogram , in addition to the following :

1 All its angles are equal in measure , the measure of each one is 90°.



m (∠ A) = m (∠ B) = m (∠ C) = m (∠ D) = 90°

2 Its diagonals are equal in length.

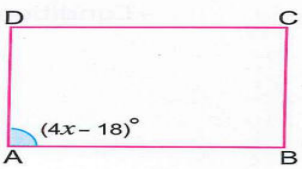


AC = BD

Example(1):

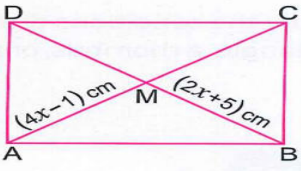
In each of the following figures , find the value of x , where ABCD is a rectangle :

1



m(A)= (4x-18) = 90
4x= 90 - 18
4x = ...
x = ...

2

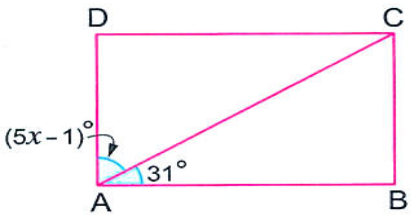


(4x-1) = (2x+5)
4x-2x = 1+5
2x = 6
x = 3

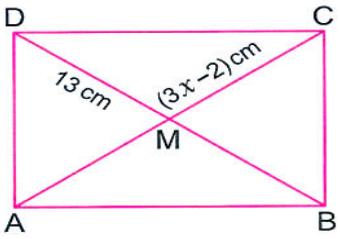
Example(2):

In each of the following figures , find the value of x where ABCD is a rectangle :

1

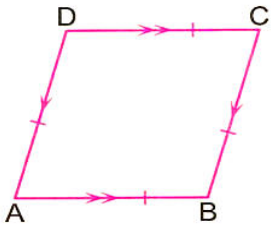


2



2 The Rhombus

The rhombus is a parallelogram in which two adjacent sides are equal in length



ABCD is a parallelogram ,

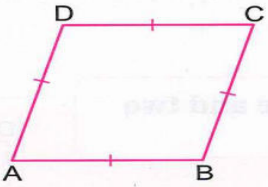
If AB = BC

then ABCD is a rhombus.

Properties of The Rhombus

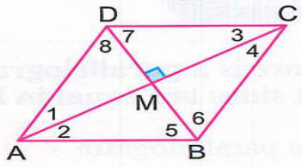
The rhombus is a parallelogram , so it has the same properties of the parallelogram , in addition to the following :

1 All its sides are equal in length.



AB = BC = CD = DA

2 The diagonals are perpendicular and bisect its interior angles.



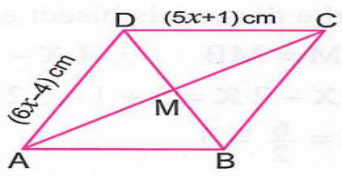
$\overline{AC} \perp \overline{BD}$

m (∠ 1) = m (∠ 2) = m (∠ 3) = m (∠ 4)

, m (∠ 5) = m (∠ 6) = m (∠ 7) = m (∠ 8)

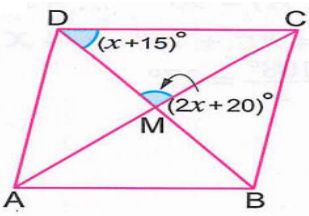
Example(3):

1



Find : the length of $\overline{BC}$

2



Find : m (∠ BCD)

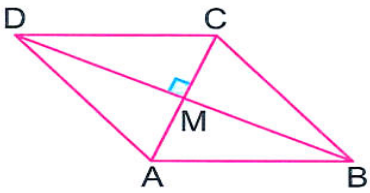
Example(4):

In the opposite figure , ABCD is a rhombus , its diagonals intersecting at point M :

1 If the length of $\overline{AB}$ is $(10\,x + 1)$ cm and the length of $\overline{DA}$ is $(x + 10)$ cm , find the length of $\overline{DC}$.

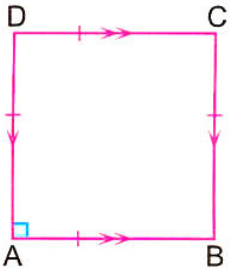
2 If $m(\angle CMB) = 2x°$, $m(\angle MDC) = (x - 15)°$

Find : $m(\angle CDA)$



3 The Square

The square is a parallelogram that has one right angle and two adjacent sides are equal in length.



ABCD is a parallelogram ,

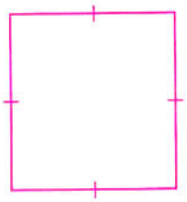
If $m(\angle A) = 90°$, $AB = BC$

then ABCD is a square.

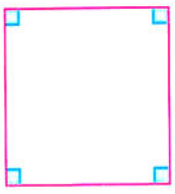
Properties of The Square

The square is a parallelogram with 4 right angles like a rectangle and 4 equal sides like a rhombus. Therefore , it has all the properties of the three shapes (parallelogram , rectangle and rhombus).

1 All its sides are equal in length.



2 All its angles are right angles.



3 Its diagonals are equal in length , perpendicular , and bisect its interior angles.

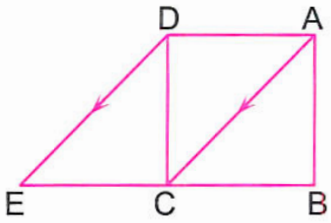
Example(5):

In the opposite figure :

ABCD is a square , $\overline{DE}$ is drawn parallel to $\overline{AC}$

and intersect $\overrightarrow{BC}$ at point E

1 **Prove that :** EC = CB 2 **Find :** m (∠ ADE)

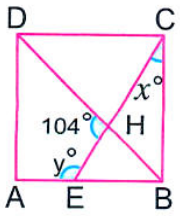


Example(6):

In the opposite figure :

ABCD is a square.

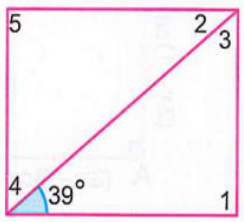
Find the values of x and y



Exercises

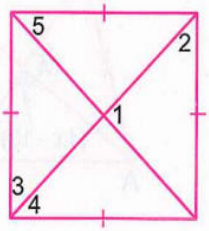
1 Find the measures of the numbered angles in each of the following :

1



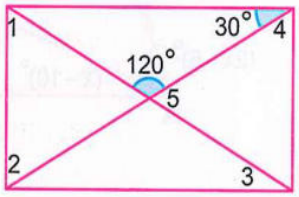
Rectangle

2



Square

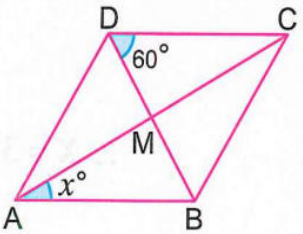
3



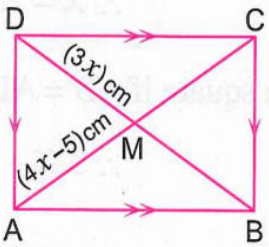
Rectangle

3 In each of the following figures , find the value of x :

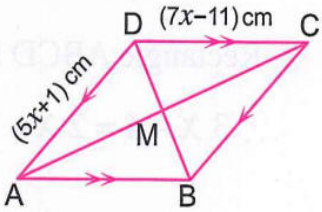
1 If ABCD is a rhombus



2 If ABCD is a rectangle

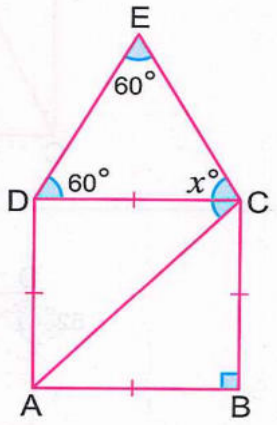


3 If ABCD is a rhombus

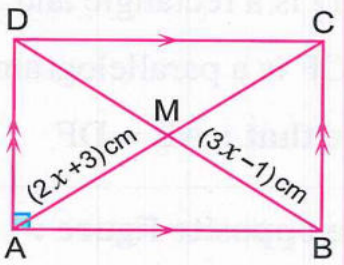


7 In each of the following figures , calculate with proof the value of x :

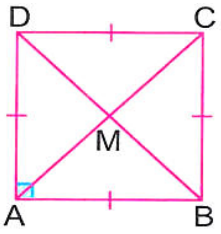
1



2



8 In the opposite figure :

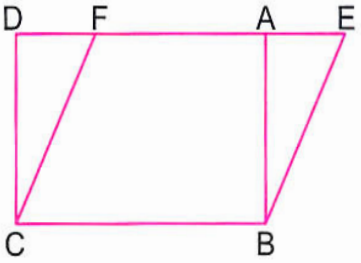


ABCD is a square

BD = 5 a – 4 , MC = 2 a – 1

Find the value of a , and then find the length of $\overline{AC}$

12 In the opposite figure :



ABCD is a rectangle and

, EBCF is a parallelogram.

Prove that : AE = DF

Homework

Choose the correct answer from the given ones :

1 The diagonals of a rectangle are .....

(a) perpendicular.

(b) equal in length.

(c) equal in length and perpendicular.

(d) bisect the interior angles.

2 The diagonals of a rhombus are .....

(a) perpendicular and unequal in length.

(b) equal in length and not perpendicular.

(c) perpendicular and equal in length.

(d) unequal in length and not perpendicular.

3 The diagonals of a square are .....

(a) only perpendicular.

(b) only equal in length.

(c) perpendicular and equal in length.

(d) neither equal in length nor perpendicular.

4 If two adjacent sides of a parallelogram are equal in length , then the shape becomes .....

(a) a square.

(b) a rhombus.

(c) a rectangle.

(d) a trapezium.

5 Which of the following sets of quadrilaterals have all their sides are equal in length ?

(a) {Square , Rectangle}

(b) {Trapezium , Rhombus}

(c) {Square , Rhombus}

(d) {Rectangle , Rhombus}

6 If ABCD is a parallelogram and m (∠ A) = m (∠ B) , then ABCD is .....

(a) a rectangle.

(b) a rhombus.

(c) a square.

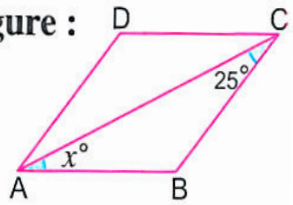
(d) a trapezium.

7 If ABCD is a parallelogram in which AC = BD , $\overline{AC} \perp \overline{BD}$, then ABCD is .....

(a) a trapezium. (b) a rhombus.

(c) a rectangle. (d) a square.

8 **In the opposite figure :**



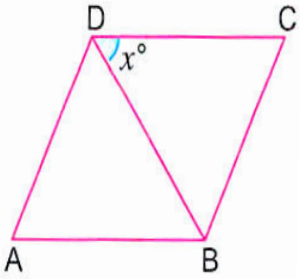
ABCD is a rhombus

What is the value of x ?

(a) 25° (b) 50°

(c) 100° (d) 130°

9 **In the opposite figure :**



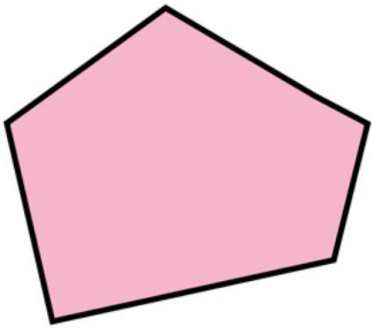
If ABCD is a rhombus , which of the following expresses m (∠ A) ?

(a) $x°$ (b) $2x°$

(c) $(180 - x)°$ (d) $(180 - 2x)°$

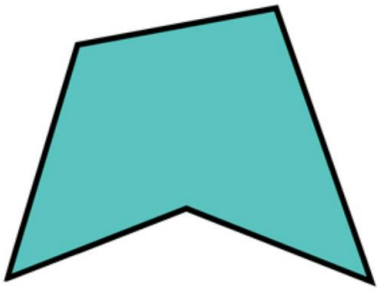
Lesson (5) Polygons

- **Concave and Convex Polygons:**



Convex

1. All of its diagonals are drawn inside the shape.
2. The measure of all interior angles is less than 180.

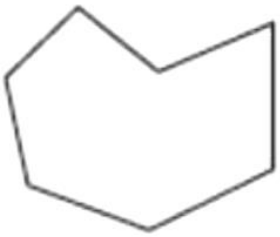


Concave

1. One diagonal (or more) is drawn outside the shape.
2. The measure of one angle (or more) is more than 180.

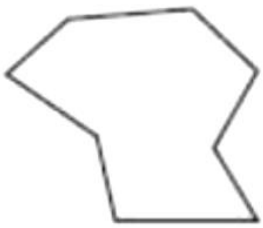
1. Identify in each of the following if the polygon is Convex or Concave:

1)

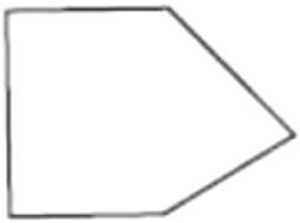


2)

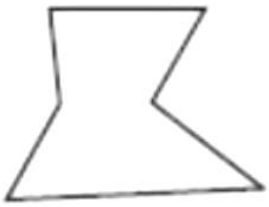
3)



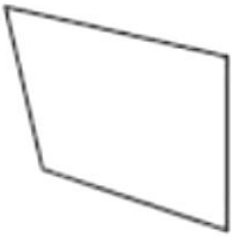
4)



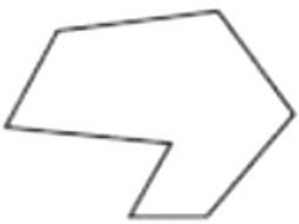
5)



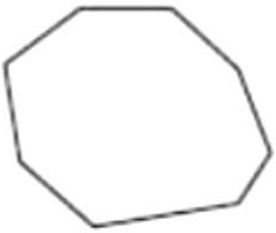
6)



7)



8)



9)

2. ABCD is a quadrilateral in which m (∠ A) = 4 x°, m (∠ B) = 5 x°, m (∠ C) = 7 x°, and m (∠ D) = 20 x°, Find the value of x, then Determine whether the quadrilateral is convex or concave ?

.....

.....

.....

.....

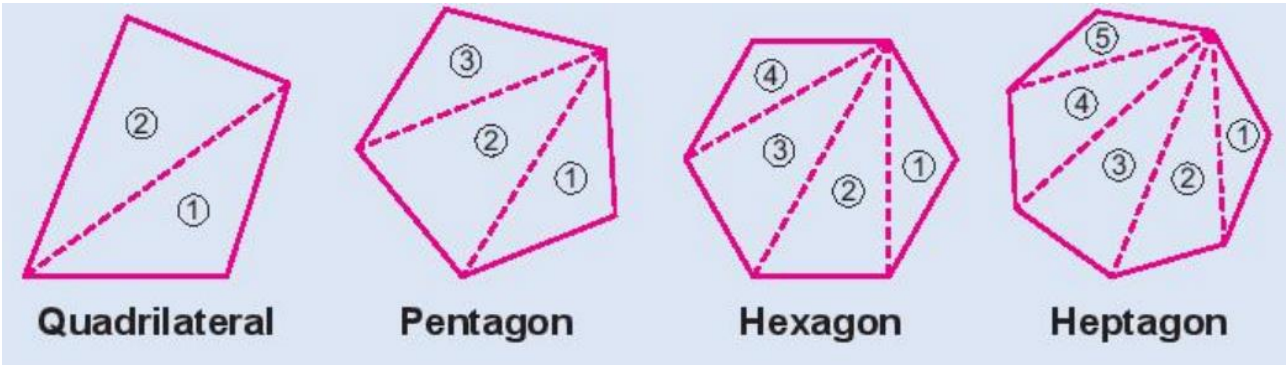
.....

.....

- **The Sum of the interior angles of a Polygon:**

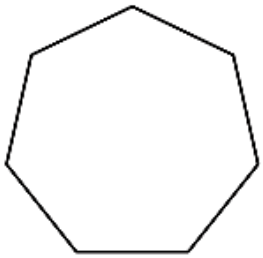
The number of all triangles that can be drawn inside a polygon of (n) sides = (n – 2)

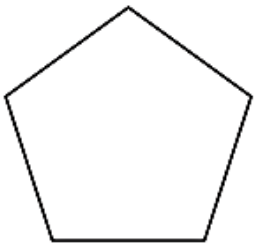
The sum of measure of the interior angles of a polygon with (n) sides = $(n-2) \times 180^0$

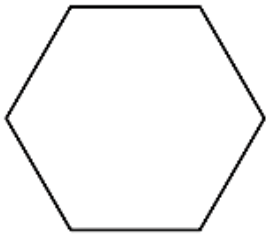


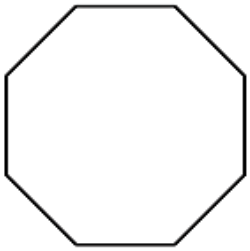
Polygon	Sides (n)	Number of Triangles = (n – 2)	Sum of measures of interior angles = (n – 2) × 180°
Triangle	3	1	1 × 180° = 180°
Quadrilateral	4	2	2 × 180° = 360°
Pentagon	5	3	3 × 180° = 540°
Hexagon	6	4	4 × 180° = 720°
Heptagon	7	5	5 × 180° = 900°
Octagon	8	6	6 × 180° = 1080°
Septagon	9	7	7 × 180° = 1260°
Decagon	10	8	8 × 180° = 1440°

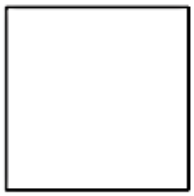
3. In each of the following Write the number of sides and name of each Polygon:

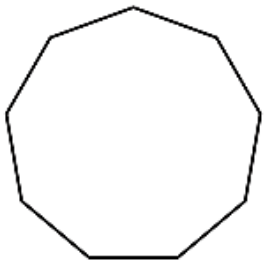
1) 
Number of sides _____
Polygon type _____

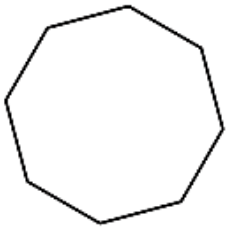
2) 
Number of sides _____
Polygon type _____

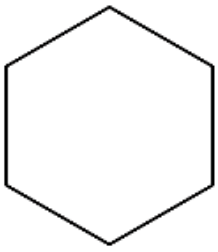
3) 
Number of sides _____
Polygon type _____

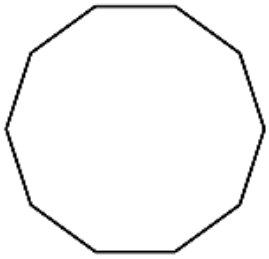
4) 
Number of sides _____
Polygon type _____

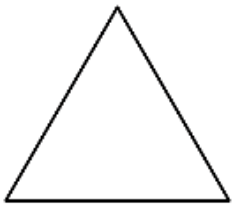
5) 
Number of sides _____
Polygon type _____

6) 
Number of sides _____
Polygon type _____

7) 
Number of sides _____
Polygon type _____

8) 
Number of sides _____
Polygon type _____

9) 
Number of sides _____
Polygon type _____

10) 
Number of sides _____
Polygon type _____

4. Choose the correct answer:

1) The polygon which has three sides is called .....

a. Triangle
c. Pentagon
b. Quadrilateral
d. Octagon

2) The Polygon which has eight sides is called .....

a. Hexagon
c. Octagon
b. Pentagon
d. Quadrilateral

3) The Polygon which has six sides is called .....

a. Octagon
c. Quadrilateral
b. Triangle
d. Hexagon

4) The polygon which has nine sides is called .....

a. Heptagon
c. Decagon
b. Septagon
d. Hexagon

5) The sum of measures of the interior angles of a Pentagon is .....

a. 360°
b. 720°
c. 1080°
d. 540°

6) The sum of measures of the interior angles of a Heptagon is .....

a. 900°
b. 360°
c. 1440°
d. 720°

7) The sum of measures of the interior angles of a Quadrilateral is .....

a. 1260°
b. 720°
c. 360°
d. 540°

8) The Polygon in which the measure of its interior angles = 1080° is called .....

a. Heptagon
c. Octagon
b. Decagon
d. Hexagon

9) The Polygon in which the measure of its interior angles = 720° is called .....

a. Heptagon
c. Decagon
b. Septagon
d. Hexagon

10) The Polygon in which the measure of its interior angles = 1440° is called .....

a. Quadrilateral
c. Septagon
b. Decagon
d. Octagon

5. **In a Quadrilateral ABCD:**

m (∠ A) = 12 x°, m (∠ C) = 50 x°, and m (∠ B) = m (∠ D) = 5 x°

Find the value of x, then Determine whether the shape is convex or concave?

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6. **In a Quadrilateral ABCD:**

m (∠ A) = m (∠ C) = 3 x°, m (∠ B) = m (∠ D) = 5 x° , and m (∠ E) = 4 x°

Find the value of x, then Determine whether the shape is convex or concave?

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7. **In a Hexagon ABCDEF:**

m (∠ A) = m (∠ D) = 90°, m (∠ B) = m (∠ E) = x° ,

and m (∠ C) = m (∠ F) = (x + 22)°

Find the value of x, then Determine whether the shape is convex or concave?

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8. **In a Pentagon:**

If the ratio among the measures of its interior angles is 4 : 3 : 2 : 3 : 3,

Find the measure of the greatest angle in this polygon, then Determine whether the shape is convex or concave?

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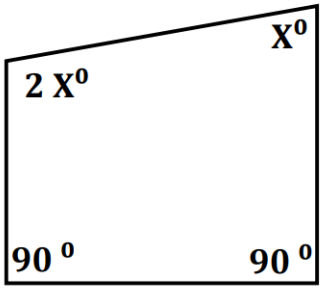
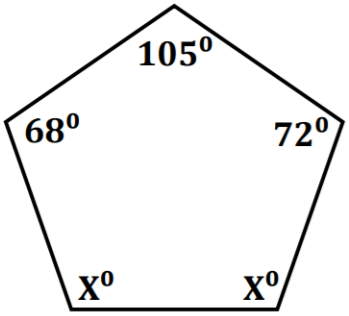
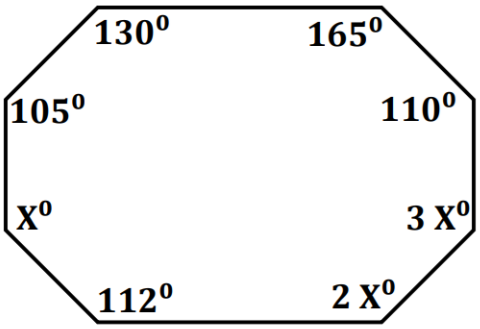
9. In each of the following figures, Calculate the value of X:







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10. In the opposite figure, Calculate the value of x:

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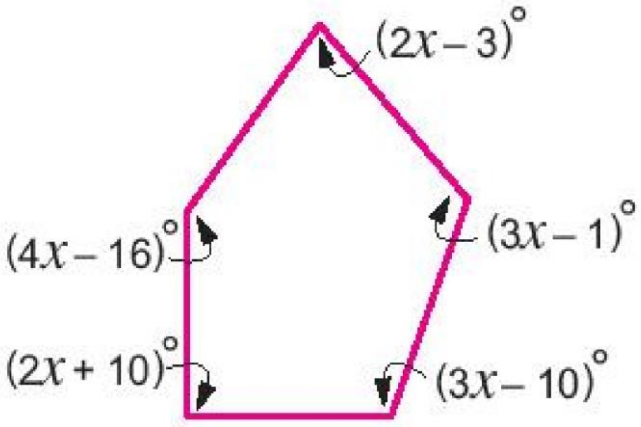

11. In the opposite figure, Calculate the value of x:

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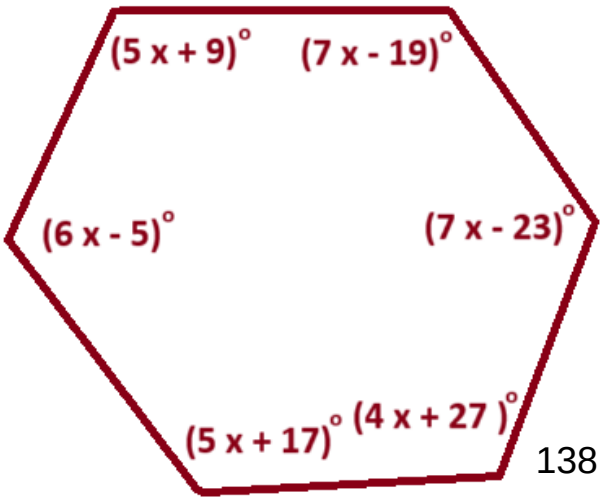

12.In the opposite figure, Calculate the value of x:

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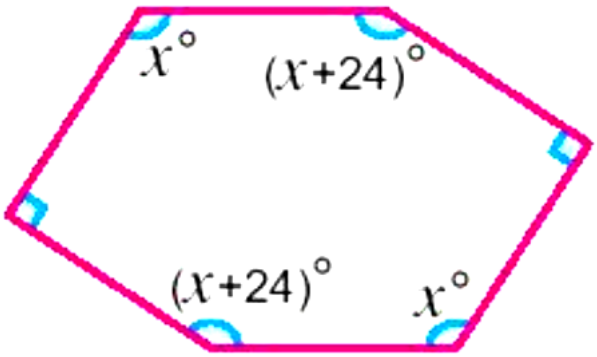

13.In the opposite figure, Calculate the value of x:

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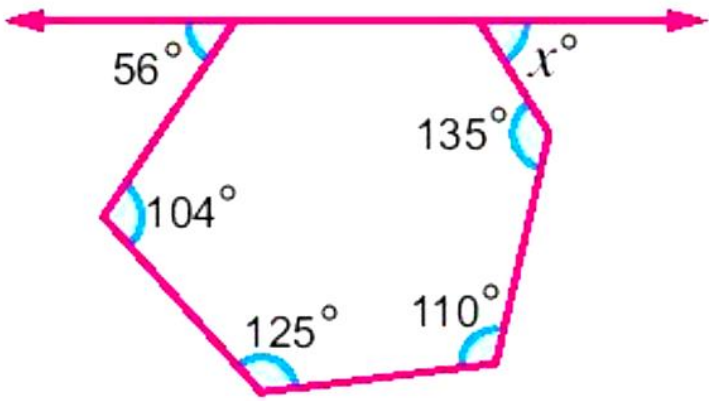

14.In the opposite figure, Calculate the value of x:

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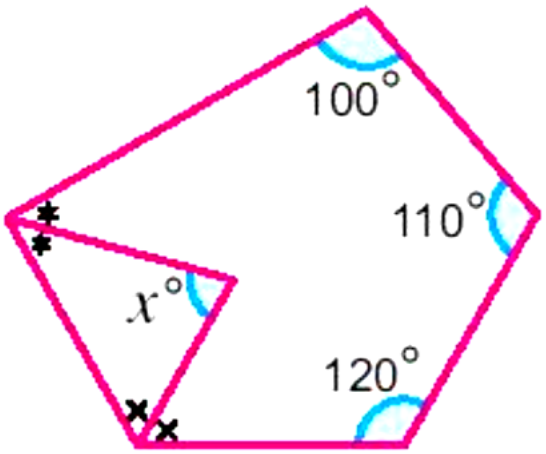
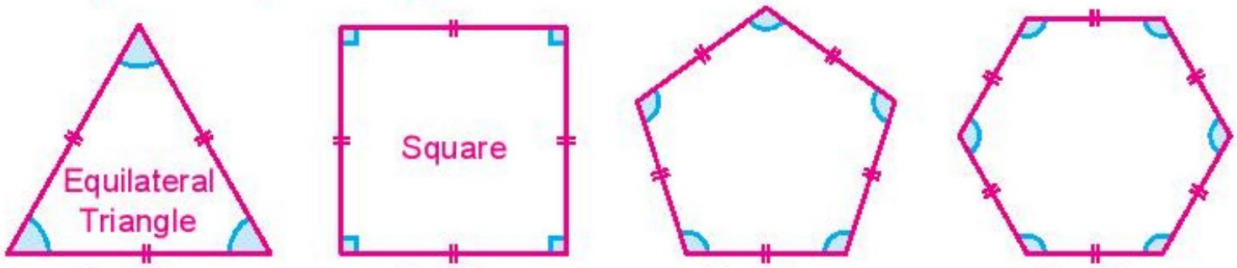


• The Regular Polygon:

A regular polygon is a polygon that has both of the following properties:

1. All its sides are equal in length.
2. All its interior angles are equal in measure.

Examples of Regular Polygons:



Regular Triangle | Regular Quadrilateral | Regular Pentagon | Regular Hexagon

Regular Polygon	Number of sides	Sum of measures of Interior angles	Measure of ONE interior angle
Regular Triangle	3	180°	180° ÷ 3 = 60°
Regular Quadrilateral	4	360°	360° ÷ 4 = 90°
Regular Pentagon	5	540°	540° ÷ 5 = 108°
Regular Hexagon	6	720°	720° ÷ 6 = 120°
Regular Heptagon	7	900°	900° ÷ 7 = 128.6°
Regular Octagon	8	1080°	1080° ÷ 8 = 135°
Regular Septagon	9	1260°	1260° ÷ 9 = 140°
Regular Decagon	10	1440°	1440° ÷ 10 = 144°

15. Choose the correct answer:

1) The Polygon which has equal sides in length, and equal angles in measure is called …….

a. Concave Polygon
c. Regular Polygon
b. Convex Polygon
d. Irregular Polygon

2) The Triangle which has equal sides in length is called …….

a. Quadrilateral
c. Scalene
b. Isosceles
d. Equilateral

3) The Quadrilateral which has equal sides in length, and equal angles in measure is called …….

a. Square
c. Rectangle
b. Rhombus
d. Parallelogram

4) The measure of each interior angle of a regular Octagon = ……. °

a. 1080°
b. 120°
c. 90°
d. 135°

5) The measure of each interior angle of a regular Septagon = ……. °

a. 140°
b. 270°
c. 128°
d. 360°

6) The measure of each interior angle of a regular Pentagon = ……. °

a. 54°
b. 120°
c. 72°
d. 108°

7) The measure of each interior angle of a regular Polygon with 10 sides = ……. °

a. 160°
b. 90°
c. 144°
d. 36°

8) The measure of each interior angle of a regular Polygon with 18 sides = ……. °

a. 120°
b. 720°
c. 360°
d. 160°

9) If the perimeter of a regular hexagon is 48 cm, then its side length = ……. cm

a. 4
b. 8
c. 6
d. 12

10) If the side length of a regular octagon is (3 x – 23) cm, and its perimeter equals 56 cm, then the value of x is …….

a. 10
b. 12
c. 30
d. 7

11) If the measure of an interior angle in a regular Decagon is (3 x – 6)°, then the value of x is …….

a. 30°
b. 36°
c. 144°
d. 50°

16. In the opposite figures, if each of the following is a regular Polygon, then Calculate the value of x:

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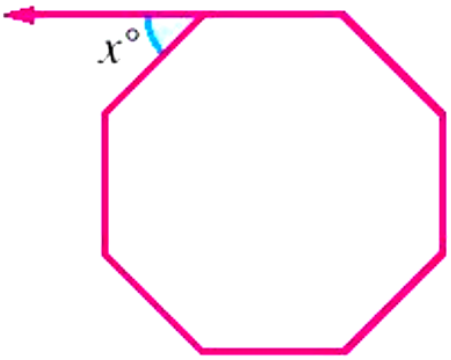

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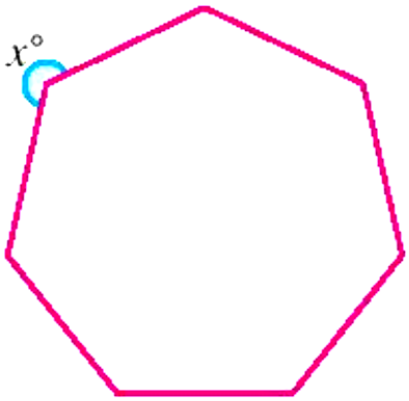

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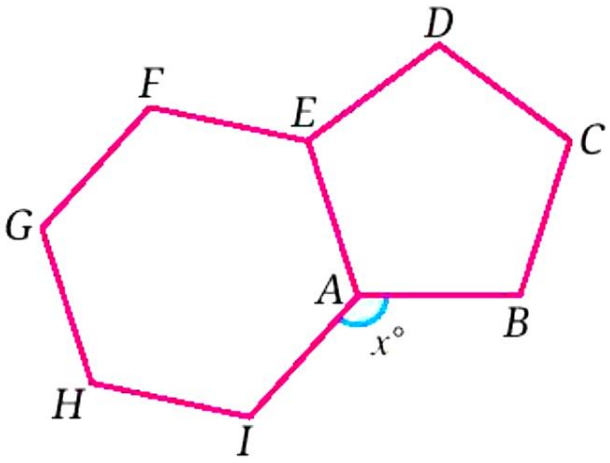

17. In the opposite figure, Calculate the measure of ∠ ABC:

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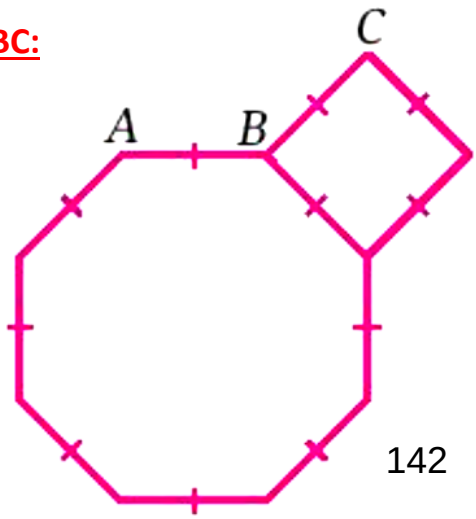
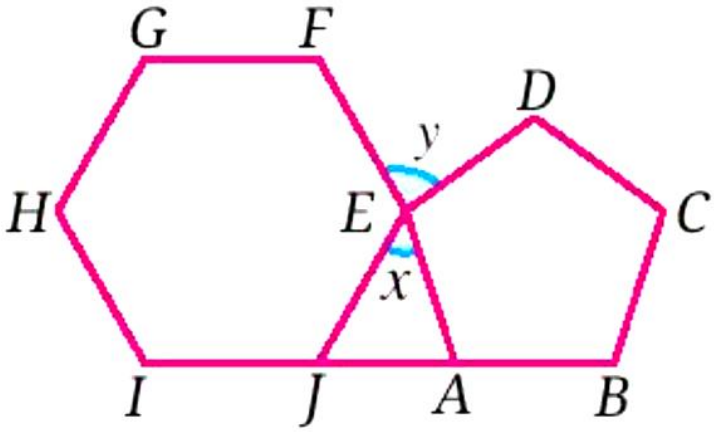


18. In the opposite figure:

ABCDE is a regular Pentagon,

EFGHIJ is a regular Hexagon,

Calculate the value of x and y.



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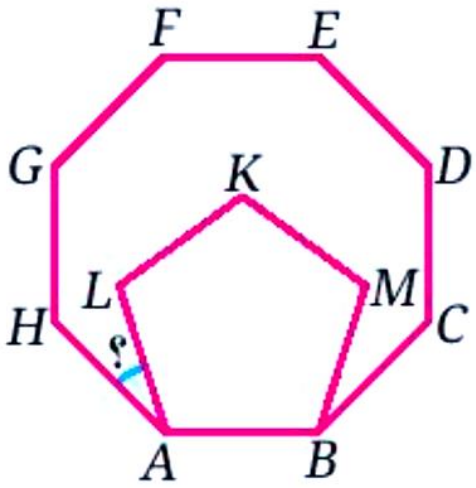
19. In the opposite figure:

ABCDEFGH is a regular Octagon,

ABMKL is a regular Pentagon,

Find with proof m (∠ HAL) ?



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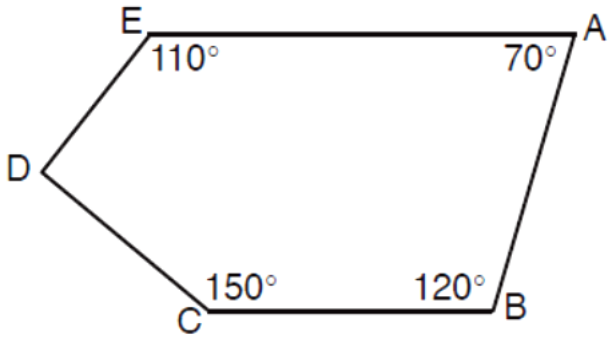
20. In the opposite figure:

ABCDE is a Pentagon in which:

m (∠ A) = 70°, m (∠ B) = 120°,

m (∠ C) = 150°, m (∠ E) = 110°,

Find with proof m (∠ D) ?



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21. In the opposite figure:

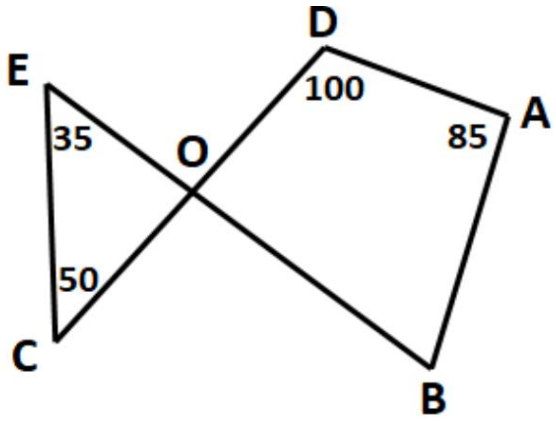
ABDO is a Quadrilateral in which:

m (∠ A) = 85°, m (∠ D) = 100°,

DC ∩ EB = { O } where

m (∠ C) = 50°, m (∠ E) = 35°,

Find with proof m (∠ B) ?



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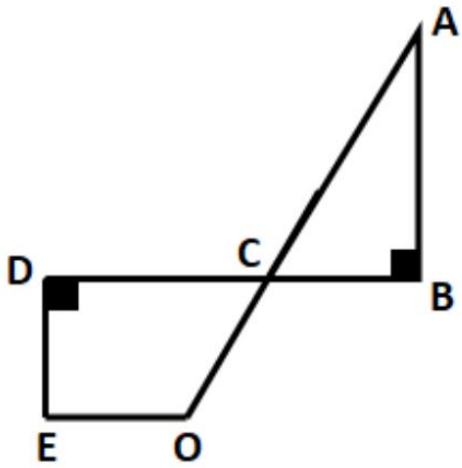
22. In the opposite figure:

AO ∩ DB = { C } , m (∠ A) = 30°,

m (∠ B) = m (∠ D) = 90°,

and m (∠ EOC) = 120°

Find with proof m (∠ E) ?



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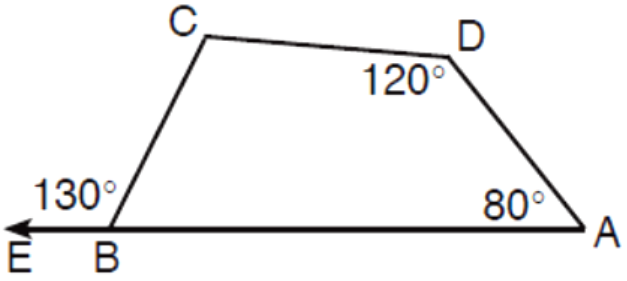
23. In the opposite figure:

ABCD is a Quadrilateral, E ∈ AB

m (∠ A) = 80°, m (∠ D) = 120°,

and m (∠ CBE) = 130°

Find with proof m (∠ C) ?



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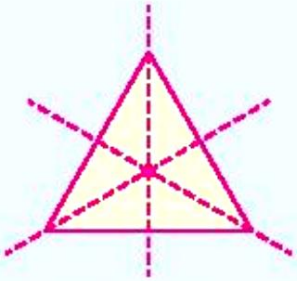
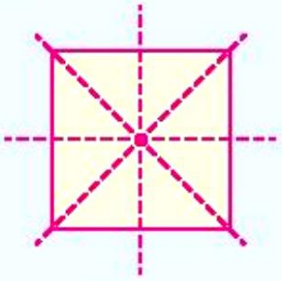
- **Axes of Symmetry of Regular Polygons:**

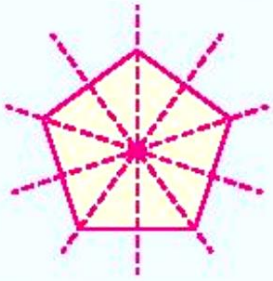
The axis of symmetry of a regular polygon is a straight line that passes through the center of the polygon and divides it into two identical polygons.



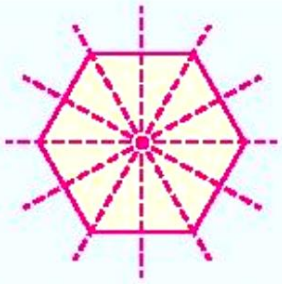
Equilateral Triangle
(3 axes of symmetry)



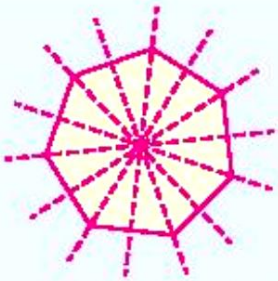
Square
(4 axes of symmetry)



Regular Pentagon
(5 axes of symmetry)



Regular Hexagon
(6 axes of symmetry)



Regular Heptagon
(7 axes of symmetry)

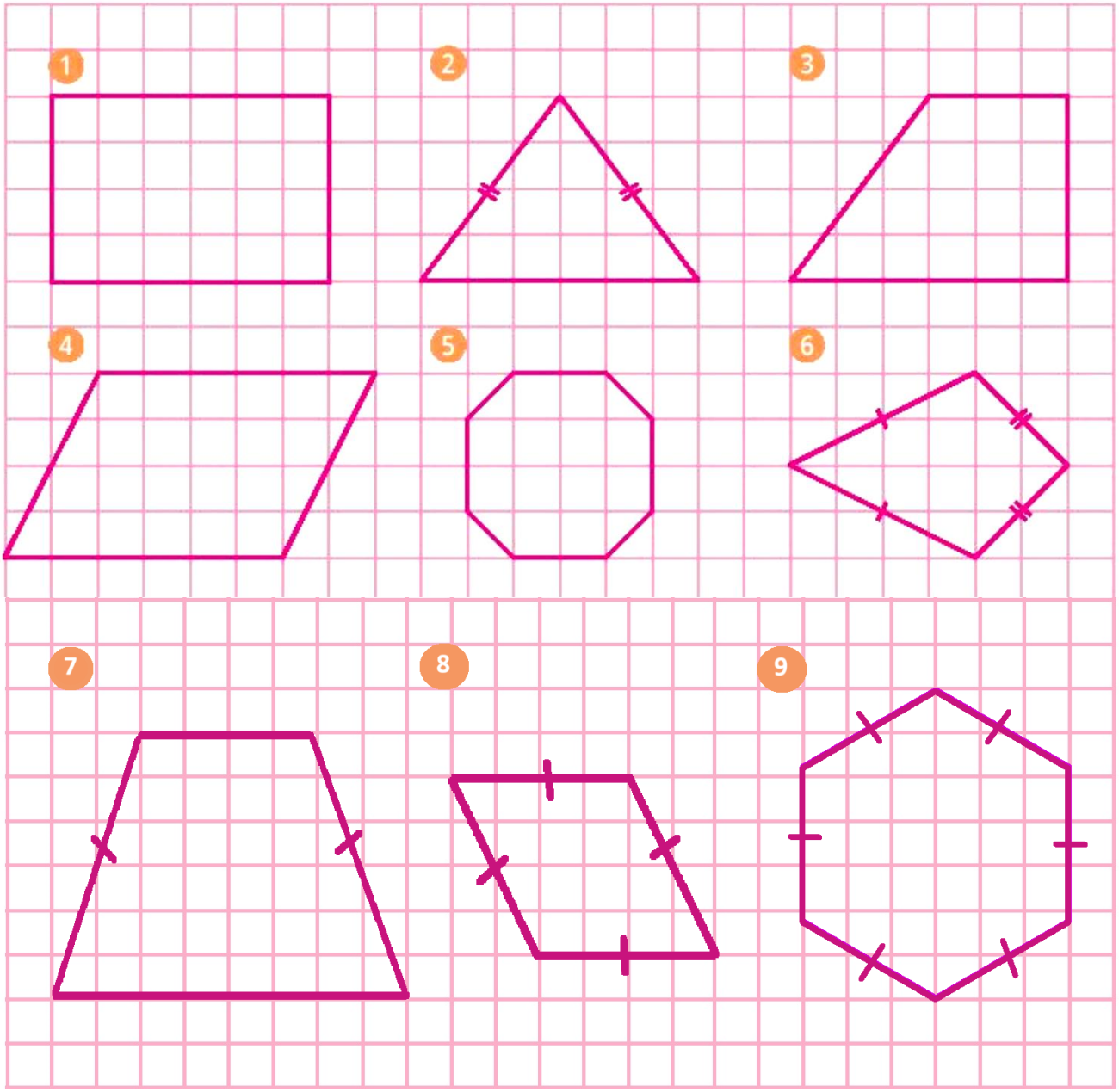
➢ **For any Regular Polygon:**

Number of Axes of Symmetry = Number of its sides

➢ **For Irregular Polygons:**

Polygon Name	Number of axes of Symmetry
Rhombus	2
Rectangle	2
Isosceles Triangle	1
Isosceles Trapezium	1
Scalene Triangle	0
Parallelogram	0
Trapezium	0

24. In the opposite figures, how many axes of symmetry in each:



Homework

1. Choose the correct answer:

1) Which of the following could be an interior angle in a concave polygon?

a. Acute
c. Reflex
b. Straight
d. Right

2) The triangle which has 3 axes of symmetry is called .....

a. Isosceles
c. Scalene
b. Equilateral
d. Right – angled

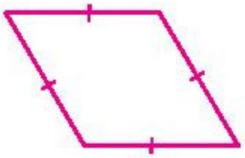
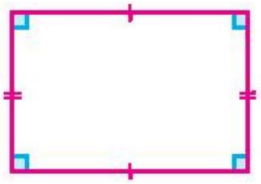
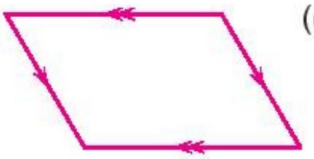
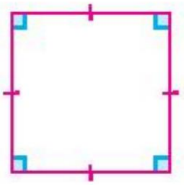
3) The polygon which has no diagonals is .....

a. Concave
c. Triangle
b. Convex
d. Square

4) How many axes of symmetry does a regular polygon with 9 sides have?

a. 6
b. 7
c. 8
d. 9

5) Which of the following does not have an axis of symmetry?

(a) 
(b) 
(c) 
(d) 

6) What is the measure of the interior angle of a regular polygon with 10 sides?

a. 90°
b. 150°
c. 206°
d. 144°

7) The number of diagonals in a quadrilateral is .....

a. 1
b. 2
c. 3
d. 4

8) The number of diagonals in a Pentagon is .....

a. 3
b. 4
c. 5
d. 6

9) The number of axes of symmetry of a square is .....

a. 2
b. 3
c. 0
d. 4

10) The number of axes of Symmetry of a parallelogram is .....

a. 0
b. 1
c. 2
d. 3

11) The number of axes of symmetry of an Isosceles Trapezium is .....

a. 0 b. 1 c. 2 d. 3

12) The measure of each interior angle in a regular octagon equal .....

a. 100° b. 108° c. 135° d. 120°

13) The sum of measure of the interior angles of a Heptagon equals .....

a. 1080° b. 900° c. 720° d. 540°

14) When all possible diagonals are drawn from one vertex of a convex octagon, then the number of the formed triangles is .....

a. 8 b. 10 c. 6 d. 4

15) If all possible diagonals are drawn from one vertex of a convex polygon is 4 diagonals, then the number of the formed triangles equals .....

a. 4 b. 8 c. 16 d. 6

16) If the number of possible triangles formed by drawing all the possible diagonals from one vertex of a convex polygon is 7, then the sum of the interior angles of this polygon equals .....

a. 900° b. 720° c. 1440° d. 1260°

2. In a quadrilateral ABCD:

m (∠ A) = 6 x°, m (∠ B) = 7 x°, m (∠ C) = 12 x° and m (∠ D) = 11 x°

Find the value of x, then Determine whether the shape is convex or concave?

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25. In a Quadrilateral ABCD:

m (∠ A) = 4 x°, m (∠ B) = 5 x°, m (∠ C) = 7 x° and m (∠ D) = 20 x°

Find the value of x, then Determine whether the shape is convex or concave?

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3. In the opposite figure, Calculate the value of x:

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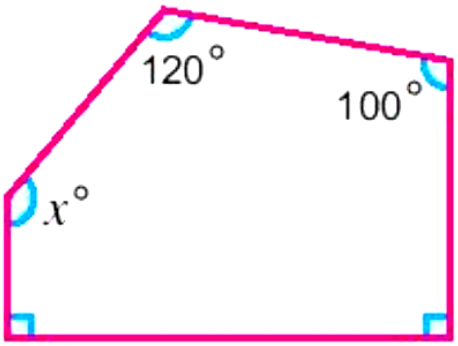

4. In the opposite figure, Calculate the value of x:

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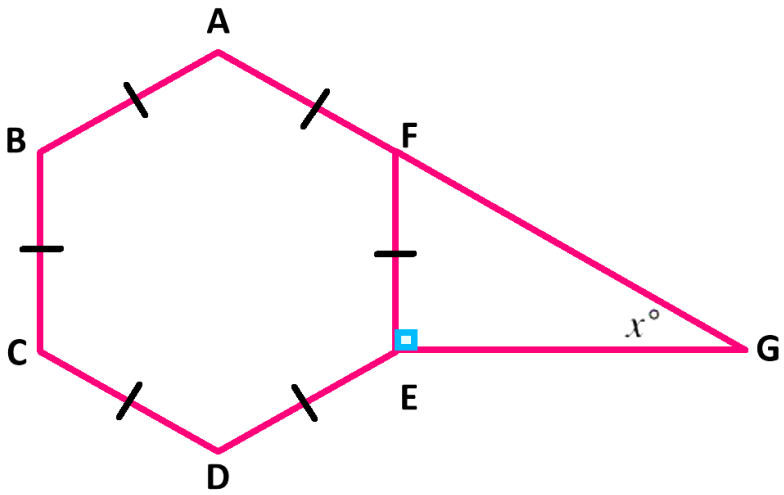

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5. In the opposite figure, Calculate the value of x:

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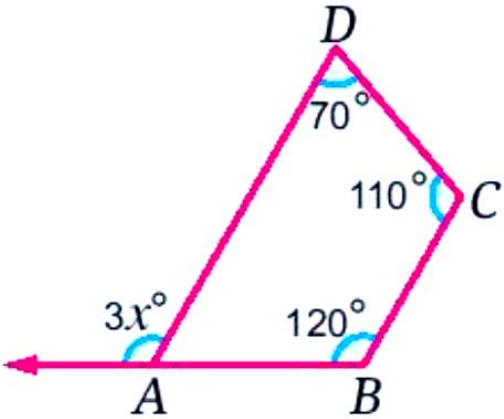

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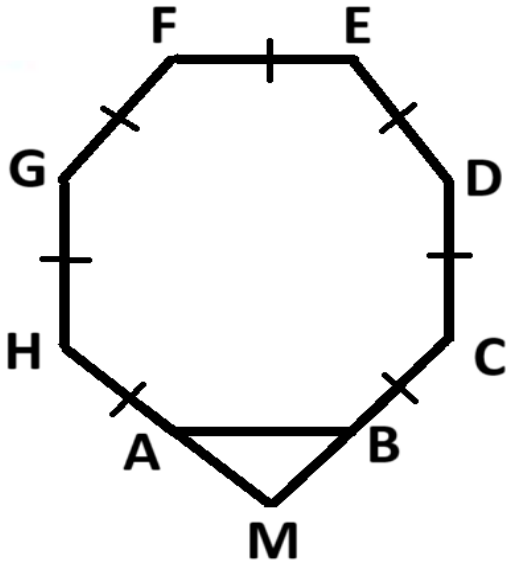
6. In the opposite figure, Calculate m (∠ AMB):



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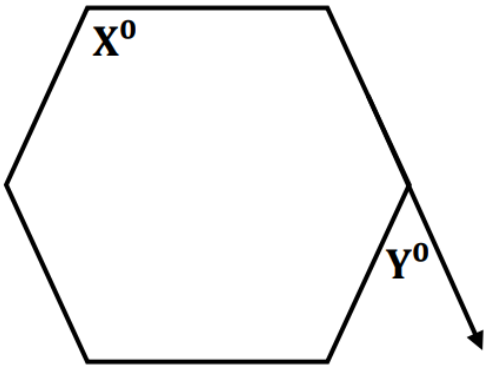
7. In the opposite figure:

Calculate the value of (3 Y – X)



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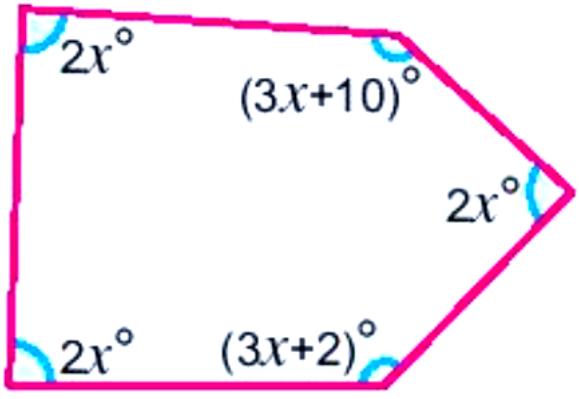
8. In the opposite figure, Calculate the value of x:



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Coordinates
Lesson (6)

- **Ordered Pairs:**

(a , b)

- First Projection — Represents X - axis
- Second Projection — Represents Y - axis

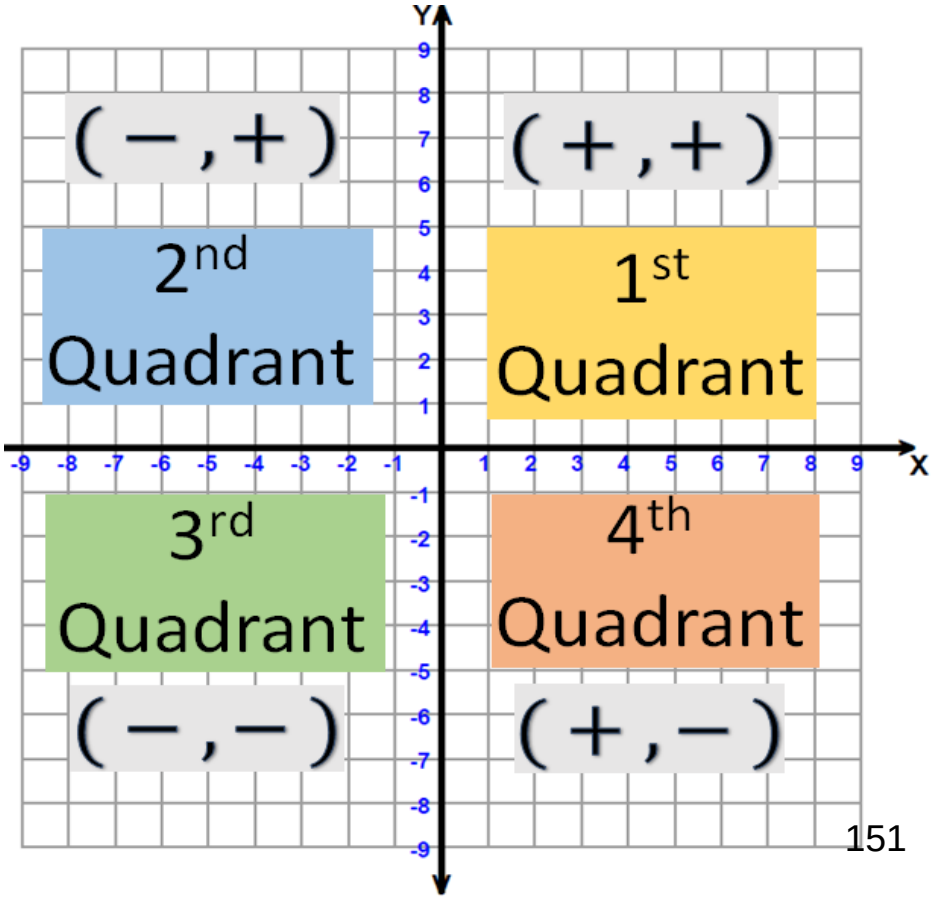
✓ Defines the location of a point on graph.
✓ (2 , 5) ≠ (5 , 2)

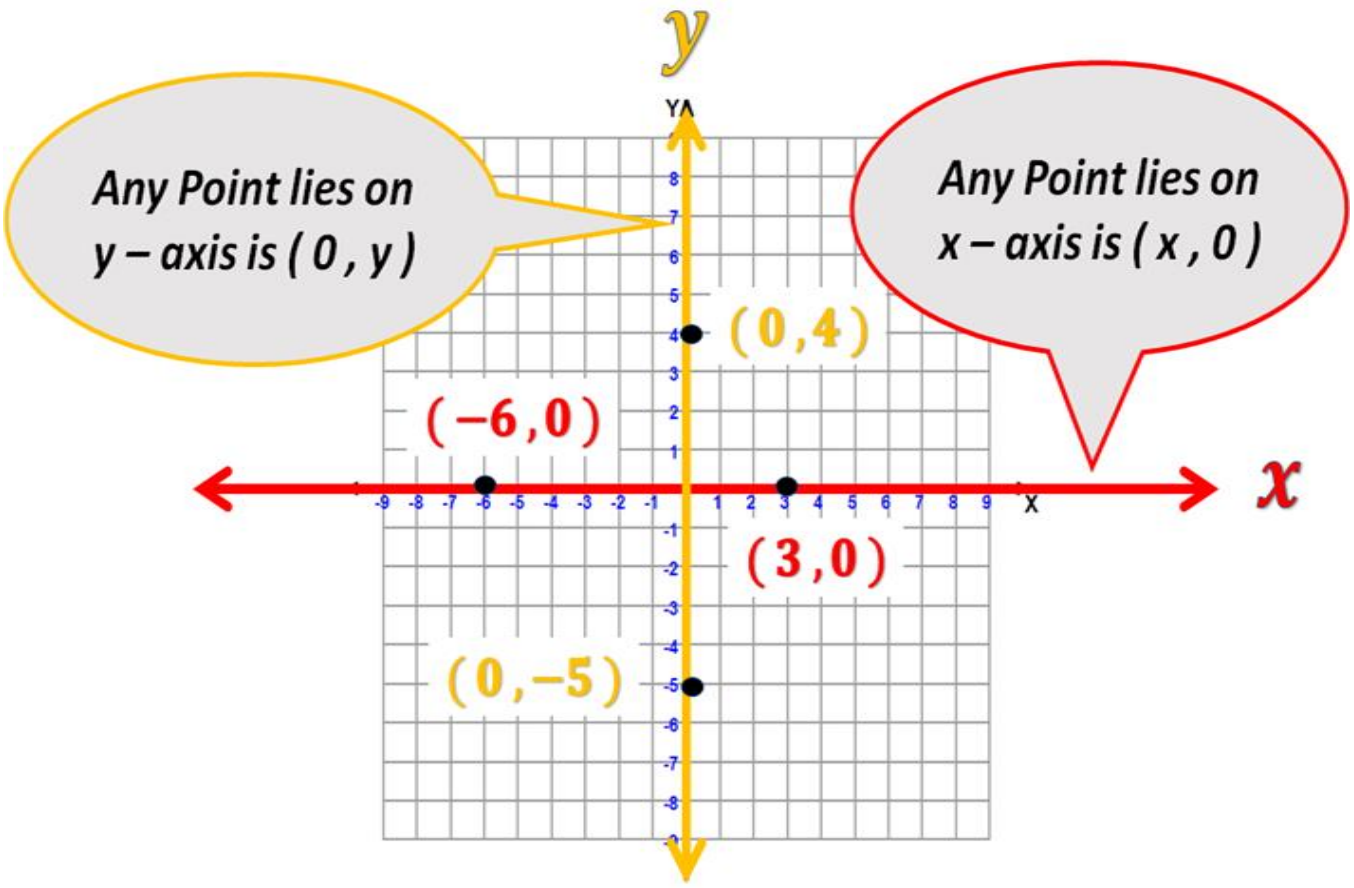
- **Square Lattice:**

✓ The square lattice is divided into 4 quadrants using two axes.
✓ Each quadrant has a special Signs property as shown.

- **X and Y axes:**

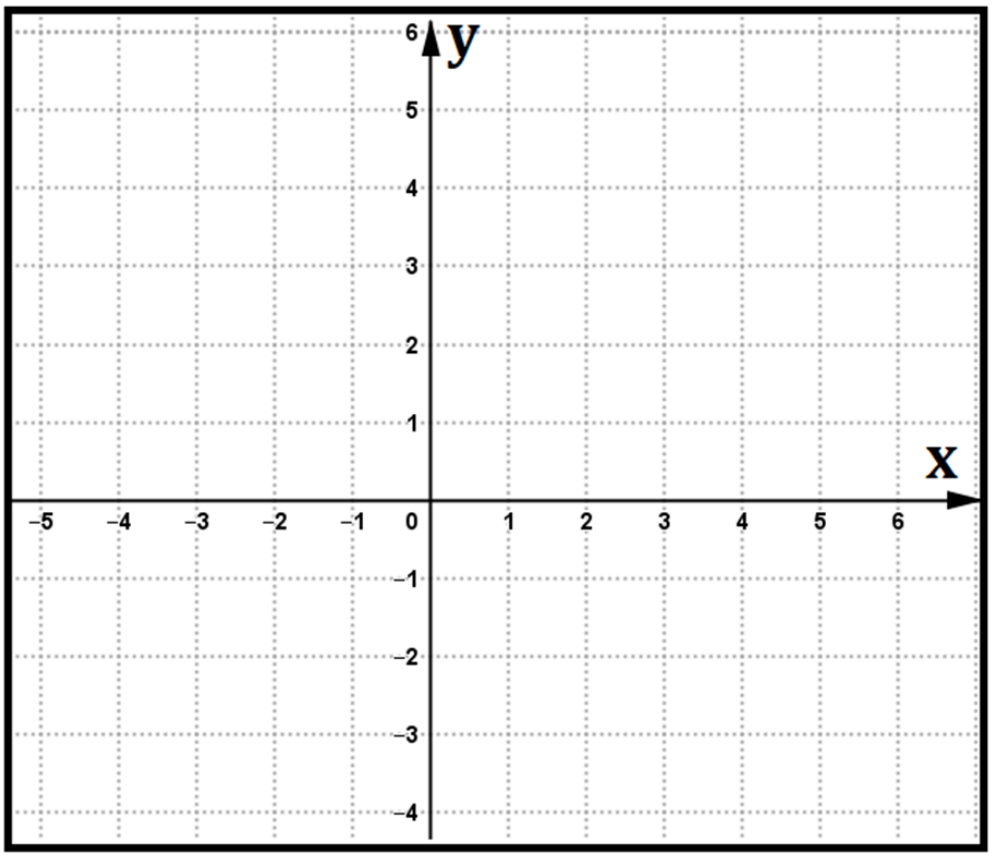
✓ X – axis is the horizontal number line, and the second projection of any point lies on x – axis is zero.
✓ Y – axis is the vertical number line, and the first projection of any point lies on y – axis is zero.
✓ The two axes intersects at the origin point O (0 , 0).





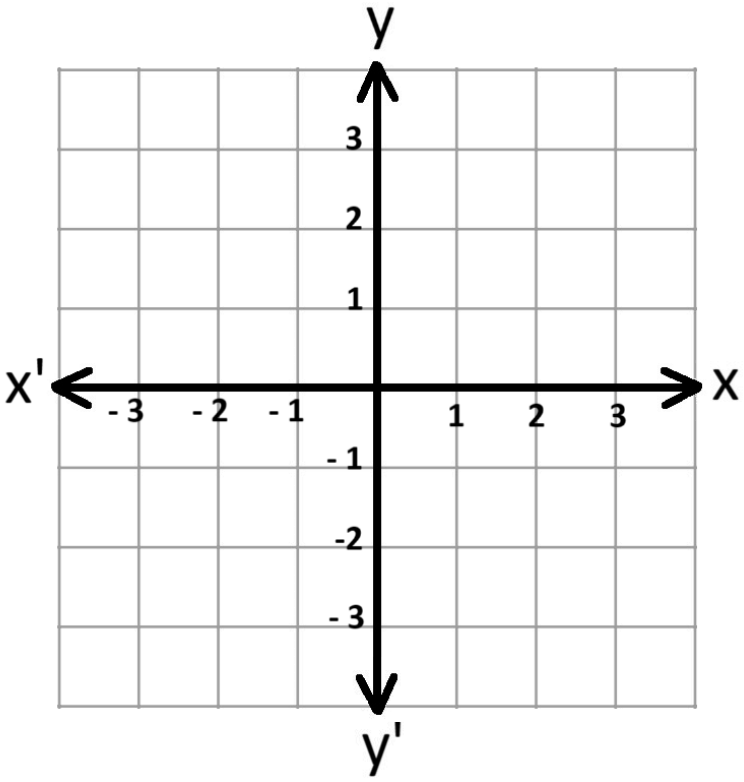
9. Identify on graph the following points, and mention the quadrant or axis belongs to the point:

1. A (4 , 5)
2. B (6 , - 3)
3. C (- 2 , 4)
4. D (- 1 , 2)
5. E (- 3 , - 4)
6. F (5 , - 1)
7. M (0 , - 3)
8. N (5 , 0)
9. K (0 , 3)
10. H (- 4 , 0)



10. Plot the following points, then calculate the area of figure:

A (3 , 2) , B (- 1 , 2) ,
C (- 3 , 0) , D (1 , 0)



11. In each of the following graphs:
Determine the coordinates of the vertices of Δ ABC, then calculate its area:

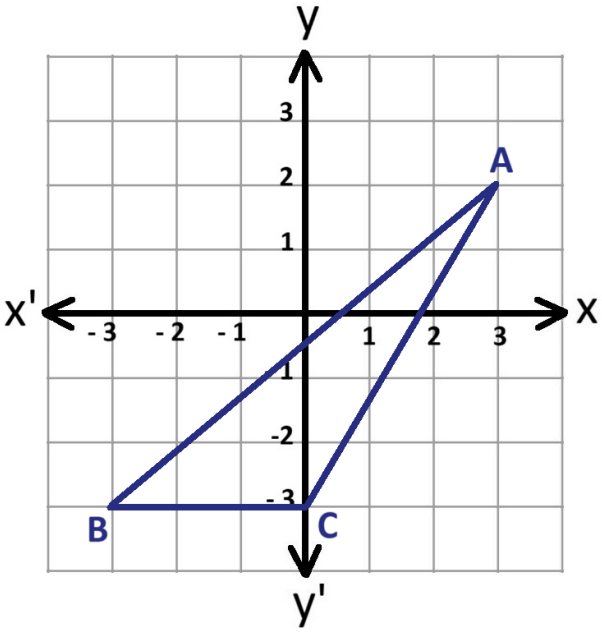


Figure (1)

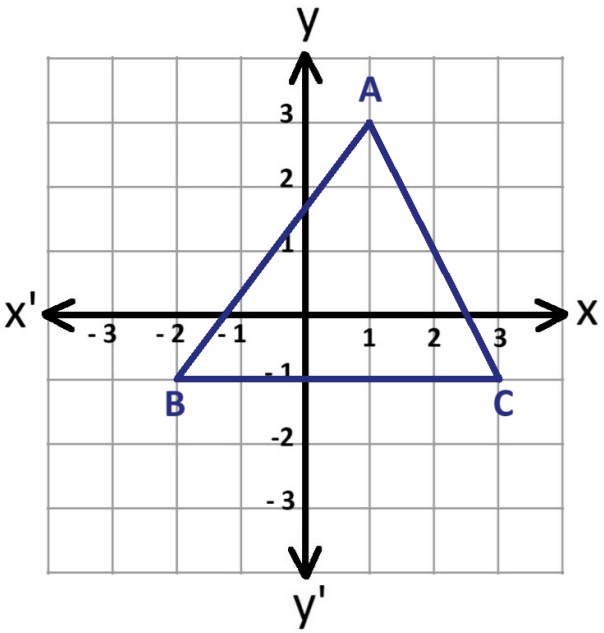


Figure (2)

12. Choose the correct answer:

1. In which quadrant is the point (- 3 , 5) located ?

   a. First b. Second c. Third d. Fourth

2. In which quadrant is the point (| x | , x^2) located ?

   a. First b. Second c. Third d. Fourth

3. If x < 0 , then in which quadrant does the point (- 3 x , 2 – x) lies in ?

   a. First b. Second c. Third d. Fourth

4. If x is a positive integer, and the point (x – 2 , x – 4) lies in 4^{th} quadrant, then x may be = …….

   a. 1 b. 2 c. 3 d. 4

5. If the point (a , b) lies in 3^{rd} quadrant, then the quadrant which the point (- 2a , b – 6) lies in is ……

   a. First b. Second c. Third d. Fourth

6. On which axis does the point (0 , - 1) lie ?

   a. X – axis c. Origin Point
   b. Y – axis d. 1^{st} Quadrant

7. If the point (b – 7 , 5) lies on y – axis, then what is the value of b ?

   a. 7 b. – 7 c. 5 d. – 5

8. If x < 0 , y > 0, in which quadrant is the point (x , - y) located ?

   a. First b. Second c. Third d. Fourth

9. If the point (3 , k – 2) lies on x – axis, then what is the value of k ?

   a. 3 b. – 3 c. 2 d. – 2

10. If the point A (4 k + 4 , - k + 3) lies on y – axis, then in which quadrant is the point B (- 2 k , 4 k + 1) located ?

    a. First b. Second c. Third d. Fourth

11. If the point (2 k + 4 , - 3 k + 6) lies on x – axis, then in which quadrant is the point B (3 – k , 1 – 2k) located ?

    a. First b. Second c. Third d. Fourth

Homework

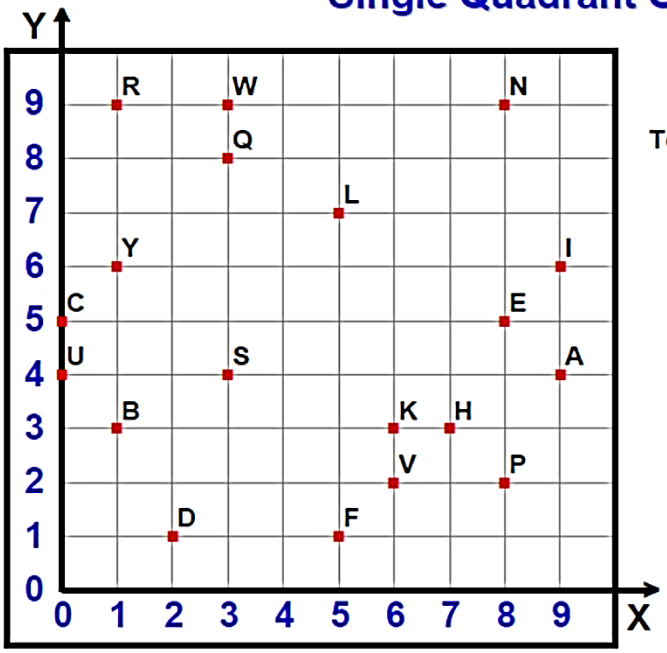
1. Identify on graph the following points, and then mention the quadrant or the axis belongs to the point:

1) A (2 , 5)
2) P (7 , - 3)
3) N (- 5 , 2)
4) H (1 , - 1)
5) D (- 2 , - 3)
6) B (3 , - 4)
7) M (0 , 3)
8) J (4 , 0)
9) L (- 2 , 0)
10) Q (0 , 2)

2. In the previous graph plot the points: X (6 , 1) , Y (6 , 4) , Z (- 2 , 4)
And then find the coordinates of the point D that makes ABCD a rectangle.

3. Mention the letter of the ordered pair given:

Single Quadrant Ordered Pairs



Tell what point is located at each ordered pair.

1) (3,9) _____ 6) (1,9) _____

2) (8,9) _____ 7) (2,1) _____

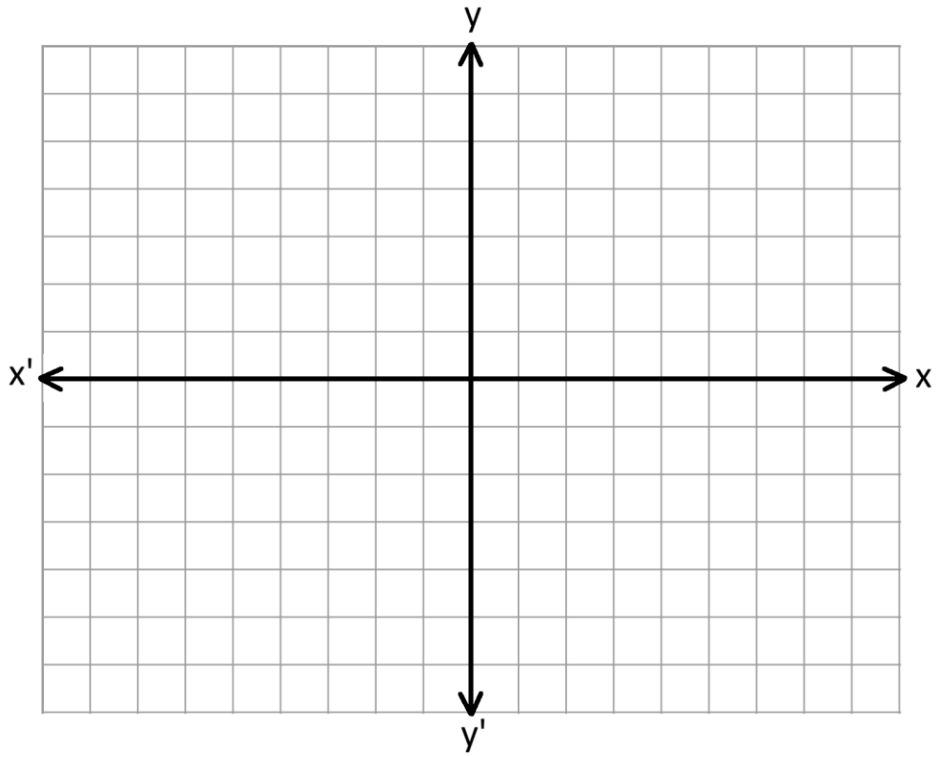
3) (1,3) _____ 8) (6,2) _____

4) (5,1) _____ 9) (5,7) _____

5) (6,3) _____ 10) (9,4) _____

4. Plot the following points, then calculate the area of figure:

X (- 4 , 2) , Y (- 4 , 7) , Z (3 , 7) , L (3 , 2)



5. In each of the following graphs:
Determine the coordinates of the vertices of Δ ABC, then calculate its area:

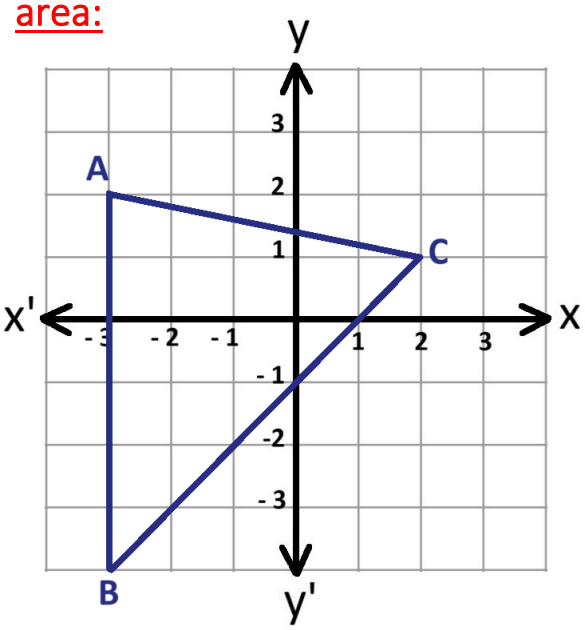


Figure (1)

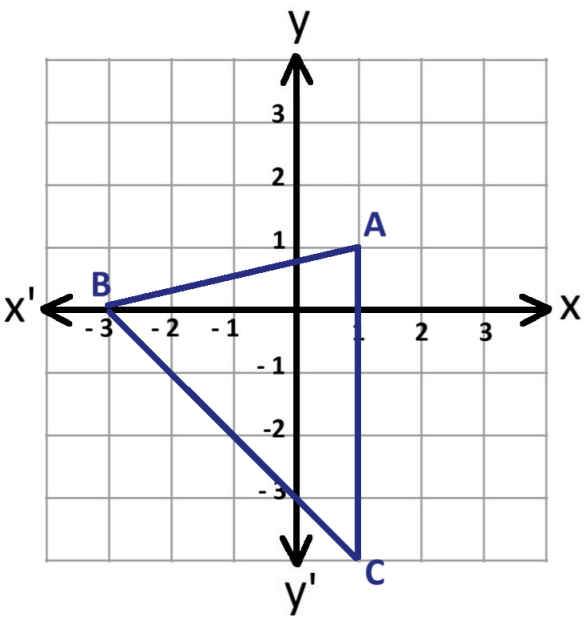


Figure (2)

6. Choose the correct answer:

1. **In which quadrant is the point (3 , - 4) located ?**

   a. First b. Second c. Third d. Fourth

2. **If x is a positive integer, and the point (2 x – 10 , 3 x – 9) lies in 2nd quadrant, then x may be …….**

   a. 2 b. 3 c. 4 d. 5

3. **If the point (a – 2 , 2a + 3) lies on the x – axis, then in which quadrant is the point (a , 6 – a) located?**

   a. First b. Second c. Third d. Fourth

4. **If the point (b , 2) lies on y – axis, then what is the value of 5b + 1 ?**

   a. 5 b. – 1 c. 6 d. 1

5. **If the point (3 , k – 2) lies on x – axis, then what is the value of k + 3 ?**

   a. 7 b. 5 c. 2 d. – 3

6. **If the point (- 3 k , 2 k – 4) lies on the x – axis, then in which quadrant is the point B (k – 6 , – 3k) located ?**

   a. First b. Second c. Third d. Fourth

- **Projection of a point on the coordinate axes:**

✓ The projection of point (a , b) on x – axis is (a , 0)

✓ The projection of point (a , b) on y – axis is (0 , b)

✓ If the point lies on x – axis its projection on x – axis is itself.

✓ If the point lies on y – axis its projection on y – axis is itself.

7. Find the projection of each of the following points on both axes:

Points	Projection on x – axis	Projection on y – axis
A (3 , 4)		
B (- 2 , 1)		
C (0 , 4)		
D (2 , - 3)		
E (- 5 , 0)		

8. Choose the correct answer:

1. **The projection of the point A (4 , 5) on x – axis is A' .....**

   a. (4 , 0) b. (5 , 4) c. (- 4 , - 5) d. (0 , 5)

2. **If the projection of the point A on x – axis is the point A' (- 3 , 0), then Point A may be .....**

   a. (2 , 3) b. (- 1 , - 3) c. (- 3 , 5) d. (0 , - 3)

3. **If the projection of the point A on y – axis is itself, then the point A may be ...**

   a. (1 , 2) b. (3 , 0) c. (0 , 5) d. (3 , 5)

4. **The distance between the point (3 , 5) and x – axis equals ..... Length units**

   a. 3 b. 5 c. -3 d. -5

5. **The distance between the point (- 2 , - 4) and y – axis equals ... Length units**

   a. 2 b. 4 c. -2 d. -4

6. **If the point A' (x – 1 , y + 3) is the projection of the point A (3 , 5) on x – axis, then the value of x + y = ...**

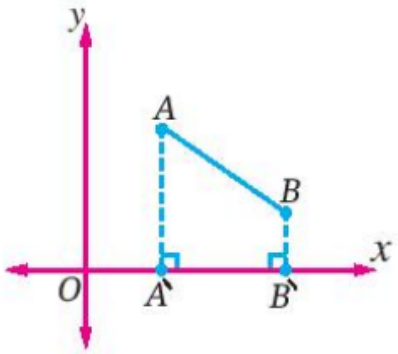
   a. 7 b. 4 c. 3 d. 1

7. **If the point (k – 7 , k + 3) is the projection of a point on x – axis, then k = ...**

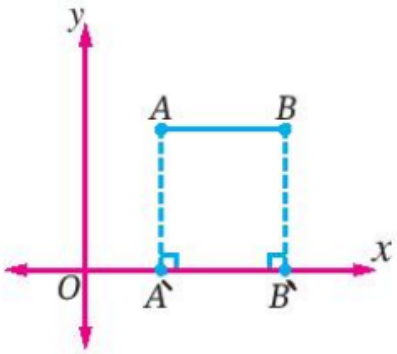
   a. 7 b. 3 c. -3 d. -7

- **Projection of a line segment on the coordinate axes:**

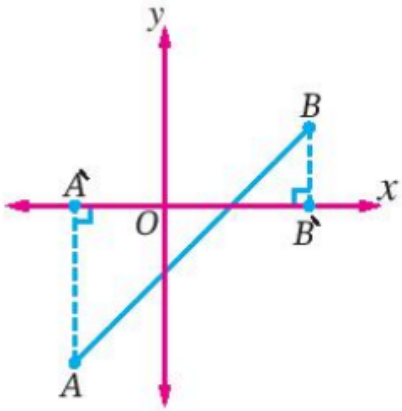
1. $\overline{A'B'}$ is the projection of $\overline{AB}$ on the x-axis.



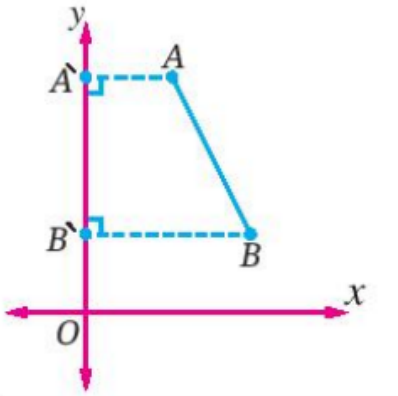
2. $\overline{A'B'}$ is the projection of $\overline{AB}$ on the x-axis.



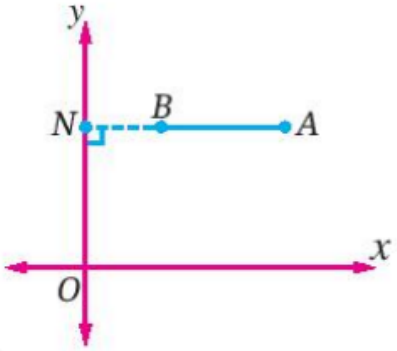
3. $\overline{A'B'}$ is the projection of $\overline{AB}$ on the x-axis.



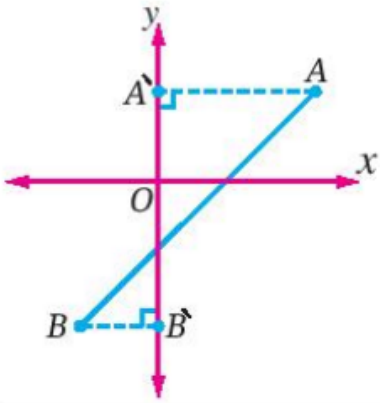
4. $\overline{A'B'}$ is the projection of $\overline{AB}$ on the y-axis.



5. Point N is the projection of $\overline{AB}$ on the y-axis.



6. $\overline{A'B'}$ is the projection of $\overline{AB}$ on the y-axis.



9. Determine the length of the projection of the line segment AB on the x – axis in each of the following cases:

Projection of Point A	Projection of Point B	Length of projection of AB
A (- 2 , 1) →	B (3 , 6) →	
A (- 4 , 3) →	B (- 2 , 3) →	
A (5 , 5) →	B (5 , - 1) →	
A (0 , 3) →	B (6 , - 1) →	

- **Midpoint of a line segment:**

Example: Find the midpoint of the line segment with the endpoints (4, 1) and (2, –5).

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{4 + 2}{2}, \frac{1 - 5}{2}\right)$$

$$= (3, -2)$$

10. Find the midpoint between the following pairs of points:

Point A	Point B	Midpoint of AB
A (3 , 5)	B (7 , 1)	
A (2 , 4)	B (6 , 0)	
A (- 5 , 4)	B (5 , - 4)	
A (2 , - 2)	B (- 6 , 8)	

11. Answer the following:

1) If (x , 0) is the midpoint of AB where A (- 1 , 5) and B (2 , - 5), Find x ?

.....

.....

2) If (2 , 1) is the midpoint of AB where A (3 , - 4) and B (m , 6), Find m ?

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.....

3) If (2 , 1) is the midpoint of AB where B (3 , 0), Find coordinates of A ?

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4) If ABCD is a rhombus where A (3 , 2) and C (- 1 , - 2) , Find the coordinates of the point of intersection of its diagonals M ?

.....

.....

5) ABCD is a square where A (0 , 5) , B (3 , 2) , C (0 , - 1) , Find the coordinates of D ?

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12. If C is the midpoint of AB, Find the value of x and y:

1) C (x , - 3) A (- 3 , y) B (9 , 11)

.....

.....

2) C (- 4 , 2) A (x , 0) B (0 , y)

.....

.....

3) C (- 3 , y) A (x , - 6) B (9 , - 11)

.....

.....

4) C (4 , 6) A (x , 3) B (6 , y)

.....

.....

5) C (x , y) A (1 , 5) B (3 , 7)

.....

.....

6) C (- 2 , 7) A (4 , y) B (x , - 2)

.....

.....

13. If ABCD is a parallelogram with vertices A (- 4 , 2) , B (4 , - 2) , C (9 , 1)
Find:

a. Coordinates of the diagonals intersection.

b. The coordinates of point D.

.....

.....

.....

14. If ABCD is a rhombus with vertices A (3 , 5) , B (12 , - 3) , C (13 , 9)
Find:

a. Coordinates of the diagonals intersection.

b. The coordinates of point D.

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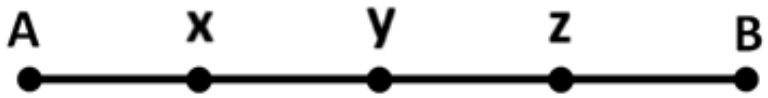
15. Answer the following:

1) If A (- 7 , 13) and B (3 , 5), then Find the coordinates of the points that divide AB into four equal parts ?



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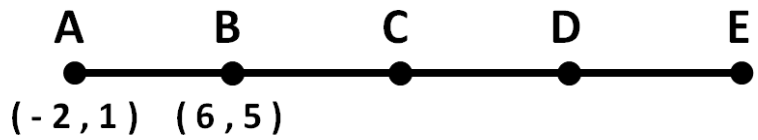
2) If A , B , C , D , E are five collinear points and AB = BC = CD = DE where A (- 2 , 1) , B (6 , 5) , then Find the coordinates of C , D and E ?



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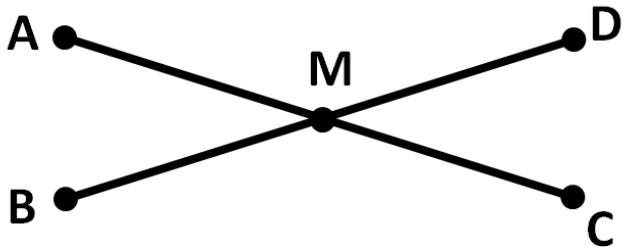
3) If A (- 1 , - 1) , B (2 , 3) , C (6 , 0) , D (3 , - 4) are four points, Prove that AC and BD bisect each other ?



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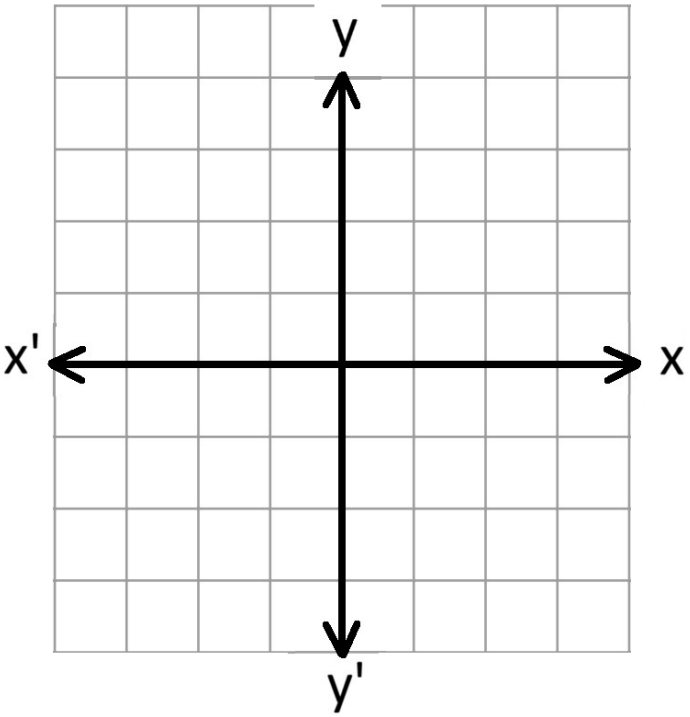
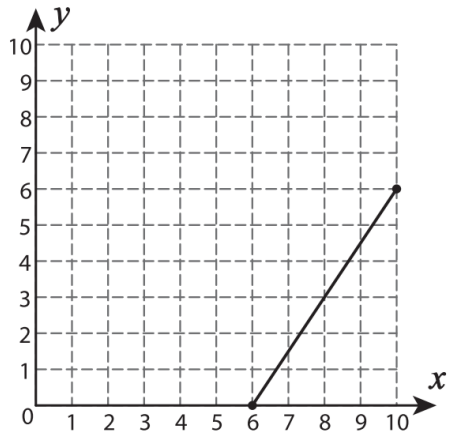
4) **If A (3 , - 1) , B (- 1 , - 1), then Draw the square ABCD such that point C is located in the second quadrant ?**



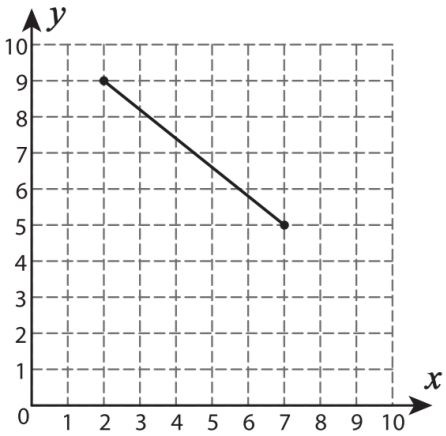
5) **In the opposite figure:**

In each of the following graphs Find the midpoint of the line segment drawn:

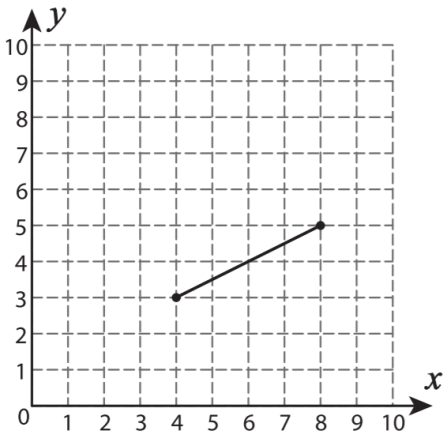
1)



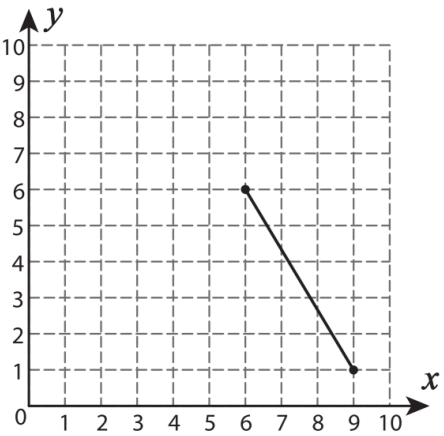
2)



3)



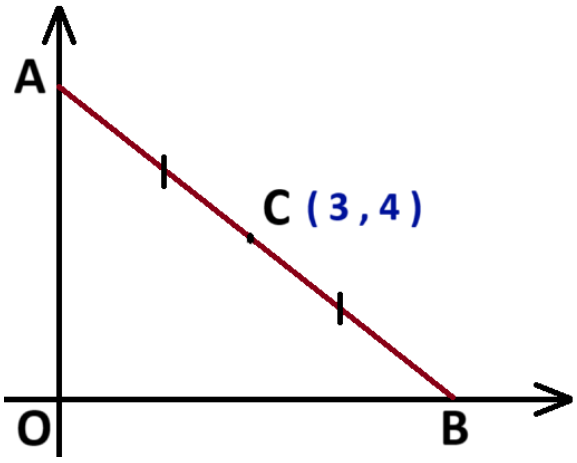
4)



6) **In the opposite figure:**

Find the coordinates of the points A and B.

Calculate the area of Δ AOB



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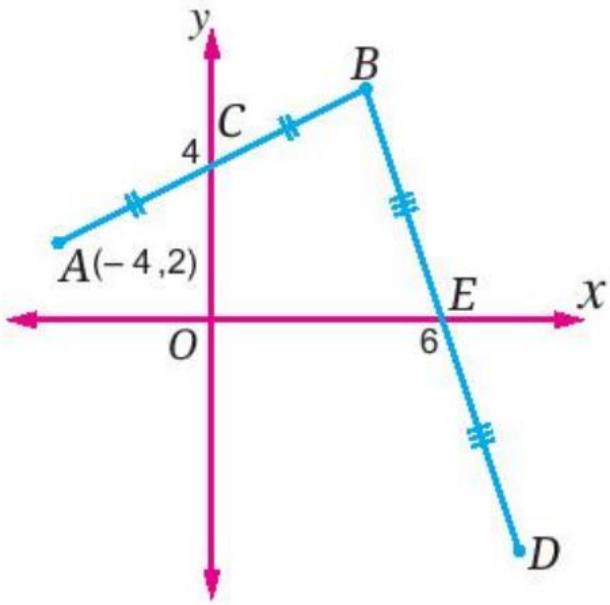
7) **In the opposite figure:**

If C and E are the midpoints of AB and BD respectively, then Determine the coordinates of point D ?



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Homework

1. If (4 ,- 3) is the midpoint of AB where A (3 , - 4), Find coordinates of B ?

   .....

   .....

2. If the origin point is midpoint of AB where A (5 ,- 2), Find coordinates of B ?

   .....

   .....

3. If A (3 , 2) , B (4 , - 3) , C (- 1 , - 2) , and D (- 2 , 3) are vertices of a rhombus , Find the coordinates of the point of intersection of its diagonals M ?

   .....

   .....

4. If AB = BC , and A (0 , 5) and C (- 4 , - 1) , then Find coordinates of B ?

   .....

   .....

5. If AB is a diameter in circle M, where M (5 , 7) and B (8 , 11), then Find coordinates of A ?

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   .....

6. If M (1 , 2) is the point of intersection of diagonals of the parallelogram ABCD , where A (2 , 5), then Find the coordinates of the point C ?

   .....

   .....

7. If M (1 , - 5) is the midpoint between A (x , - 12) and B (9 , y), Find the values of x and y ?

   .....

   .....

8. ABCD is a parallelogram in which A (5 , 4) , B (3 , 1) and C (- 4 , 1), Find:

   a. Coordinates of the diagonals intersection.

   b. The coordinates of point D.

   .....

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