Algebra

FINAL REVISION

GRADE 9 / PREP 3.SECOND TERM

SIMPLEST MATHS | Mr.Mohamed El-Shourbagy/01093149109

Revision

LESSON [1] Solving Tow Equations Of The First Degree In Two Variables Graphically and algebraically

Prelude

- The equations : $x + y = 3$, $3x = y - 7$, $y = 2x - 1$
  - each of them contains two variables which are x and y
  - each of these two variables is of the first degree (the index of each of them is 1)
  therefore they are called equations of the first degree in two variables.

- Solving the equation of the first degree in two variables in $\mathbb{R} \times \mathbb{R}$ means :
  Finding an ordered pair from the real numbers satisfying this equation.

- Assuming an equation as : $x + y = 3$
  It can be solved by making one of its two variables in an independent side as follows :
  $x = 3 - y$ or $y = 3 - x$
  Then by giving one of the two variables a value and calculating the value of the other ,
  then we get the ordered pair which represents a solution of the equation.

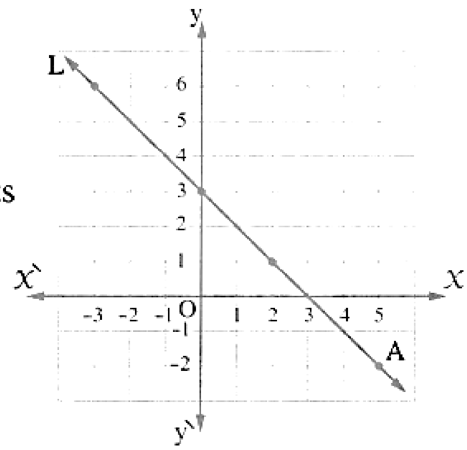
For Example :

With respect to the equation $y = 3 - x$

- as $x = 2$ $\therefore y = 3 - 2 = 1$ $\therefore (2 , 1)$ is a solution of the equation.
- as $x = 0$ $\therefore y = 3 - 0 = 3$ $\therefore (0 , 3)$ is a solution of the equation.
- as $x = -3$ $\therefore y = 3 - (-3) = 6$ $\therefore (-3 , 6)$ is a solution of the equation.

Thus we can get an infinite number of ordered pairs , each of them represents a solution of the equation in $\mathbb{R} \times \mathbb{R}$

- We can represent these ordered pairs on a perpendicular square net as shown in the opposite figure. In which we notice that all of these points lie on one straight line.
  By drawing this straight line which passes through these points , we get the graphical solution of the equation : $x + y = 3$



Notice that :

Each point belongs to this straight line determines a solution of the equation.

, The equation of the first degree in two variables has an infinite number of solutions in $\mathbb{R} \times \mathbb{R}$

First | Solving two equations of the first degree in two variables graphically

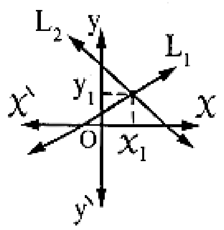
- The meaning of solving two equations graphically is finding the ordered pair or ordered pairs which satisfy the two equations simultaneously.

  Since the set of solution of the equation of the first degree in two variables in $\mathbb{R} \times \mathbb{R}$ is represented graphically by a straight line.

Then to solve the two equations graphically, we do as follows :

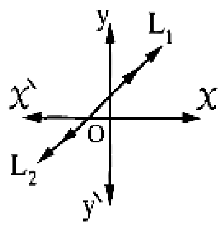
In the Cartesian plane draw the two straight lines which represent the two equations to be L_1 and L_2 , then the S.S. is the point of intersection of the two straight lines L_1 and L_2 , then we have three cases.

1 L_1 and L_2 intersect at the point (x_1 , y_1)



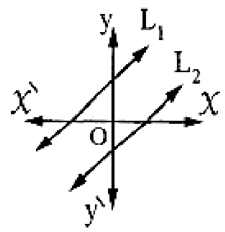
- There is a unique solution (x_1 , y_1)
- The S.S. = $\{(x_1 , y_1)\}$

2 L_1 and L_2 are coincident



- There is an infinite number of solutions

3 L_1 and L_2 are parallel



- There is no solution
- The S.S. = $\varnothing$

The following examples in the following table show each case of the previous cases.

Example (1)

$L_1 : 2x - y = 5$

$L_2 : x + 3y + 1 = 0$

$\because L_1 : y = 2x - 5$

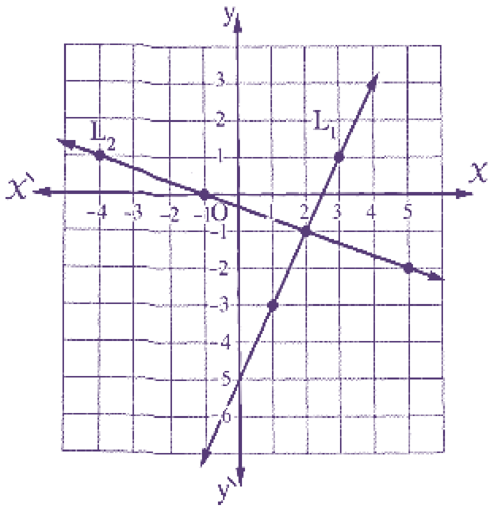
$\therefore$

x	1	2	3
y	−3	−1	1

$\because L_2 : x = -3y - 1$

$\therefore$

x	−1	−4	5
y	0	1	−2



The solution set in $\mathbb{R}^2 = \{(2, -1)\}$

Example (2)

$L_1 : y = 2x - 4$

$L_2 : 4x = 2y + 8$

$\because L_1 : y = 2x - 4$

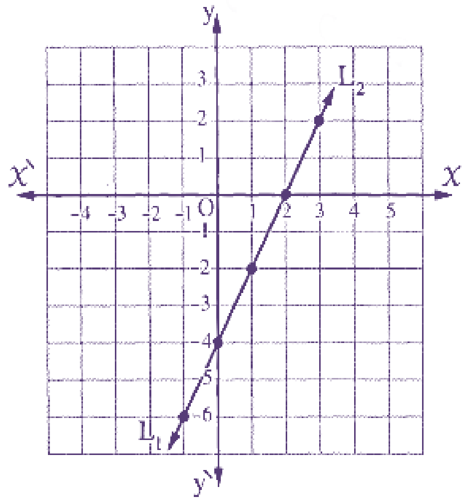
$\therefore$

x	0	1	−1
y	−4	−2	−6

$\because L_2 : x = \frac{2y+8}{4} = \frac{1}{2}y + 2$

$\therefore$

x	2	3	1
y	0	2	−2



The solution set in $\mathbb{R}^2$

$= \{(x, y) : y = 2x - 4, (x, y) \in \mathbb{R}^2\}$

Example (3)

$L_1 : y = 2x - 2$

$L_2 : 2y - 4x - 2 = 0$

$\because L_1 : y = 2x - 2$

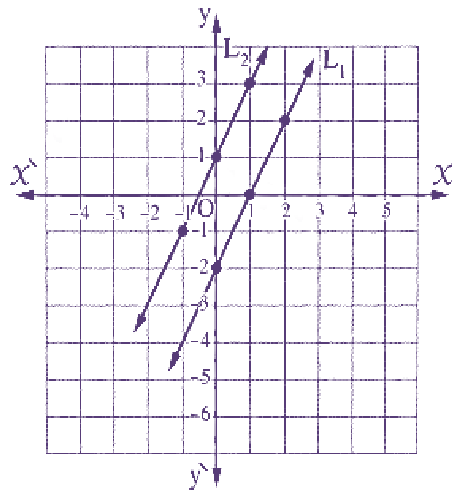
$\therefore$

x	2	1	0
y	2	0	−2

$\because L_2 : y = 2x + 1$

$\therefore$

x	0	1	−1
y	1	3	−1



The solution set in $\mathbb{R}^2 = \varnothing$

Remark

We can recognise the number of solutions of any two equations of the first degree in two variables by knowing the slope of the straight line and determining the point of its intersection with y-axis as follows :

We find the slopes of the two straight lines

If

- $m_1 = m_2$: We find the points of intersection of the two straight lines with y-axis
  If
  - The two points are equals: Then the two straight lines are coincident and the number of solutions is an infinite number.
  - The two points are different: Then the two straight lines are parallel and the number of solutions = 0
- $m_1 \neq m_2$: The two straight lines intersect at one point, then we say the number of solutions = 1

Example 4 Find the number of solutions of each two pairs of the following equations :

1 $2x - 3y = 6$, $y = \frac{2}{3}x + 3$

2 $x + 2y = 1$, $2x + 3y = 12$

3 $4x - y + 7 = 0$, $2y - 8x = 14$

Solution

1 $\because m_1 = \frac{-\text{ the coefficient of } x}{\text{the coefficient of } y} = \frac{-2}{-3} = \frac{2}{3}$

, $m_2 = \text{the coefficient of } x = \frac{2}{3}$

$\therefore m_1 = m_2$

2 $\because m_1 = -\frac{1}{2}$, $m_2 = \frac{-2}{3}$

$\therefore m_1 \neq m_2$

$\therefore$ The two straight lines intersect at one point.

$\therefore$ The number of solutions = 1

Second | Solving two equations of the first degree in two variables algebraically

This method depends on removing one of the two variables to get an equation of the first degree in one variable , then we get the value of this variable by solving this equation.

Then we substitute by this value in any of the two equations to get the value of the other variable which we have removed before.

For that purpose, we follow one of the two methods :

1 **Substituting method.** **2** **Omitting method.**

In the following, we will explain each of the two methods.

1 Substituting method

The following example shows how to use the substituting method to solve two equations of the first degree in two variables algebraically.

Example 5 **Find by using the substituting method the solution set of the following equations in $\mathbb{R} \times \mathbb{R}$:**

$2x - y = 5$, $x + 3y + 1 = 0$

Solution **To use the substituting method, we do the following steps :**

- From the first equation

$\because 2x - y = 5$ $\therefore y = 2x - 5$

- Substituting by $y = 2x - 5$ in the other equation

$\therefore x + 3(2x - 5) + 1 = 0$ $\therefore x + 6x - 15 + 1 = 0$

$\therefore 7x - 14 = 0$ $\therefore 7x = 14$ $\therefore \boxed{x = 2}$

3 Substituting by the value of x in the equation which we got in the first step we get the value of y

$\therefore y = 2 \times 2 - 5$ $\therefore \boxed{y = -1}$ $\therefore$ The S.S. = $\{(2 , -1)\}$

UNIT [2]

LESSON [1] Set of zeroes Of Polynomial Function

Generally

If f is a polynomial function in X , then the set of values of X which makes $f(X) = 0$ is called the set of zeroes of the function f and is denoted by $z(f)$

i.e. $z(f)$ is the solution set of the equation $f(X) = 0$ in $\mathbb{R}$

Notice the difference among f , $f(X)$, $z(f)$:

- f denotes to the function
- $f(X)$ denotes to the rule of the function or the image of X by the function f
- $z(f)$ denotes to the set of zeroes of the function f and it is the solution set of the equation $f(X) = 0$ in $\mathbb{R}$

LESSON [2] Algebraic fractional Function

Definition

If p and k are two polynomial functions , z (k) is the set of zeroes of the function k ,

then the function n where $n : \mathbb{R} - z(k) \longrightarrow \mathbb{R}$, $n(X) = \dfrac{p(X)}{k(X)}$

n is called a real algebraic fractional function or briefly it is called an algebraic fraction.

Remark

The set of zeroes of the algebraic fractional function is the set of values which makes its numerator equals zero and its denominator does not equal zero.

i.e. The set of zeroes of the algebraic fractional function

= the set of zeroes of the numerator – the set of zeroes of the denominator.

For example:

- If the function $n : n(X) = \dfrac{X^2 + 3X}{X^2 - 9}$, then $n(X) = \dfrac{X(X+3)}{(X-3)(X+3)}$

  i.e. $z(n) = \{0 , -3\} - \{3 , -3\} = \{0\}$

- If the function $n : n(X) = \dfrac{3X + 6}{X^2 + X - 2}$, then $n(X) = \dfrac{3(X+2)}{(X-1)(X+2)}$

  i.e. $z(n) = \{-2\} - \{1 , -2\} = \varnothing$

The common domain of two algebraic fractions or more

- The common domain of two algebraic fractions is the set of real numbers that makes the two algebraic fractions identified together (at the same time)
- **Assume that we have the two algebraic fractions n_1 and n_2 where :**

$n_1(X) = \frac{3}{X-2}$ and $n_2(X) = \frac{5X}{X^2-1}$,

then the domain of n_1 (say) $m_1 = \mathbb{R} - \{2\}$ (because n_1 is undefined when $X = 2$)

and the domain of n_2 (say) $m_2 = \mathbb{R} - \{1, -1\}$ (because n_2 is undefined when $X = 1$ or $X = -1$)

According to that :

$= \mathbb{R} -$ the set of zeroes of the two denominators

(because n_1 and n_2 are undefined together when $X = 2$ or $X = 1$ or $X = -1$)

LESSON [3] Equality Of two algebraic Functions

Reducing the algebraic fraction

Definition

It is said that the algebraic fraction is in its simplest form if there are no common factors between its numerator and denominator.

From the previous , to reduce the algebraic fraction , we do as follows :

1. Factorize each of the numerator and denominator perfectly.
2. Identify the domain of the algebraic fraction before removing the common *factors between the numerator and denominator.*
3. Remove the common factors between the numerator and denominator to get the simplest form of the algebraic fraction.

Equality of two algebraic fractions

It is said that the two algebraic fractions n_1 and n_2 are equal (i.e. $n_1 = n_2$) if the two following conditions are satisfied together :

1. The domain of n_1 = the domain of n_2
2. $n_1(X) = n_2(X)$ for each $X \in$ the common domain.

LESSON [4] Operations On the algebraic fractions

First : Adding and subtracting the algebraic fractions

1 Adding and subtracting two algebraic fractions having the same denominator :

If $x \in$ the common domain of the two algebraic fractions n_1 and n_2 where

$n_1(x) = \frac{f(x)}{k(x)}$ and $n_2(x) = \frac{p(x)}{k(x)}$, then :

- $n_1(x) + n_2(x) = \frac{f(x)}{k(x)} + \frac{p(x)}{k(x)} = \frac{f(x) + p(x)}{k(x)}$
- $n_1(x) - n_2(x) = \frac{f(x)}{k(x)} - \frac{p(x)}{k(x)} = \frac{f(x) - p(x)}{k(x)}$

2 Adding and subtracting two algebraic fractions having different denominators :

If $x \in$ the common domain of the two algebraic fractions n_1 and n_2 where

$n_1(x) = \frac{f(x)}{r(x)}$ and $n_2(x) = \frac{p(x)}{k(x)}$, then :

- $n_1(x) + n_2(x) = \frac{f(x)}{r(x)} + \frac{p(x)}{k(x)} = \frac{f(x) \times k(x) + p(x) \times r(x)}{r(x) \times k(x)}$
- $n_1(x) - n_2(x) = \frac{f(x)}{r(x)} - \frac{p(x)}{k(x)} = \frac{f(x) \times k(x) - p(x) \times r(x)}{r(x) \times k(x)}$

The steps of adding or subtracting two algebraic fractions :

1. Arrange the terms of each of the numerator and denominator of each fraction descendingly or ascendingly according to the powers of any variable in it.
2. Factorize the numerator and the denominator of each fraction if possible.
3. Find the common domain which will be the domain of the result.
4. Reduce each fraction separately to make the operations of addition or subtraction easier.
5. Unify the denominators.
6. Perform the operations of addition or subtraction of the terms of the numerators.
7. Put the final result in the simplest form if possible.

The properties of the operations of the addition and subtraction of the algebraic fractions :

- The addition operation of the algebraic fractions has the following properties :

1. Commutation.
2. Association.
3. Zero is the additive neutral (additive identity) of any algebraic fraction.
4. The additive inverse of any algebraic fraction is available.

i.e. the additive inverse of the algebraic fraction : $\frac{g(x)}{k(x)}$ is $-\frac{g(x)}{k(x)}$, $\frac{-g(x)}{k(x)}$ or $\frac{g(x)}{-k(x)}$

LESSON [5] The Operations On the algebraic fractions

Second : Multiplying and Dividing the algebraic fractions

(1) Multiplying the algebraic fractions

Remark

Notice the reduction of the numerator of the first number with the denominator of the second number and the numerator of the second number with the denominator of the first number.

- The following shows how to multiply two algebraic fractions :

Multiplying two algebraic fractions

If $x \in$ the common domain of the two algebraic fractions n_1 and n_2 where :

$n_1(x) = \frac{f(x)}{r(x)}$, $n_2(x) = \frac{p(x)}{k(x)}$

, then : $n_1(x) \times n_2(x) = \frac{f(x)}{r(x)} \times \frac{p(x)}{k(x)} = \frac{f(x) \times p(x)}{r(x) \times k(x)}$

For example :

If : $n_1(x) = \frac{2}{x}$, $n_2(x) = \frac{x}{x-1}$,

then : $n_1(x) \times n_2(x) = \frac{2}{x} \times \frac{x}{x-1} = \frac{2 \times \cancel{x}}{\cancel{x}(x-1)}$

where the domain of the product $= \mathbb{R} - \{0 , 1\}$

, $n_1(x) \times n_2(x) = \frac{2}{x-1}$

Note that :

The domain of the product is the common domain of the two algebraic fractions before reduction.

The steps of multiplying the algebraic fractions :

1. Arrange the terms of each of the numerator and the denominator of each fraction alone descendingly or ascendingly according to the powers of any symbol in it.
2. Factorize the numerator and the denominator of each fraction alone if it is possible.
3. Find the common domain.
4. Remove the common factors between the numerator and the denominator of each fraction and between the numerator of a fraction and the denominator of another fraction.
5. Perform the operation of multiplication and put the result in the simplest form.

The properties of the operation of multiplying the algebraic fractions :

The operation of multiplying the algebraic fractions has the following properties :

1. Commutation.
2. Association.
3. One is the multiplicative neutral (the multiplicative identity).
4. Existing the multiplicative inverses.

The multiplicative inverse of the algebraic fraction :

If n is an algebraic fraction where $n(x) = \frac{p(x)}{k(x)} \neq 0$

, then n has a multiplicative inverse which is the algebraic fraction n^{-1} where $n^{-1}(x) = \frac{k(x)}{p(x)}$

and the domain of n^{-1} is $\mathbb{R}$ – the set of zeroes of each of the numerator and the denominator of any of the two fractions.

Note that :

$n(x)$ and $n^{-1}(x)$ each of them is the reciprocal of the other

i.e. the numerator of each of them is a denominator for the other.

For example:

If $n(x) = \frac{x+1}{x-5}$, then : $n^{-1}(x) = \frac{x-5}{x+1}$

where the domain of $n = \mathbb{R} - \{5\}$

and the domain of $n^{-1} = \mathbb{R} - \{5, -1\}$

Example (2) If $n(x) = \frac{x^3 - 4x^2 - 5x}{x^2 - 25}$

1. **Find :** $n^{-1}(x)$ and state its domain.
2. **Find :** $n^{-1}(-1)$
3. If $n^{-1}(x) = \frac{1}{3}$ **Find the value of** x

Solution

1 $\because n(x) = \frac{x(x^2 - 4x - 5)}{(x-5)(x+5)} = \frac{x(x-5)(x+1)}{(x-5)(x+5)}$

$\therefore$ The domain of $n^{-1} = \mathbb{R} - \{0, 5, -1, -5\}$

, $n^{-1}(x) = \frac{(x-5)(x+5)}{x(x-5)(x+1)}$

$= \frac{x+5}{x(x+1)}$

2 $n^{-1}(-1)$ is undefined because $-1 \notin$ the domain of n^{-1}

3 $\because n^{-1}(x) = \frac{1}{3}$

$\therefore \frac{x+5}{x(x+1)} = \frac{1}{3}$

$\therefore x(x+1) = 3(x+5)$

$\therefore x^2 + x - 3x - 15 = 0$

$\therefore x^2 - 2x - 15 = 0$

$\therefore (x-5)(x+3) = 0$

$\therefore x = 5$ refused because $5 \notin$ the domain of n^{-1} or $x = -3$

(2) Dividing an algebraic fractions by another

Dividing an algebraic fraction by another :

If n_1 and n_2 are two algebraic fractions where :

$n_1(x) = \frac{f(x)}{r(x)}$, $n_2(x) = \frac{p(x)}{k(x)}$, then : $n_1(x) \div n_2(x) = n_1(x) \times n_2^{-1}(x) = \frac{f(x)}{r(x)} \times \frac{k(x)}{p(x)}$

where the domain of $n_1 \div n_2$ = the common domain of each of n_1, n_2 and n_2^{-1}

$= \mathbb{R}$ – the set of zeroes of the denominator of n_1 or the denominator of n_2

or the numerator of n_2

$= \mathbb{R} - \{z(r) \cup z(p) \cup z(k)\}$

Example 3 **Find $n(x)$ in the simplest form showing the domain of n :**

$n(x) = \frac{x^2 - 7x + 10}{x^2 - 4x - 5} \div \frac{x^3 - 8}{x^2 + 2x + 4}$

, then find n (2) and n (3) if it is possible.

Solution

$\because n(x) = \frac{(x-2)(x-5)}{(x-5)(x+1)} \div \frac{(x-2)(x^2 + 2x + 4)}{x^2 + 2x + 4}$

$\therefore$ The domain of $n = \mathbb{R} - \{5, -1, 2\}$

, $n(x) = \frac{x-2}{x+1} \times \frac{1}{x-2} = \frac{1}{x+1}$

n (2) is undefined because $2 \notin$ the domain of n

, $n(3) = \frac{1}{3+1} = \frac{1}{4}$

UNIT [3] THE PROBABILITY

- We can calculate the probability of an event (say A) from the relation :

$$P(A) = \frac{\text{The number of elements of the event A}}{\text{The number of elements of the sample spaces}} = \frac{n(A)}{n(S)}$$

For example:

In the experiment of rolling a fair die once and observing the number appears on the upper face , if S is the sample space of the experiment and A is the event of getting an even number , then :

$S = \{1, 2, 3, 4, 5, 6\}$, $n(S) = 6$, $A = \{2, 4, 6\}$, $n(A) = 3$

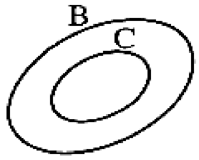
, then $P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$ (i.e. The probability of occurring the event $A = \frac{1}{2}$)

Remarks

- Zero $\le$ the probability of any event ≤ 1
- Probability can be written as a fraction or percentage.

Remarks

From the previous example we notice that :



1 $C \subset B$ therefore $B \cap C = C$, then we deduce that :

The probability of occurring the two events B and C together

= the probability of occurring the event C

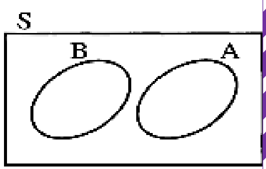
i.e. $P(B \cap C) = P(C) = \frac{n(C)}{n(S)}$

2 $A \cap C = \varnothing$ therefore it is said that the two events A and C are two mutually exclusive events

, then we can deduce that :

The probability of occurring the event A or C = $P(A \cup C) = P(A) = \frac{n(A)}{n(S)}$

Mutually exclusive events

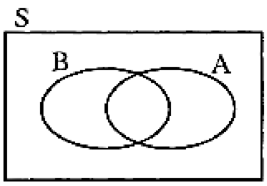


- It is said that the two events A and B are mutually exclusive if

$A \cap B = \varnothing$, then $P(A \cap B) = 0$

i.e. The probability of their occurring together = the probability of the impossible event = 0

Rule :



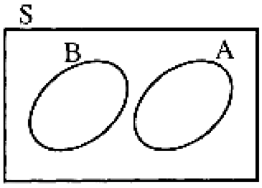
- For any two events from the sample space S of a random experiment :

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- If A and B are two mutually exclusive events , then :

$P(A \cap B) = \text{zero}$, then :

$P(A \cup B) = P(A) + P(B)$



Remarks

For any event A of the sample space S it will be :

1 $A \cap \grave{A} = \varnothing$

i.e. The two events A and $\grave{A}$ are two mutually exclusive events

i.e. Occurring one of them prevents the occurring of the other , then $P(A \cap \grave{A}) = \text{zero}$

2 $A \cup \grave{A} = S$

i.e. The union of any event and the complementary event of it = the set of sample space S ,

then $P(A \cup \grave{A}) = P(A) + P(\grave{A}) = P(S) = 1$

From that we deduce that :

$P(A) = 1 - P(\grave{A})$, $P(\grave{A}) = 1 - P(A)$

Note that :

$$P(S) = \frac{n(S)}{n(S)} = 1$$

Remarks

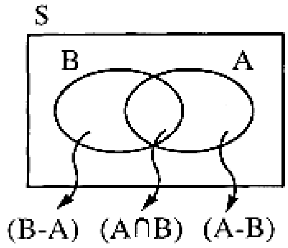
If A and B are two events of a sample space (S) of a random experiment ,

then $(A - B) \cup (A \cap B) = A$

i.e. $P(A - B) + P(A \cap B) = P(A)$

Also : $(B - A) \cup (A \cap B) = B$

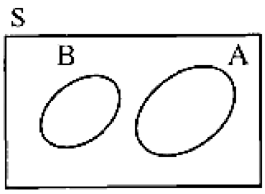
i.e. $P(B - A) + P(A \cap B) = P(B)$



Remarks

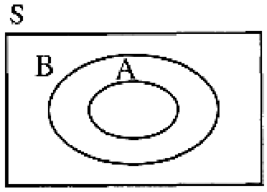
1 If A and B are two mutually exclusive of the sample space (S) , then :

- $A - B = A$ ***i.e.*** $P(A - B) = P(A)$
- $B - A = B$ ***i.e.*** $P(B - A) = P(B)$



2 If A and B are two events of the sample space (S) and $A \subset B$, then :

- $A - B = \varnothing$
- $P(A - B) = P(\varnothing) = \frac{n(\varnothing)}{n(S)} = \text{zero}.$



Examinations models from the school book

Model 1

Answer the following questions :

1 Complete each of the following :

(1) If x is a negative number, the greatest number of the following :

$5 + x$, $5 - x$, $5x$, and $\frac{5}{x}$ is ……………

(2) If : $x \in \mathbb{R} - \{0 , 1\}$, then $\frac{1 - x}{x} \div \frac{x - 1}{x}$ in its simplest form equals ……………

(3) If A , B are two events in a random experiment , and $B \subset A$

, then $P(A \cap B) =$ ……………

(4) If the sum of two positive numbers is 4 , and the sum of their squares is 10 , then the two numbers are ……………

(5) If the solution set of the equation : $x^2 + m x + 9 = 0$ is $\{-3\}$, then m = ……………

(6) If : $n(x) = \frac{x + 7}{x - 2}$, then the domain of n^{-1} is ……………

---

2 Choose the correct answer from those given :

(1) The common domain of the two fractions $\frac{2}{x - 3}$, $\frac{7}{x - 6}$ is ……………

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{3\}$ (c) $\mathbb{R} - \{6\}$ (d) $\mathbb{R} - \{3 , 6\}$

(2) The probability of the impossible event equals ……………

(a) $\varnothing$ (b) zero (c) 1 (d) – 1

(3) If : $2x = 1$, then $\frac{1}{5} x =$ ……………

(a) $\frac{2}{5}$ (b) $\frac{1}{5}$ (c) $\frac{1}{10}$ (d) $\frac{1}{2}$

(4) If : $x^2 - y^2 = 2(x + y)$ such that $x + y \neq 0$, then $x - y =$ ……………

(a) 2 (b) 4 (c) 6 (d) 8

(5) The set of zeroes of the function f where $f(x) = \frac{x - 3}{x + 2}$ is ……………

(a) {zero} (b) $\{3\}$ (c) $\{-2\}$ (d) $\{3 , -2\}$

(6) The solution set of the two equations : $x - 2y = 1$, $3x + y = 10$ is ……………

(a) $\{(5 , 2)\}$ (b) $\{(3 , 1)\}$ (c) $\{(4 , 2)\}$ (d) $\{(1 , 3)\}$

---

3 [a] Solve the equation : $3x^2 = 5x + 4$ approximating to the nearest two decimals.

[b] Find n in its simplest form showing its domain where : $n(x) = \frac{x^2 + 2x + 4}{x^3 - 8} + \frac{x^2 + x - 2}{x^2 - 4}$

4 [a] Graph the function f where $f(x) = x^2 - 2x + 1$ over the interval $[-2, 4]$, then from the graph find the solution set of the equation : $x^2 - 2x + 1 = 0$

[b] If : $n(x) = \frac{x^2 - 3x}{x^2 - 9} \div \frac{2x}{x + 3}$, find n in its simplest form showing its domain.

---

5 **[a] Find the solution set of the two equations :** $y = x - 3$, $x^2 + y^2 = 17$ in $\mathbb{R} \times \mathbb{R}$

[b] A card is drawn randomly from 30 identical cards numbered from 1 to 30, find the probability that the number on the drawn card is :

First : Divisible by 4 **Second :** Prime number

Model 2

Answer the following questions :

1 **Complete each of the following :**

(1) If : $x = 2$, $y = 3$, then $(y - 2x)^{10} = \dots\dots$

(2) If : $x \in \mathbb{R} - \{0, 3\}$, then $\frac{x}{x - 3} \div \frac{x}{3 - x} = \dots\dots$ in its simplest form.

(3) If A, B are two events in a random experiment, and $A \subset B$, then $P(A \cup B) = \dots\dots$

(4) If : $x - y = 3$, $x + y = 9$, then $y = \dots\dots$

(5) The common domain of the two fractions $\frac{x}{x^2 - 1} \div \frac{3}{x^2 + x}$ is $\dots\dots$

(6) If : $\{-2, 2\}$ is the set of zeros of the function $f(x) = x^2 + a$, then $a = \dots\dots$

---

2 **Choose the correct answer from those given :**

(1) If the sum of two numbers is 8, and their product is 15, then the two numbers are $\dots\dots$

(a) 2, 6 (b) 3, 5 (c) 4, 4 (d) 1, 15

(2) If the fraction $\frac{x - a}{x + 5}$ is the multiplicative inverse of $\frac{x + 5}{x + 3}$, then $a = \dots\dots$

(a) 3 (b) -5 (c) -3 (d) 5

(3) If a coin is tossed once, then the probability that the head appears = $\dots\dots$

(a) 1 (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

(4) If : $x + y = 0$, $x^2 = 25$, then $y = \dots\dots$

(a) 20 (b) -5 (c) 5 (d) ± 5

(5) If the two equations : $x + 3y = 6$, $2x + ky = 12$ have an infinite number of solutions, then k = ……………

(a) 2 (b) 6 (c) 3 (d) 1

(6) If the probability that Hamdy succeeds is 95 % , then the probability that he does not succeed is ……………

(a) 20 % (b) 10 % (c) 5 % (d) zero

---

3 **[a] Solve the two equations :** $x - 2y = 1$, $x^2 - xy = 0$ in $\mathbb{R} \times \mathbb{R}$

[b] Find n in its simplest form showing its domain where : $n(x) = \frac{x - 3}{x^2 - 7x + 12} - \frac{4}{x^2 - 4x}$

---

4 **[a] Solve the equation :** $2x^2 - 5x + 1 = 0$ in $\mathbb{R}$ approximating to the nearest two decimals.

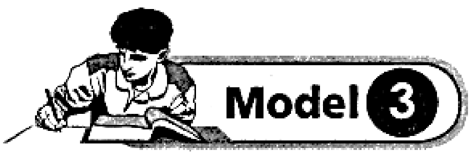
[b] If : $n(x) = \frac{x^2 - 3x + 2}{x^2 - 1} \div \frac{3x - 15}{x^2 - 4x - 5}$, find n in its simplest form showing its domain.

---

5 **[a]** If the set of zeroes of the function f where $f(x) = ax^2 + bx + 8$ is $\{2 , 4\}$. Find the value of a and b

[b] A ball is drawn randomly from 25 identical balls of the same volume and weight , 10 of them are red , 8 are white and the rest are green , find the probability that the drawn ball is :

First : White **Second :** Green or white **Third :** Not green



Answer the following questions :

1 **Complete each of the following :**

(1) If : $x \in \mathbb{R} - \{2\}$, then $\frac{x}{x - 2} + \frac{2}{2 - x} =$ …………… (in its simplest form).

(2) If : $xy = 3$, $x^2y - xy^2 = 6$, then $x - y =$ ……………

(3) If A , B are two mutually exclusive events in a random experiment , then $P(A \cap B) =$ ……………

(4) If the length of a rectangle is 3 cm. and its diagonal is of length 4 cm. , then its width is …………… cm.

(5) If : $(5 , x - 4) = (y + 2 , 3)$, then $x + y =$ ……………

(6) The common domain of the two fractions : $\frac{5}{x - 3}$, $\frac{x + 1}{x^2 - 3x}$ is ……………

2 **Choose the correct answer from those given :**

(1) If x is a negative number, then the greatest number is ……………

(a) $7 + x$ (b) $7 - x$ (c) $7x$ (d) $\frac{7}{x}$

(2) If : $n(x) = \frac{x-1}{x+3}$, then the domain of n^{-1} is ……………

(a) $\mathbb{R} - \{-3\}$ (b) $\mathbb{R} - \{1\}$ (c) $\mathbb{R} - \{1, -3\}$ (d) $\{1, -3\}$

(3) If a die is tossed once, then the probability of appearance of an odd number equals ……………

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) 1 (d) 3

(4) If the solution set of the equation : $x^2 - a x + 4 = 0$ is $\{-2\}$, then a = ……………

(a) zero (b) -1 (c) -2 (d) -4

(5) If the two equations : $x + 2y = 4$, $2x + ky = 11$ represent two parallel lines, then k = ……………

(a) 4 (b) -4 (c) 1 (d) -1

(6) The solution set of the two equations : $x - y = 0$, $xy = 16$ in $\mathbb{R} \times \mathbb{R}$ is ……………

(a) $\{(0, 0)\}$ (b) $\{(4, 4)\}$ (c) $\{(-4, -4)\}$ (d) $\{(4, 4), (-4, -4)\}$

---

3 **[a] Find n in its simplest form showing its domain where :** $n(x) = \frac{x^3 - 8}{x^2 + x - 6} \times \frac{x + 3}{x^2 + 2x + 4}$

[b] Find the solution set of the equation : $(x - 3)^2 - 5x = 0$ in $\mathbb{R}$, approximating to the nearest two decimals.

---

4 **[a]** If : $n(x) = \frac{x+5}{x^2 + 7x + 10} - \frac{x-1}{x^2 + 5x + 6}$, find n in its simplest form showing its domain, find n (– 2)

[b] Find the solution set of the two equations : $x - y = 4$, $3x + 2y = 7$ in $\mathbb{R} \times \mathbb{R}$ graphically, then verify algebraically.

---

5 **[a]** Graph the function f where $f(x) = 4x - x^2 - 3$ on the interval $[0, 4]$.

Find the solution set of the equation : $x^2 - 4x + 3 = 0$ and write the equation of the axis of symmetry.

[b] If A, B are two events in a random experiment, P (A) = 0.7, P (B) = 0.6 and P $(A \cap B) = 0.4$, find the probability of :

First : Non occurrence of the event A.

Second : Occurrence of one event without the other.

Model 4

Answer the following questions :

1 Complete each of the following :

(1) If : $2^5 \times 3^5 = m \times 6^5$, then m = ……………

(2) The set of zeroes of the function f where $f(X) = \frac{X^2 - X - 2}{X^2 - 4}$ is ……………

(3) If A , B are two mutually exclusive events in a random experiment , then P (A ∩ B) = ……………

(4) The simplest form of $\frac{-3}{X^2 + 4} + \frac{X^2 + 7}{X^2 + 4}$ = ……………

(5) If the curve of the function f : $f(X) = X^2 - a$ passes through the point (2 , 0) , then a = ……………

(6) The solution set of the two equation : $X + 2y = 3$, $4X + 8y = 7$ is ……………

2 Choose the correct answer from those given :

(1) If : $n(X) = \frac{X - 2}{X + 1}$, then $n^{-1}(2)$ = ……………

(a) zero (b) 2 (c) – 1 (d) undefined

(2) If : $\frac{1}{X} + \frac{1}{y} + \frac{1}{X y} = \frac{\cdots\cdots}{y X}$

(a) 2 (b) 3 (c) $X + y + 1$ (d) $X + y$

(3) The solution set of the two equations : $X + y = 0$, $X^2 + y^2 = 2$ in $\mathbb{R} \times \mathbb{R}$ is ……………

(a) {(0 , 0)} (b) {(1 , – 1)}
(c) {(– 1 , 1)} (d) {(1 , – 1) , (– 1 , 1)}

(4) The common domain of the two fractions : $\frac{2}{X - 3}$, $\frac{7}{2X - 6}$ is ……………

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{3\}$ (c) $\mathbb{R} - \{2, 3\}$ (d) $\mathbb{R} - \{3, -3\}$

(5) If the probability of occurrence the event A is 75 % , then the probability of non-occurrence the event A equals ……………

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) 1

(6) If $X = 3$ belongs to the solution set of the equation : $X^2 - aX - 6 = 0$, then a = ……………

(a) 3 (b) 2 (c) 1 (d) – 1

3 **[a] Find the solution set of the two equations :** $x - 2y = 0$, $2x + 3y = 7$ graphically , then verify algebraically.

[b] Find n in its simplest form showing its domain where :

$$n(x) = \frac{x^3 - 8}{x^3 - 7x^2 + 10x} \div \frac{x^2 + 2x + 4}{3x^2 - 15x}$$

---

4 [a] A classroom consists of 40 students , 30 of them play football , 20 play basketball and 15 play football and basketball , if a student is chosen randomly. Find :

First : The probability that this student is playing one of the two games at least.

Second : The probability that this student is playing only one of the two games.

[b] Solve the two equations : $x - y = 1$, $x^2 + y^2 = 25$ in $\mathbb{R} \times \mathbb{R}$

---

5 **[a] Find n in its simplest form showing its domain where :** $n(x) = \frac{x^2 - 2x}{x^2 - 3x + 2} - \frac{4 - x^2}{x^2 + x - 2}$

[b] Graph the function f where $f(x) = x^2 + 2x + 3$ on the interval $[-3 , 1]$, from the graph **find :**

First : The maximum or minimum value of the function as well as the vertex point of the curve.

Second : The solution set of the equation : $x^2 + 2x + 3 = 0$

Model 5

Answer the following questions :

1 **Complete each of the following :**

(1) If $x \in \mathbb{R} - \{0\}$, then $\frac{1 - x}{x} + \frac{x - 1}{x} =$ .... (in its simplest form)

(2) The sum of two positive numbers is 5 , and the sum of their squares is 13 , then the two numbers are .... , ...

(3) If 3 is a zero of the function $f : f(x) = x^3 - 3x^2 + a$, then a = ...

(4) A , B are two mutually exclusive events in a random experiment , $P(A) = \frac{1}{2}$ and $P(A \cup B) = \frac{5}{6}$, then $P(B) =$ ....

(5) The common domain of the two fractions $n_1 = \frac{2}{x^2 - 1}$, $n_2 = \frac{5x}{x^2 - x}$ is ....

(6) The solution set of the two equations : $x - y = 2$, $y - x = 3$ is ....

2 **Choose the correct answer from those given :**

(1) The value of the expression $\left(\frac{5x}{x^2+1} \div \frac{x}{x^2+1}\right)$ is equal to ....

(a) 5 (b) – 5 (c) x (d) – x

(2) The solution set of the two equations : $x - y = 1$, $x + y = 7$ in $\mathbb{R} \times \mathbb{R}$ is ....

(a) $\{(1 , 0)\}$ (b) $\{(2 , 1)\}$ (c) $\{(2 , 5)\}$ (d) $\{(4 , 3)\}$

(3) If P (A) = 4 P (À) , then P (A) = ...

(a) 0.8 (b) 0.6 (c) 0.4 (d) 0.2

(4) If the length of a rectangle is 2 cm. more than its width , and its area is 24 cm^2, then its perimeter = ...

(a) 10 cm. (b) 20 cm. (c) 30 cm. (d) 40 cm.

(5) If : $n(x) = \frac{x-7}{x+3}$, then the domain of $n^{-1}(x)$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{-3\}$ (c) $\mathbb{R} - \{-3 , 7\}$ (d) $\mathbb{R} - \{7\}$

(6) If : $x - 3 = 0$, $y^2 = x + 6$, then y = ...

(a) 9 (b) 3 (c) – 3 (d) 3 , – 3

---

3 **[a] Find n in its simplest form showing its domain where :** $n(x) = \frac{x^2 - 2x}{x^2 - 4} + \frac{2x + 6}{x^2 + 5x + 6}$

[b] Solve the equation : $2x(x - 5) = 1$ approximating to the nearest one decimal.

---

4 **[a] Solve the two equations :** $x + y = 7$, $x^2 + y^2 = 25$ in $\mathbb{R} \times \mathbb{R}$

[b] A bag contains 15 identical balls numbered from 1 to 15 , one ball is chosen randomly , if the event A is getting an odd number and the event B is getting a prime number. **Find :**

(1) P (A) (2) P (B) (3) P (A – B)

---

5 **[a]** If : $n(x) = \frac{x^3 - 8}{x^2 - 6x + 5} \div \frac{x^3 - 2x^2 + 4x}{2x^2 + x - 3}$, find n in its simplest form showing its domain , find n (– 1)

[b] A number formed from two digits , their sum equals 5 , and if the two digits exchange then the resulting number exceeds than the original number by 9 , find the original number.

Some governorates' examinations

1 Cairo Governorate

Answer the following questions : *(Calculators are permitted)*

1 Choose the correct answer from those given :

(1) If : $n_1(X) = \frac{X+2}{X-1}$, $n_2(X) = \frac{X-5}{X+3}$, then the common domain of the two functions n_1 and n_2 is ....

(a) $\mathbb{R} - \{1 , -2\}$ (b) $\mathbb{R} - \{-3 , 5\}$ (c) $\mathbb{R}$ (d) $\mathbb{R} - \{1 , -3\}$

(2) The set of zeroes of the function f where $f(X) = 2X^2$ is ....

(a) $\{0\}$ (b) $\mathbb{R} - \{0\}$ (c) $\mathbb{R} - \{2\}$ (d) $\mathbb{R}$

(3) If (2 , 1) is a solution of the equation : $2X + ay = 6$, then a = ...

(a) 2 (b) 6 (c) 1 (d) 3

(4) If A and B are two mutually exclusive events , then $P(A \cap B)$ = ...

(a) 1 (b) 0 (c) $\varnothing$ (d) $\frac{1}{2}$

(5) The point of intersection of the two straight lines which equations are $X + y = 3$ and $X - y = 1$ is ....

(a) (1 , 2) (b) (4 , – 1) (c) (2 , 1) (d) (5 , – 2)

(6) If A and B are two events from the sample space of a random experiment and if P (B) = 0.7 and P (A) = 0.2 and $A \subset B$, then $P(A \cup B)$ = ...

(a) zero (b) 0.2 (c) 0.7 (d) 0.5

---

2 **[a] Find :** n (X) in its simplest form showing the domain of n where :

$$n(X) = \frac{3X-4}{X^2-5X+6} + \frac{2X+6}{X^2+X-6}$$

[b] Find algebraically the S.S. in $\mathbb{R} \times \mathbb{R}$ of the two equations : $X - 3y = 6$ and $2X + y = 5$

---

3 **[a] Find the solution set in $\mathbb{R}$ of the equation :** $X^2 - 5X + 3 = 0$ approximating the roots to the nearest tenth.

[b] The perimeter of a rectangle is 14 cm. and its area 12 cm^2.
Find each of its two dimensions.

---

4 **[a]** If : $n(X) = \frac{X^2+X+1}{X^2-9} \div \frac{X^3-1}{X^2-4X+3}$, then find n (X) in its simplest form showing the domain of n.

[b] Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations : $X + y = 3$ and $Xy + y^2 = 6$

5 [a] If A and B are two events from the sample space of a random experiment ,

p (A) = 0.7 , p (B) = 0.4 and p (A ∩ B) = 0.2 , then find

(1) p (Á) (2) p (A ∪ B)

[b] Graph the quadratic function f where $f(x) = x^2 - 4x + 3$, $x \in [-1 , 5]$, then from the graph deduce :

(1) The coordinates of the vertex of the curve.

(2) The minimum value of the function.

(3) The S.S. in ℝ of the equation : $x^2 - 4x + 3 = 0$

2 Giza Governorate

Answer the following questions : *(Calculators are permitted)*

1 Choose the correct answer from those given :

(1) If the sum of two positive numbers is 9 and their product is 8 , then the two numbers are ....

(a) 2 , 7 (b) 3 , 6 (c) 4 , 5 (d) 1 , 8

(2) The S.S. of the two equations : $x + y = 0$, $x - 2 = 0$ in ℝ × ℝ is ....

(a) {(0 , 2)} (b) {(2 , 2)} (c) {(– 2 , 2)} (d) {(2 , – 2)}

(3) If a regular dice is rolled once , then the probability of getting an even number equal ....

(a) 3 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

(4) The simplest form of the function f where : $f(x) = \frac{2x^2 + x}{x}$ and $x \neq 0$ is ....

(a) $3x$ (b) $2x^2 + 1$ (c) $x^2 + 1$ (d) $2x + 1$

(5) If : p (A) = $\frac{1}{3}$, then p (Á) = ...

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) 1 (d) $\frac{1}{2}$

(6) If the domain of the function : $n(x) = \frac{1}{x} + \frac{9}{x + b}$ is ℝ – {0 , 4} , then b = ...

(a) 0 (b) 4 (c) – 4 (d) 3

---

2 **[a] Find algebraically the S.S. of the two equations :** $2x - y + 3 = 0$ and $x + 2y + 4 = 0$ in ℝ × ℝ

[b] The difference between two numbers is 5 and the product of them is 36 find the two numbers.

3 [a] If A and B are two events in the sample space of a random experiment and P (A) = 0.6 , P (B) = 0.3 , P (A ⋂ B) = 0.2 , then Find : (1) P (A ⋃ B) (2) P (A – B)

[b] Simplify to its simplest form showing the domain of n where :

$$n(x) = \frac{3x}{x^2 - 2x} - \frac{12}{x^2 - 4}$$

---

4 **[a] Find the S.S. of the two equations :** $3x + 4y = 24$ and $x - 2y = -2$ in ℝ × ℝ

[b] Find by using the general formula the solution set of the equation :

$3x^2 - 6x + 1 = 0$

---

5 [a] Find : n (x) in the simplest form showing the domain where :

$$n(x) = \frac{x^2 - 3x + 2}{x^2 - 49} \div \frac{x - 2}{x + 7}$$

[b] Graph the function f : $f(x) = x^2 - 1$ taking $x \in [-2, 2]$ and from the graph deduce :

(1) The coordinates of the vertex of the curve.

(2) The minimum or maximum value of the function.

(3) The two roots of the equation $f(x) = 0$

3 Alexandria Governorate

Answer the following questions : *(Calculators are permitted)*

1 **Choose the correct answer from those given :**

(1) If A and B are mutually exclusive events and if P (A) = $\frac{1}{3}$, P (A ⋃ B) = $\frac{7}{12}$, then P (B) = ...

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

(2) The set of zeroes of f where : $f(x) = -3x$ is ....

(a) {0} (b) {– 3} (c) {– 3 , 0} (d) ℝ

(3) If A and B are two events from S where B ⊂ A , then P (A ⋂ B) = ...

(a) zero (b) P (B) (c) P (A) (d) P (A – B)

(4) The solution set of the two equations : $x + 3y = 4$, $3y + x = 1$ is ....

(a) {(3 , 1)} (b) {(1 , 3)} (c) ∅ (d) {(1 , 0)}

(5) If : P (A) = P (A`) , then P (A) = ...

(a) zero (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

(6) The domain of the function n : $n(x) = \frac{x}{x^2 + 9}$ is ....

(a) ℝ (b) ℝ – {3} (c) ℝ – {– 3} (d) ℝ – {3 , – 3}

2 [a] Find the S.S. of the equation : $x^2 - 2x - 4 = 0$ in $\mathbb{R}$ approximating the result to the nearest tenth.

[b] Find n (x) in the simplest form showing the domain of n where :

$$n(x) = \frac{x^2 + x + 1}{x} \times \frac{x^2 - x}{x^3 - 1}$$

---

3 [a] **Find graphically , then verify algebraically the S.S. in $\mathbb{R} \times \mathbb{R}$ to the equations :**

$y = x + 4$ and $x + y = 4$

[b] **Put in the simplest form with determining the domain of the function n :**

$$n(x) = \frac{x^2 - 4}{x^2 + 3x + 2} - \frac{x^2 - 2x}{x^2 - x - 2}$$ then , find n (1)

---

4 [a] 12 cards numbered from 1 to 12 , if a card is picked randomly , what's the probability of getting an odd number divisible by 3

[b] **Find algebraically the solution set of the two equations :**

$y - x = 2 , x^2 + xy - 4 = 0$

---

5 [a] **Represent graphically the function f : $f(x) = 4 - x^2$** on the interval $[-3 , 3]$ and from the drawing deduce the :

(1) Roots of the equation : $f(x) = 0$

(2) Equation of symmetric axis.

[b] A rectangle with a length more than its width by 4 cm. If the perimeter of the rectangle is 28 cm. , find area of the rectangle.

Answers of examination's models of algebra and probability

Model 1

1

(1) $5 - x$ (2) -1 (3) P (B)

(4) 1 and 3 (5) 6 (6) $\mathbb{R} - \{-7, 2\}$

2

(1) (d) (2) (b) (3) (c) (4) (a) (5) (b) (6) (b)

3

[a] $\because 3x^2 - 5x - 4 = 0$

$\therefore a = 3, b = -5$ and $c = -4$

$\therefore x = \frac{5 \pm \sqrt{25 - 4 \times 3 \times -4}}{2 \times 3} = \frac{5 \pm \sqrt{73}}{6}$

$\therefore x \simeq 2.26$ or $x \simeq -0.59$

$\therefore$ The S.S. = {2.26 , – 0.59}

[b] $n(x) = \frac{x^2 + 2x + 4}{(x-2)(x^2 + 2x + 4)} + \frac{(x+2)(x-1)}{(x-2)(x+2)}$

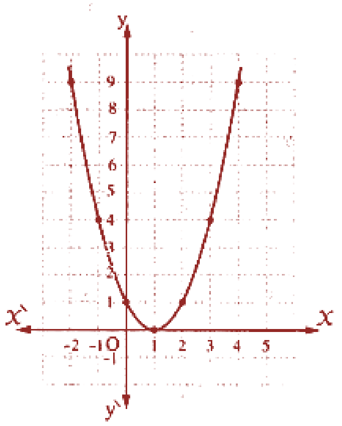
The domain of n = $\mathbb{R} - \{2, -2\}$

, $n(x) = \frac{1}{x-2} + \frac{x-1}{x-2} = \frac{x}{x-2}$

4

[a] $f(x) = x^2 - 2x + 1$

x	-2	-1	0	1	2	3	4
y	9	4	1	0	1	4	9



From the graph : The S.S. = {1}

[b] $n(x) = \frac{x(x-3)}{(x-3)(x+3)} \div \frac{2x}{x+3}$

$\therefore$ The domain of n = $\mathbb{R} - \{3, -3, 0\}$

, $n(x) = \frac{x}{x+3} \times \frac{x+3}{2x} = \frac{1}{2}$

5

[a] Substituting by the 1st equation in the second equation

$\therefore x^2 + (x-3)^2 = 17$

$\therefore x^2 + x^2 - 6x + 9 = 17$ $\therefore 2x^2 - 6x - 8 = 0$

$\therefore x^2 - 3x - 4 = 0$ $\therefore (x-4)(x+1) = 0$

$\therefore x = 4$ and hence y = 1

or $x = -1$ and hence y = – 4

$\therefore$ The S.S. = {(4 , 1) , (– 1 , – 4)}

[b] **First :** $\frac{7}{30}$ **Second :** $\frac{1}{3}$

Model 2

1

(1) 1 (2) – 1 (3) P (B)

(4) 3 (5) $\mathbb{R} - \{-1, 0, 1\}$ (6) – 4

2

(1) (b) (2) (c) (3) (c) (4) (d) (5) (b) (6) (c)

3

[a] From the first equation : $x = 2y + 1$

, substituting in the second equation

$\therefore (2y+1)^2 - (2y+1)y = 0$

$\therefore 4y^2 + 4y + 1 - 2y^2 - y = 0$

$\therefore 2y^2 + 3y + 1 = 0$ $\therefore (2y+1)(y+1) = 0$

$\therefore y = -\frac{1}{2}$ and hence $x = 0$

or y = – 1 and hence $x = -1$

[b] $n(x) = \frac{x-3}{(x-3)(x-4)} - \frac{4}{x(x-4)}$

$\therefore$ The domain of n = $\mathbb{R} - \{3, 4, 0\}$

, $n(x) = \frac{1}{x-4} - \frac{4}{x(x-4)} = \frac{x-4}{x(x-4)} = \frac{1}{x}$

4

[a] $\because 2x^2 - 5x + 1 = 0$ $\therefore a = 2, b = -5$ and $c = 1$

$\therefore x = \frac{5 \pm \sqrt{(-5)^2 - 4 \times 2 \times 1}}{2 \times 2} = \frac{5 \pm \sqrt{17}}{4}$

$\therefore x \simeq 2.28$ or $x \simeq 0.22$

$\therefore$ The S.S. = {2.28 , 0.22}

Answers

[b] $\because$ $n(x) = \frac{(x-2)(x-1)}{(x-1)(x+1)} \div \frac{3(x-5)}{(x-5)(x+1)}$

$\therefore$ The domain of $n = \mathbb{R} - \{1, -1, 5\}$

, $n(x) = \frac{x-2}{x+1} \div \frac{3}{x+1} = \frac{x-2}{x+1} \times \frac{x+1}{3} = \frac{x-2}{3}$

5

[a] $\because f(2) = 0$ $\therefore 4a + 2b = -8$

$\therefore 2a + b = -4$ (1)

, $\because f(4) = 0$ $\therefore 16a + 4b = -8$

$\therefore 4a + b = -2$ (2)

, Subtracting (1) from (2) :

$\therefore 2a = 2$ $\therefore a = 1$ and hence $b = -6$

[b] First : $\frac{8}{25}$ Second : $\frac{3}{5}$ Third : $\frac{18}{25}$

Model 3

1

(1) 1 (2) 2 (3) zero

(4) $\sqrt{7}$ (5) 10 (6) $\mathbb{R} - \{3, 0\}$

2

(1) (b) (2) (c) (3) (b) (4) (d) (5) (a) (6) (d)

3

[a] $n(x) = \frac{(x-2)(x^2+2x+4)}{(x-2)(x+3)} \times \frac{x+3}{x^2+2x+4}$

$\therefore$ The domain of $n = \mathbb{R} - \{2, -3\}$, $n(x) = 1$

[b] $\because (x-3)^2 - 5x = 0$ $\therefore x^2 - 6x + 9 - 5x = 0$

$\therefore x^2 - 11x + 9 = 0$

$\therefore a = 1$, $b = -11$ and $c = 9$

$\therefore x = \frac{11 \pm \sqrt{(-11)^2 - 4 \times 1 \times 9}}{2 \times 1} = \frac{11 \pm \sqrt{85}}{2}$

$\therefore x \simeq 10.11$ or $x \simeq 0.89$

$\therefore$ The S.S. $= \{10.11, 0.89\}$

4

[a] $n(x) = \frac{x+5}{(x+5)(x+2)} - \frac{x-1}{(x+2)(x+3)}$

$\therefore$ The domain of $n = \mathbb{R} - \{-5, -2, -3\}$

, $n(x) = \frac{1}{x+2} - \frac{x-1}{(x+2)(x+3)}$

$= \frac{x+3-x+1}{(x+2)(x+3)} = \frac{4}{(x+2)(x+3)}$

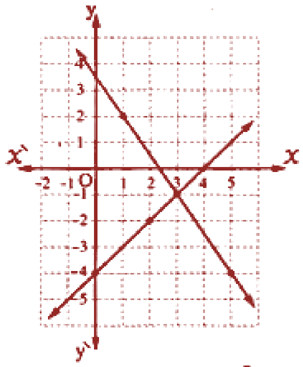
, $n(-2)$ is undefined

[b] • **Graphically :**

$y = x - 4$, $y = \frac{1}{2}(7 - 3x)$

x	0	2	4
y	-4	-2	0

x	1	3	5
y	2	-1	-4



From the graph : The S.S. $= \{(3, -1)\}$

• **Algebraically :**

$\because x - y = 4$ $\therefore 2x - 2y = 8$ (1)

, $\because 3x + 2y = 7$ (2)

, Adding (1) and (2) :

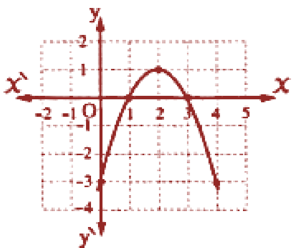
$\therefore 5x = 15$ $\therefore x = 3$, substituting in (1)

$\therefore y = -1$ $\therefore$ The S.S. $= \{(3, -1)\}$

5

[a] $f(x) = -x^2 + 4x - 3$

x	0	1	2	3	4
y	-3	0	1	0	-3



, $\because$ The S.S. of the equation : $-x^2 + 4x - 3 = 0$ is $\{1, 3\}$

$\therefore$ The S.S. of the equation : $x^2 - 4x + 3 = 0$ is $\{1, 3\}$

$\therefore$ The equation of the axis of symmetry is $x = 2$

[b] $P(\acute{A}) = 1 - P(A) = 1 - 0.7 = 0.3$

, Probability of occurence of one event without the other $= P(A - B) + P(B - A)$

$= P(A) - P(A \cap B) + P(B) - P(A \cap B)$

$= 0.7 - 0.4 + 0.6 - 0.4 = 0.5$

Model 4

1

(1) 1 (2) {– 1} (3) zero
(4) 1 (5) 4 (6) ∅

2

(1) (d) (2) (c) (3) (d) (4) (b) (5) (a) (6) (c)

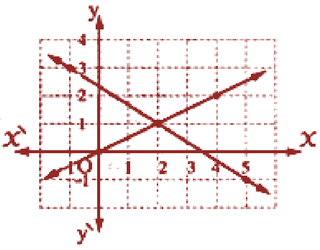
3

[a] • **Graphically :**

$y = \frac{1}{2} x$, $y = \frac{1}{3}(7 - 2x)$

x	0	2	4
y	0	1	2

x	– 1	2	5
y	3	1	– 1



From the graph : The S.S. = {(2 , 1)}

• **Algebraically :**

$\because x = 2y$, Substituting in the other equation

$\therefore 2(2y) + 3y = 7$ $\therefore 7y = 7$

$\therefore y = 1$ and hence $x = 2$

$\therefore$ The S.S. = {(2 , 1)}

[b] $n(x) = \frac{(x-2)(x^2+2x+4)}{x(x^2-7x+10)} \div \frac{x^2+2x+4}{3x(x-5)}$

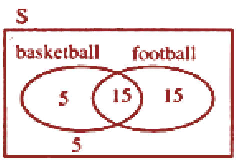
$= \frac{(x-2)(x^2+2x+4)}{x(x-2)(x-5)} \div \frac{x^2+2x+4}{3x(x-5)}$

$\therefore$ The domain of n = $\mathbb{R}$ – {0 , 2 , 5} , n (x) = 3

4

[a] **First :** $\frac{7}{8}$

Second : $\frac{1}{2}$



[b] $\because x = y + 1$ (1) , substituting in the other equation

$\therefore (y+1)^2 + y^2 = 25$

$\therefore y^2 + 2y + 1 + y^2 = 25$ $\therefore 2y^2 + 2y - 24 = 0$

$\therefore y^2 + y - 12 = 0$ $\therefore (y+4)(y-3) = 0$

$\therefore y = -4$ and hence $x = -3$

or $y = 3$ and hence $x = 4$

$\therefore$ The S.S. = {(– 3 , – 4) , (4 , 3)}

5

[a] $n(x) = \frac{x(x-2)}{(x-2)(x-1)} + \frac{(x-2)(x+2)}{(x+2)(x-1)}$

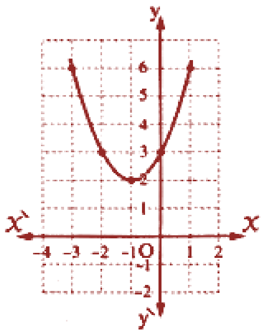
$\therefore$ The domain of n = $\mathbb{R}$ – {2 , 1 , – 2}

, $n(x) = \frac{x}{x-1} + \frac{x-2}{x-1}$

$= \frac{2x-2}{x-1} = \frac{2(x-1)}{x-1} = 2$

[b] $f(x) = x^2 + 2x + 3$

x	– 3	– 2	– 1	0	1
$f(x)$	6	3	2	3	6



From the graph :

• The minimum value of the function = 2

, the vertex point is (– 1 , 2)

• The S.S. of the equation : $x^2 + 2x + 3 = 0$ is ∅

Model 5

1

(1) 0 (2) 2 , 3 (3) 0
(4) $\frac{1}{3}$ (5) $\mathbb{R}$ – {1 , – 1 , 0} (6) ∅

2

(1) (a) (2) (d) (3) (a) (4) (b) (5) (c) (6) (d)

3

[a] $n(x) = \frac{x(x-2)}{(x-2)(x+2)} + \frac{2(x+3)}{(x+2)(x+3)}$

$\therefore$ The domain of n = $\mathbb{R}$ – {2 , – 2 , – 3}

, $n(x) = \frac{x}{x+2} + \frac{2}{x+2} = \frac{x+2}{x+2} = 1$

[b] $\because 2x(x-5) = 1$ $\therefore 2x^2 - 10x - 1 = 0$

$\therefore a = 2$, $b = -10$ and $c = -1$

$\therefore x = \frac{10 \pm \sqrt{(-10)^2 - 4 \times 2 \times -1}}{2 \times 2}$

$\therefore x \simeq 5.1$ or $x \simeq -0.1$

4

[a] $X = 7 - y$, substituting in the other equation

$\therefore (7 - y)^2 + y^2 = 25$

$\therefore 49 - 14y + y^2 + y^2 = 25$

$\therefore 2y^2 - 14y + 24 = 0$

$\therefore y^2 - 7y + 12 = 0$

$\therefore (y - 4)(y - 3) = 0$

$\therefore y = 4$ and hence $X = 3$

or $y = 3$ and hence $X = 4$

$\therefore$ The S.S. $= \{(3, 4), (4, 3)\}$

[b] $P(A) = \frac{8}{15}$, $P(B) = \frac{2}{5}$

, $\because P(A \cap B) = \frac{5}{15}$

$\therefore P(A - B) = \frac{8}{15} - \frac{5}{15} = \frac{3}{15} = \frac{1}{5}$

5

[a] $n(X) = \frac{(X-2)(X^2+2X+4)}{(X-1)(X-5)} \div \frac{X(X^2+2X+4)}{(2X+3)(X-1)}$

$\therefore$ The domain of $n = \mathbb{R} - \{1, 5, 0, -\frac{3}{2}\}$

, $n(X) = \frac{(X-2)(X^2+2X+4)}{(X-1)(X-5)} \times \frac{(2X+3)(X-1)}{X(X^2+2X+4)}$

$= \frac{(X-2)(2X+3)}{X(X-5)}$

, $n(-1) = \frac{(-1-2)(2(-1)+3)}{-1(-1-5)} = -\frac{1}{2}$

[b] Let the units digit be X and the tens one be y

$\therefore X + y = 5$ (1)

, $(y + 10X) - (X + 10y) = 9$

$\therefore 9X - 9y = 9$

$\therefore X - y = 1$ (2)

, Adding (1) and (2) :

$\therefore 2X = 6$ $\therefore X = 3$ and hence $y = 2$

$\therefore$ The original number is 23

Answers of governorats' examinations of algebra and probability

Cairo 1

1

(1) (d) (2) (a) (3) (a)

(4) (b) (5) (c) (6) (c)

2

[a] $n(x) = \frac{3x-4}{(x-3)(x-2)} + \frac{2(x+3)}{(x-2)(x+3)}$

$\therefore$ The domain of $n = \mathbb{R} - \{3, 2, -3\}$

, $n(x) = \frac{3x-4}{(x-3)(x-2)} + \frac{2}{x-2}$

$= \frac{3x-4+2x-6}{(x-3)(x-2)} = \frac{5x-10}{(x-3)(x-2)}$

$= \frac{5(x-2)}{(x-3)(x-2)} = \frac{5}{x-3}$

[b] $x - 3y = 6$ (1)

, $2x + y = 5$ i.e. $6x + 3y = 15$ (2)

, Adding (1) and (2) : $\therefore 7x = 21$

$\therefore x = 3$, substituting in (1)

$\therefore 3 - 3y = 6$ $\therefore -3y = 3$ $\therefore y = -1$

$\therefore$ The S.S. $= \{(3, -1)\}$

3

[a] $\because y^2 - 5x + 3 = 0$ $\therefore a = 1$, $b = -5$ and $c = 3$

$\therefore x = \frac{5 \pm \sqrt{(-5)^2 - 4 \times 1 \times 3}}{2 \times 1}$

$\therefore x \simeq 4.3$ or $x \simeq 0.7$ $\therefore$ The S.S. $= \{4.3, 0.7\}$

[b] Let the length be x cm. and the width be y cm.

$\therefore 2(x + y) = 14$ $\therefore x + y = 7$

$\therefore x = 7 - y$ (1)

, $xy = 12$, substituting by (1)

$\therefore (7 - y)y = 12$ $\therefore 7y - y^2 = 12$

$\therefore y^2 - 7y + 12 = 0$ $\therefore (y - 3)(y - 4) = 0$

$\therefore y = 3$, from (1) : $\therefore x = 4$

or $y = 4$, from (1) : $\therefore x = 3$

$\therefore$ The two dimensions are 3 cm. and 4 cm.

4

[a] $n(x) = \frac{x^2+x+1}{(x-3)(x+3)} \div \frac{(x-1)(x^2+x+1)}{(x-1)(x-3)}$

$\therefore$ The domain of $n = \mathbb{R} - \{3, -3, 1\}$

, $n(x) = \frac{x^2+x+1}{(x-3)(x+3)} \times \frac{(x-1)(x-3)}{(x-1)(x^2+x+1)}$

$= \frac{1}{x+3}$

[b] $\because xy + y^2 = 6$ $\therefore y(x + y) = 6$

, $\because x + y = 3$ $\therefore 3y = 6$

$\therefore y = 2$ $\therefore x = 1$

$\therefore$ The S.S. $= \{(1, 2)\}$

5

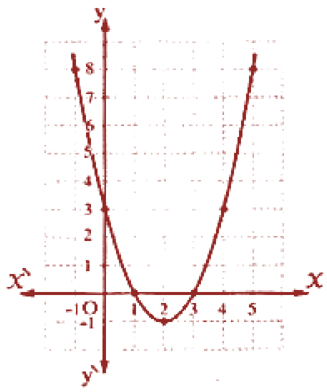
[a] (1) $P(\acute{A}) = 1 - P(A) = 1 - 0.7 = 0.3$

(2) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.7 + 0.4 - 0.2 = 0.9$

[b] $f(x) = x^2 - 4x + 3$

x	−1	0	1	2	3	4	5
y	8	3	0	−1	0	3	8



From the graph :

- The vertex point is $(2, -1)$
- The minimum value $= -1$
- The S.S. of the equation : $x^2 - 4x + 3 = 0$ is $\{1, 3\}$

Giza 2

1

(1) (d) (2) (d) (3) (c) (4) (d) (5) (b) (6) (c)

2

[a] $2x - y = -3$ (1) , $x + 2y = -4$

i.e. $2x + 4y = -8$ (2)

Subtracting (1) from (2) : $\therefore 5y = -5$

$\therefore y = -1$, substituting in (1)

$\therefore x = -2$ $\therefore$ The S.S. $= \{(-2 , -1)\}$

[b] Let the two numbers be x and y where $x > y$

$\therefore x - y = 5$ i.e. $x = 5 + y$ (1)

, $x y = 36$, from (1) :

$\therefore (5 + y)\, y = 36$ $\therefore 5y + y^2 = 36$

$\therefore y^2 + 5y - 36 = 0$ $\therefore (y + 9)(y - 4) = 0$

$\therefore y = -9$ and hence $x = -4$

or $y = 4$ and hence $x = 9$

3

[a] (1) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.6 + 0.3 - 0.2 = 0.7$

(2) $P(A - B) = P(A) - P(A \cap B)$

$= 0.6 - 0.2 = 0.4$

[b] $n(x) = \dfrac{3x}{x(x-2)} - \dfrac{12}{(x-2)(x+2)}$

$\therefore$ The domain of $n = \mathbb{R} - \{0 , 2 , -2\}$

, $n(x) = \dfrac{3}{x-2} - \dfrac{12}{(x-2)(x+2)}$

$= \dfrac{3x + 6 - 12}{(x-2)(x+2)} = \dfrac{3x - 6}{(x-2)(x+2)}$

$= \dfrac{3(x-2)}{(x-2)(x+2)} = \dfrac{3}{x+2}$

4

[a] $3x + 4y = 24$ (1)

$\because x - 2y = -2$ i.e. $2x - 4y = -4$ (2)

, Adding (1) and (2) : $\therefore 5x = 20$ $\therefore x = 4$

, Substituting in (1) : $\therefore y = 3$

$\therefore$ The S.S. $= \{(4 , 3)\}$

[b] $\because 3x^2 - 6x + 1 = 0$

$\therefore a = 3$, $b = -6$ and $c = 1$

$\therefore x = \dfrac{6 \pm \sqrt{(-6)^2 - 4 \times 3 \times 1}}{2 \times 3} = \dfrac{6 \pm \sqrt{24}}{6}$

$\therefore x \simeq 1.82$ or $x \simeq 0.18$ $\therefore$ The S.S. $= \{1.82 , 0.18\}$

5

[a] $n(x) = \dfrac{(x-2)(x-1)}{(x-7)(x+7)} \div \dfrac{x-2}{x+7}$

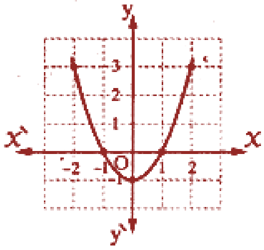
$\therefore$ The domain of $n = \mathbb{R} - \{7 , -7 , 2\}$

, $n(x) = \dfrac{(x-2)(x-1)}{(x-7)(x+7)} \times \dfrac{(x+7)}{(x-2)}$

, $n(x) = \dfrac{x-1}{x-7}$

[b] $f(x) = x^2 - 1$

x	-2	-1	0	1	2
y	3	0	-1	0	3



(1) The vertex point $= (0 , -1)$

(2) The minimum value $= -1$

(3) The two roots of the equation :

$f(x) = 0$ are -1 , 1

Alexandria 3

1

(1) (b) (2) (a) (3) (b)

(4) (c) (5) (c) (6) (a)

2

[a] $\because x^2 - 2x - 4 = 0$

$\therefore a = 1$, $b = -2$ and $c = -4$

$\therefore x = \dfrac{2 \pm \sqrt{(-2)^2 - 4 \times 1 \times -4}}{2 \times 1} = \dfrac{2 \pm \sqrt{20}}{2}$

$\therefore x \simeq 3.2$ or $x \simeq -1.2$

$\therefore$ The S.S. $= \{3.2 , -1.2\}$

[b] $n(x) = \dfrac{x^2 + x + 1}{x} \times \dfrac{x(x-1)}{(x-1)(x^2 + x + 1)}$

$\therefore$ The domain of $n = \mathbb{R} - \{0 , 1\}$

, $n(x) = 1$

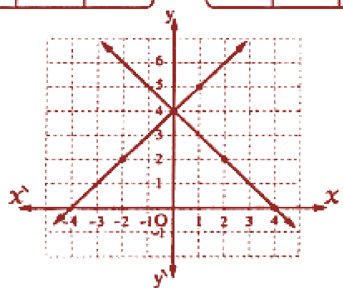
3

[a] • **Graphically :**

$y = x + 4$, $y = 4 - x$

x	0	1	-2
y	4	5	2

x	0	2	4
y	4	2	0



From the graph : The S.S. $= \{(0 , 4)\}$

[b] $n(x) = \frac{(x-2)(x+2)}{(x+2)(x+1)} - \frac{x(x-2)}{(x-2)(x+1)}$

$\therefore$ The domain of $n = \mathbb{R} - \{-2 , -1 , 2\}$

, $n(x) = \frac{x-2}{x+1} - \frac{x}{x+1} = \frac{-2}{x+1}$

, $n(1) = \frac{-2}{2} = -1$

4

[a] $\frac{1}{6}$

[b] $\because y = x + 2$ (1)

, Substituting in the other equation

$\therefore x^2 + x(x+2) - 4 = 0$

$\therefore x^2 + x^2 + 2x - 4 = 0$

$\therefore 2x^2 + 2x - 4 = 0$ $\therefore x^2 + x - 2 = 0$

$\therefore (x+2)(x-1) = 0$

$\therefore x = -2$ and hence $y = 0$

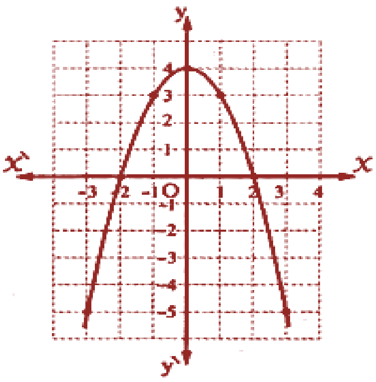
or $x = 1$ and hence $y = 3$

$\therefore$ The S.S. $= \{(-2 , 0) , (1 , 3)\}$

5

[a] $f(x) = 4 - x^2$

x	-3	-2	-1	0	1	2	3
y	-5	0	3	4	3	0	-5



(1) Roots of the equation : $f(x) = 0$ are $-2 , 2$

(2) The axis of symmetry is : $x = 0$

[b] $\because L - W = 4$ (1) , $\because 2(L + W) = 28$

$\therefore L + W = 14$ (2)

, Adding (1) and (2) : $\therefore 2L = 18$

$\therefore L = 9$, then $W = 5$

$\therefore$ Area of the rectangle $= L \times W = 9 \times 5 = 45 \text{ cm}^2$