Geometry

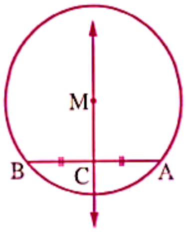
FINAL REVISION

GRADE 9 / PREP 3. SECOND TERM

SIMPLEST MATHS | Mr.Mohamed El-Shourbagy/01093149109

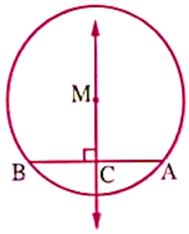
Revision

The straight line passing through the centre of the circle and the midpoint of any chord of it (not passing through the centre) is perpendicular to this chord.



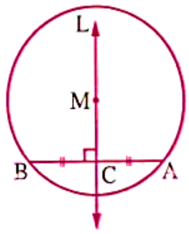
If $\overline{AB}$ is a chord of the circle M and C is the midpoint of $\overline{AB}$, then : $\overleftrightarrow{MC} \perp \overline{AB}$

The straight line passing through the centre of the circle and perpendicular to any chord of it bisects this chord.



If $\overline{AB}$ is a chord of the circle M and $\overleftrightarrow{MC} \perp \overline{AB}$, where $C \in \overline{AB}$, then : C is the midpoint of $\overline{AB}$

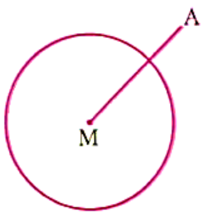
The perpendicular bisector of any chord of a circle passes through the centre of the circle.

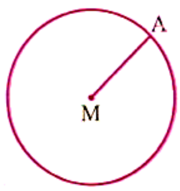


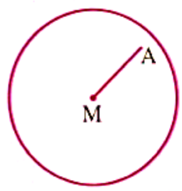
If $\overline{AB}$ is a chord of the circle M, C is the midpoint of $\overline{AB}$ and the straight line $L \perp \overline{AB}$ from the point C, then $M \in$ the straight line L

Position of a point with respect to a given circle

A is outside the circle M	A is on the circle M	A is inside the circle M
If MA > r	If MA = r	If MA < r



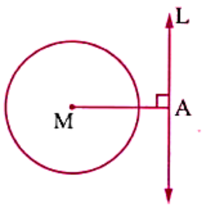




Position of a straight line L with respect to a circle M which is at a distance MA from its centre

L lies outside the circle M

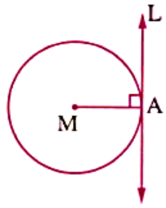
If MA > r



- L ∩ the circle M = ∅
- L ∩ the surface of the circle M = ∅

L touches the circle M

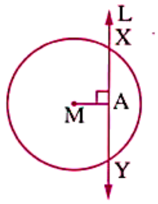
If MA = r



- L ∩ the circle M = {A}
- L ∩ the surface of the circle M = {A}

L is a secant to the circle M

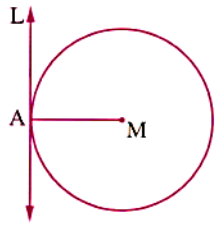
If MA < r



- L ∩ the circle M = {X , Y}
- L ∩ the surface of the circle M = $\overline{XY}$

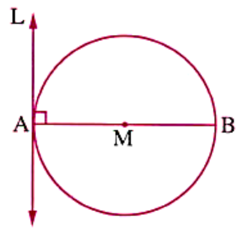
  $\overline{XY}$ is called the chord of intersection

The tangent to a circle is perpendicular to the radius drawn from the point of tangency.



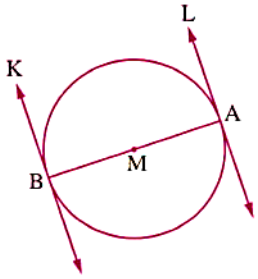
If L is a tangent to the circle M at the point A , then $\overline{MA}$ ⊥ L

The straight line which is perpendicular to the diameter of a circle at one of its endpoints is a tangent to the circle.



If $\overline{AB}$ is a diameter of the circle M , L ⊥ $\overline{AB}$ at the point A , then L is a tangent to the circle M at the point A

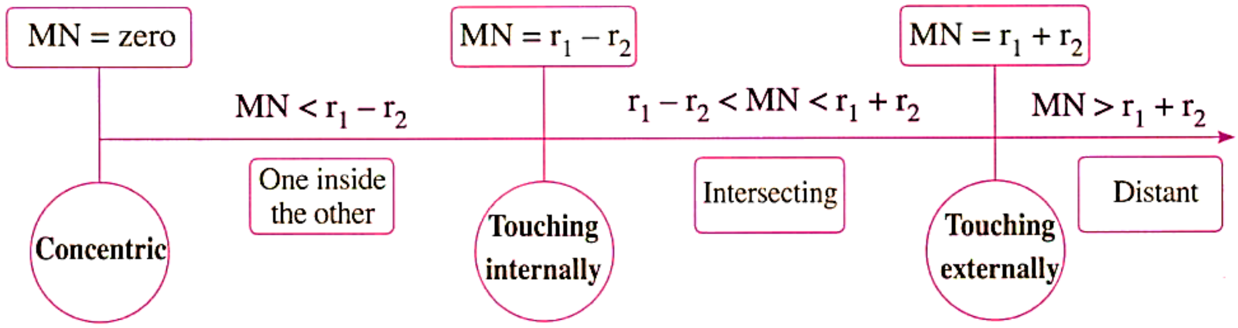
The two tangents which are drawn from the two endpoints of a diameter of a circle are parallel.



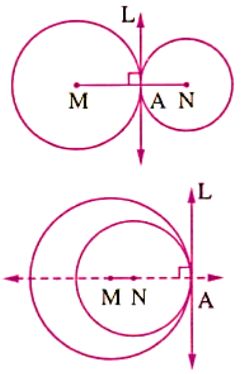
If $\overline{AB}$ is a diameter in the circle M , L and K are tangents to the circle M at A , B , then L // K

Position of the circle M with respect to the circle N

To determine the position of the circle M (with radius length r_1) with respect to the circle N (with radius length r_2) , find $r_1 - r_2$, $r_1 + r_2$, then use the following diagram to determine the position (where $r_1 > r_2$)

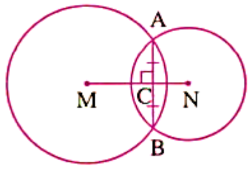


The line of centres of two touching circles passes through the point of tangency and is perpendicular to the common tangent at this point.



If the two circles M and N are touching at A , L is a common tangent to them at A , then $\overleftrightarrow{MN} \perp L$

The line of centres of two intersecting circles is perpendicular to the common chord and bisects it.



If M and N are intersecting circles at A and B , then $\overleftrightarrow{MN} \perp \overleftrightarrow{AB}$, AC = BC ($\overleftrightarrow{MN}$ is the axis of symmetry of $\overline{AB}$)

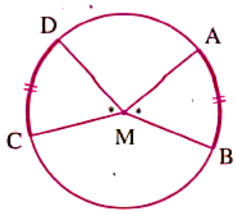
Remarks on identifying the circle

- It is possible to draw an infinite number of circles passing through a given point.
- There is an infinite number of circles that can be drawn to pass through the two points A and B and all their centres lie on the axis of symmetry of $\overline{AB}$
- The smallest circle passing through the two points A , B is the circle in which $\overline{AB}$ is a diameter in it and its centre is the midpoint of $\overline{AB}$ and the length of its radius = $\frac{1}{2}$ AB
- It is impossible to draw a circle passing through three collinear points.

- There is a unique circle passing through three points as A , B and C which are not collinear and the centre of this circle is the point of intersection of any two axes of symmetry of the axes of the line segments $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$

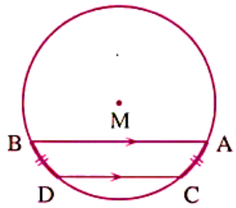
Equality of arcs in measure and length

In the same circle (or in congruent circles) , if the measures of arcs are equal , then the lengths of the arcs are equal and vice versa.



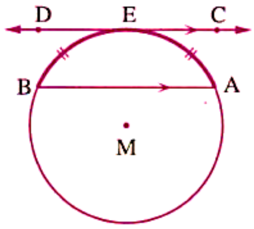
If m ($\overset{\frown}{AB}$) = m ($\overset{\frown}{CD}$) , then the length of $\overset{\frown}{AB}$ = the length of $\overset{\frown}{CD}$ and vice versa

If two parallel chords are drawn in a circle , then the measures of the two arcs between them are equal.



If $\overline{AB}$ // $\overline{CD}$, then m ($\overset{\frown}{AC}$) = m ($\overset{\frown}{BD}$)

If a chord is parallel to a tangent of a circle , then the measures of the two arcs between them are equal.



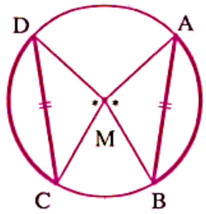
If $\overleftrightarrow{CD}$ // $\overline{AB}$, then m ($\overset{\frown}{EA}$) = m ($\overset{\frown}{EB}$)

Notice that

$$\frac{\text{The measure of the arc}}{360^\circ} = \frac{\text{The length of the arc}}{2\pi r}$$

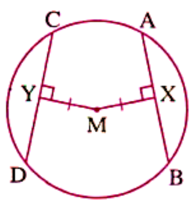
Equality of two chords in length

In the same circle (or in congruent circles) , if the measures of arcs are equal , then their chords are equal in length and vice versa.



If m ($\overset{\frown}{AB}$) = m ($\overset{\frown}{CD}$) , then AB = CD and vice versa

In the same circle (or in congruent circles) , if chords of a circle are equal in length , then they are equidistant from the centre and vice versa.

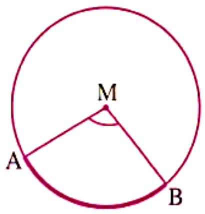


If AB = CD , $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{CD}$, then MX = MY and vice versa

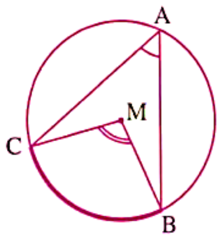
Central angle , inscribed angle and angle of tangency and relation between them

The measure of the central angle

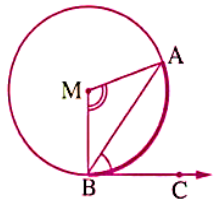
Equals the measure of the subtended arc	Equals twice the measure of the inscribed angle subtended by the same arc	Equals twice the measure of the angle of tangency subtended by the same arc



$m(\angle M) = m(\overset{\frown}{AB})$



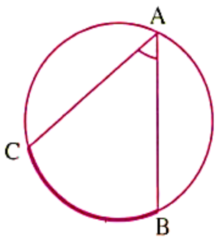
$m(\angle M) = 2\, m(\angle A)$



$m(\angle M) = 2\, m(\angle ABC)$

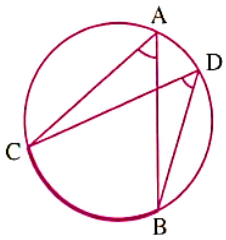
The measure of the inscribed angle

Equals half the measure of the subtended arc



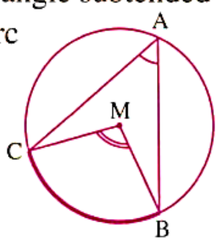
$m(\angle A) = \frac{1}{2}\, m(\overset{\frown}{BC})$

Equals the measure of the inscribed angle subtended by the same arc



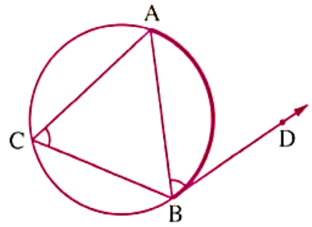
$m(\angle A) = m(\angle D)$

Equals half the measure of the central angle subtended by the same arc



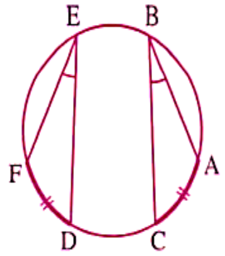
$m(\angle A) = \frac{1}{2}\, m(\angle M)$

Equals the measure of the angle of tangency subtended by the same arc



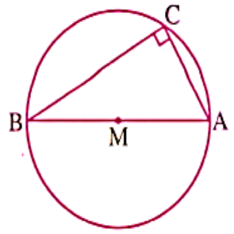
$m(\angle C) = m(\angle ABD)$

In the same circle (or in any number of circles) , the measures of the inscribed angles subtended by arcs of equal measures are equal and vice versa.



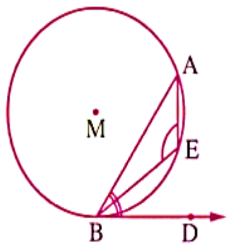
If m $(\overset{\frown}{AC})$ = m $(\overset{\frown}{DF})$
, then m (∠ B) = m (∠ E)
and vice versa

The inscribed angle in a semicircle is a right angle



If $\overline{AB}$ is a diameter , then
m (∠ C) = 90°

The angle of tangency is supplementary to the drawn inscribed angle on the chord of the angle of tangency and in one side of it.

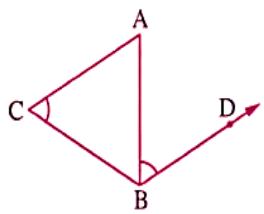


If ∠ AEB is inscribed drawn on $\overline{AB}$, ∠ ABD is angle of tangency , then
m (∠ ABD) + m (∠ AEB) = 180°

Notice that

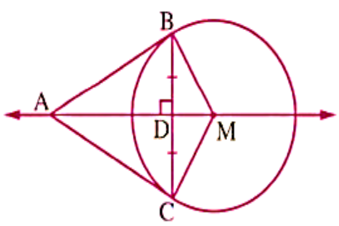
To prove that $\overleftrightarrow{BD}$ is a tangent to the circumcircle of Δ ABC

Prove that : m (∠ ABD) = m (∠ ACB)



Relation between tangents of the circle

The two tangent-segments drawn to a circle from a point outside it are equal in length

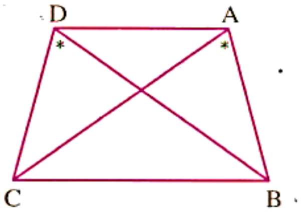


If $\overline{AB}$ and $\overline{AC}$ are tangent segments to the circle M , then
- AB = AC
- $\overleftrightarrow{AM}$ bisects ∠ BAC and ∠ BMC
- $\overleftrightarrow{AM}$ ⊥ $\overline{BC}$ and bisects it.

Cyclic quadrilateral

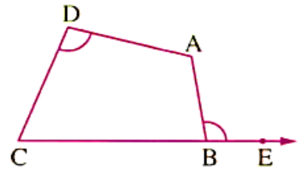
When is the quadrilateral cyclic ?

If there are two equal angles in measure and drawn on one of its sides as a base and on one side of this side.



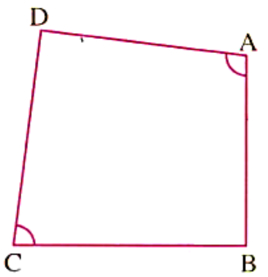
If m (∠ BAC) = m (∠ BDC)

If there is an exterior angle at any of its vertices equal in measure to the measure of the interior angle at the opposite vertex.



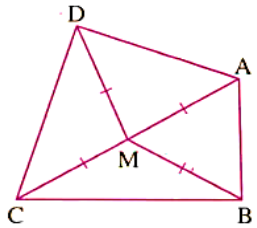
If m (∠ ABE) = m (∠ D)

If there are two opposite supplementary angles.



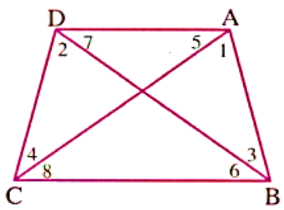
If m (∠ A) + m (∠ C) = 180°

If there is a point in the plane of the figure such that it is equidistant from its vertices.

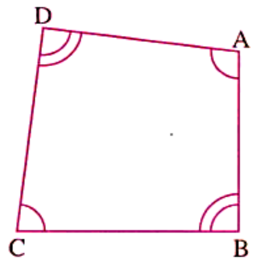


If MA = MB = MC = MD

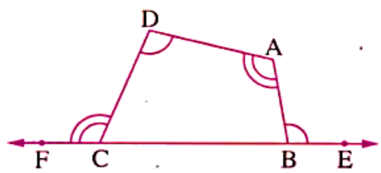
Properties of the cyclic quadrilateral



- m (∠ 1) = m (∠ 2)
- m (∠ 3) = m (∠ 4)
- m (∠ 5) = m (∠ 6)
- m (∠ 7) = m (∠ 8)



- m (∠ A) + m (∠ C) = 180°
- m (∠ B) + m (∠ D) = 180°

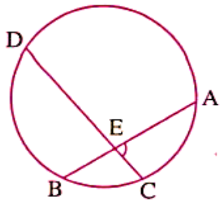


- m (∠ ABE) = m (∠ D)
- m (∠ DCF) = m (∠ A)

Well known problems

Well known problem (1)

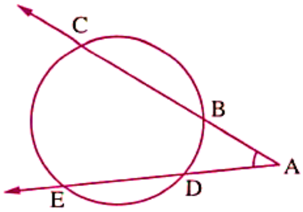
If $\overline{AB}$ and $\overline{CD}$ are two chords in a circle intersecting at the point E , then :



$m(\angle AEC) = \frac{1}{2}[m(\overset{\frown}{AC}) + m(\overset{\frown}{BD})]$

Well known problem (2)

If $\overline{CB}$ and $\overline{ED}$ are two chords in a circle , where $\overrightarrow{CB} \cap \overrightarrow{ED} = \{A\}$, then :

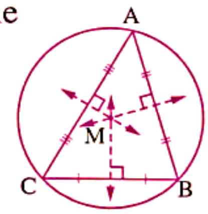


$m(\angle A) = \frac{1}{2}[m(\overset{\frown}{CE}) - m(\overset{\frown}{BD})]$

Circumcircle and inscribed circle of the triangle

The circumcircle of the triangle

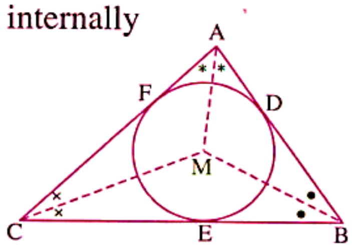
Is the circle that passes through the vertices of the triangle



and its centre is the point of intersection of the perpendicular bisectors of its sides

The inscribed circle of the triangle

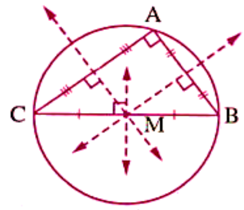
Is the circle that touches all sides of the triangle internally



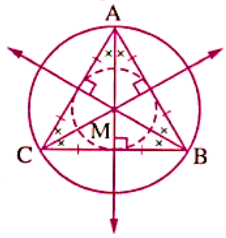
and its centre is the intersection point of the bisectors of its interior angles

Notice that

1. The centre of the circumcircle of the right-angled triangle is the midpoint of its hypotenuse.



2. The centre of the circumcircle of the equilateral triangle is the same centre of the inscribed circle to it which is the point of intersection of axes of its sides and the point of intersection of its medians and the point of intersection of the bisectors of its interior angles and also the point of intersection of its altitudes.

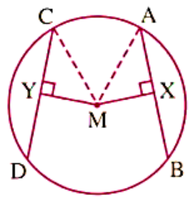


3. It's possible to draw a circumcircle to a rectangle , a square and an isosceles trapezium while it's impossible to draw a circumcircle to a parallelogram , a rhombus and not isosceles trapezium.

Important theorems and their proofs

Theorem 1

If chords of a circle are equal in length, then they are equidistant from the centre.



Given | $AB = CD$, $\overline{MX} \perp \overline{AB}$ and $\overline{MY} \perp \overline{CD}$

R.T.P. | $MX = MY$

Construction | Draw $\overline{MA}$ and $\overline{MC}$

Proof | $\because \overline{MX} \perp \overline{AB}$

$\therefore$ X is the midpoint of $\overline{AB}$ $\therefore AX = \frac{1}{2} AB$

$\because \overline{MY} \perp \overline{CD}$ $\therefore$ Y is the midpoint of $\overline{CD}$

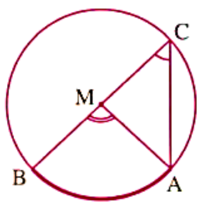
$\therefore CY = \frac{1}{2} CD$ $\because AB = CD$ (given) $\therefore AX = CY$

$\therefore \Delta\Delta$ AXM and CYM , both have $\begin{cases} AX = CY \text{ (by proof)} \\ MA = MC = r \\ m(\angle AXM) = m(\angle CYM) = 90^\circ \end{cases}$

$\therefore \Delta AXM \equiv \Delta CYM$, then we get : $MX = MY$ (Q.E.D.)

Theorem 2

The measure of the inscribed angle is half the measure of the central angle , subtended by the same arc.



Given | In the circle M : $\angle ACB$ is an inscribed angle ,

$\angle AMB$ is a central angle

R.T.P. | $m(\angle ACB) = \frac{1}{2} m(\angle AMB)$

Proof | $\because \angle AMB$ is an exterior angle of ΔAMC

$\therefore m(\angle AMB) = m(\angle A) + m(\angle C)$ (1)

, $\because MA = MC$ (two radii lengths)

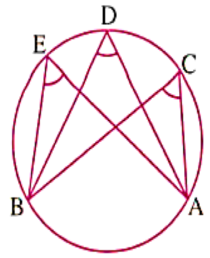
$\therefore m(\angle A) = m(\angle C)$ (2)

From (1) and (2) we get : $m(\angle AMB) = 2\, m(\angle ACB)$

$\therefore m(\angle ACB) = \frac{1}{2} m(\angle AMB)$ (Q.E.D.)

Theorem 3

In the same circle, the measures of all inscribed angles subtended by the same arc are equal.



Given $\angle C$, $\angle D$ and $\angle E$ are inscribed angles subtended by $\overset{\frown}{AB}$

R.T.P. $m(\angle C) = m(\angle D) = m(\angle E)$

Proof $\because m(\angle C) = \frac{1}{2} m(\overset{\frown}{AB})$

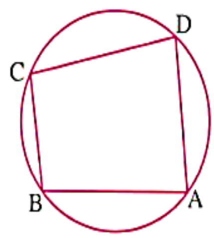
$, m(\angle D) = \frac{1}{2} m(\overset{\frown}{AB})$

$, m(\angle E) = \frac{1}{2} m(\overset{\frown}{AB})$

$\therefore m(\angle C) = m(\angle D) = m(\angle E)$ (Q.E.D.)

Theorem 4

In a cyclic quadrilateral , each two opposite angles are supplementary.



Given ABCD is a cyclic quadrilateral

R.T.P.

1. $m(\angle A) + m(\angle C) = 180°$
2. $m(\angle B) + m(\angle D) = 180°$

Proof $\because m(\angle A) = \frac{1}{2} m(\overset{\frown}{BCD})$ and $m(\angle C) = \frac{1}{2} m(\overset{\frown}{BAD})$

$\therefore m(\angle A) + m(\angle C) = \frac{1}{2} [m(\overset{\frown}{BCD}) + m(\overset{\frown}{BAD})]$

$= \frac{1}{2}$ the measure of the circle $= \frac{1}{2} \times 360° = 180°$

Similarly : $m(\angle B) + m(\angle D) = 180°$ (Q.E.D.)

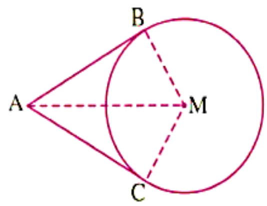
Theorem 5

The two tangent-segments drawn to a circle from a point outside it are equal in length.

Given A is a point outside the circle M

, $\overline{AB}$ and $\overline{AC}$ are two tangent-segments

to the circle at B and C respectively.



R.T.P. AB = AC

Construction Draw $\overline{MB}$, $\overline{MC}$, $\overline{MA}$

Proof ∵ $\overleftrightarrow{AB}$ is a tangent to the circle M ∴ m (∠ ABM) = 90°

, ∵ $\overleftrightarrow{AC}$ is a tangent to the circle M ∴ m (∠ ACM) = 90°

∴ In ΔΔ ABM , ACM :
- MB = MC (the lengths of two radii)
- $\overline{AM}$ is a common side.
- m (∠ ABM) = m (∠ ACM) = 90° (proved)

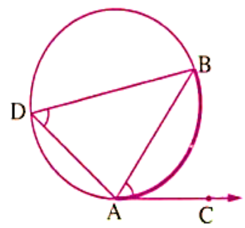
∴ Δ ABM ≡ Δ ACM and we deduce that : AB = AC (Q.E.D.)

Theorem 6

The measure of the angle of tangency is equal to the measure of the inscribed angle subtended by the same arc.

Given ∠ BAC is an angle of tangency and ∠ D is an inscribed angle.

R.T.P. m (∠ BAC) = m (∠ D)



Proof ∵ ∠ BAC is an angle of tangency.

∴ m (∠ BAC) = $\frac{1}{2}$ m ($\overset{\frown}{AB}$) (1)

, ∵ ∠ D is an inscribed angle

∴ m (∠ D) = $\frac{1}{2}$ m ($\overset{\frown}{AB}$) (2)

From (1) and (2) , we deduce that : m (∠ BAC) = m (∠ D) (Q.E.D.)

Some governorates' examinations

1 Cairo Governorate

Answer the following questions : *(Calculators are permitted)*

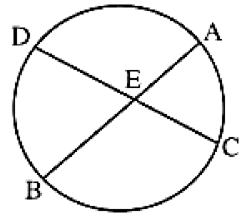
1 **Choose the correct answer from the given ones :**

(1) **In the opposite figure :**

m $(\overset{\frown}{BD}) = 80°$,

m $(\overset{\frown}{AC}) = 60°$, then

m (∠ AEC) = ...



(a) 20° (b) 30° (c) 70° (d) 140°

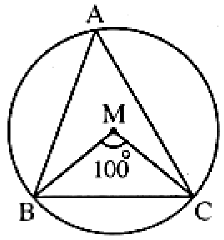
(2) The two tangents which are drawn from the two endpoints of a diameter of a circle are ....

(a) parallel. (b) intersecting. (c) perpendicular. (d) coincide.

(3) **In the opposite figure :**

M is a circle , m (∠ BMC) = 100°

, then m (∠ BAC) = ...



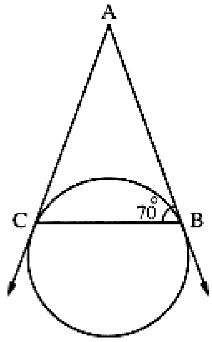
(a) 150° (b) 100°

(c) 50° (d) 25°

(4) **In the opposite figure :**

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle at B and C , m (∠ ABC) = 70°

, then m (∠ A) = ...



(a) 140° (b) 70°

(c) 40° (d) 35°

(5) Sum of the measures of any two opposite angles in the cyclic quadrilateral equals ....

(a) 90° (b) 180° (c) 270° (d) 360°

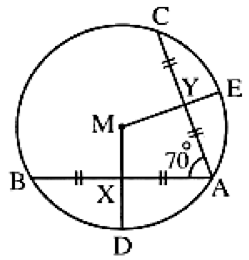
(6) Measure of an arc which represents $\frac{1}{3}$ of the measure of the circle equals = ...

(a) 60° (b) 90° (c) 120° (d) 180°

---

2 [a] **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two equal chords in length in the circle M , X is the midpoint of $\overline{AB}$ and Y is the midpoint of $\overline{AC}$, m (∠ CAB) = 70°



(1) **Calculate :** m (∠ DME) (2) **Prove that :** XD = YE

[b] In the opposite figure :

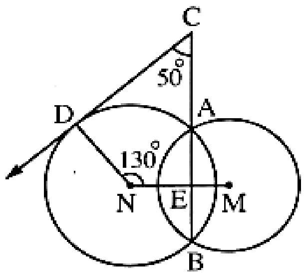
M and N are two circles intersecting at A and B.

and C $\in \overrightarrow{BA}$,

D $\in$ the circle N, m ($\angle$ MND) = 130° ,

m ($\angle$ BCD) = 50° ,

Prove that : $\overrightarrow{CD}$ is a tangent to the circle at D



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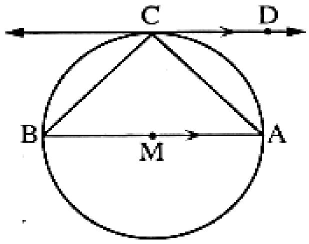
3 **[a] In the opposite figure :**

$\overline{CD}$ is a tangent to the circle M at C,

$\overrightarrow{CD}$ // $\overline{BA}$

Prove that : m ($\angle$ DCA) = 45°

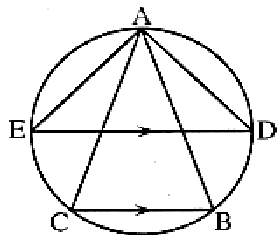


[b] In the opposite figure :

$\overline{DE}$ // $\overline{BC}$

Prove that :

m ($\angle$ DAC) = m ($\angle$ BAE)



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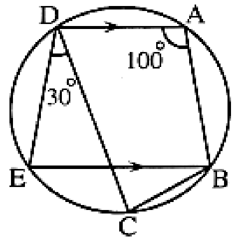
4 **[a] In the opposite figure :**

$\overline{AD}$ // $\overline{BE}$, m ($\angle$ BAD) = 100°

and m ($\angle$ CDE) = 30°

Find : m ($\angle$ ADC)



[b] In the opposite figure :

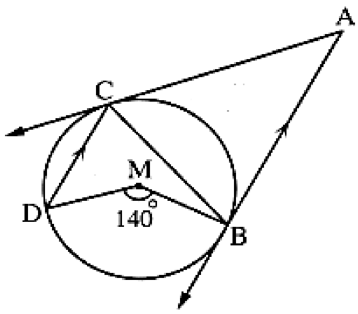
$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle

M at B and C

$\overline{AB}$ // $\overline{CD}$,

m ($\angle$ BMD) = 140°

Find : m ($\angle$ A)



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5 **[a] In the opposite figure :**

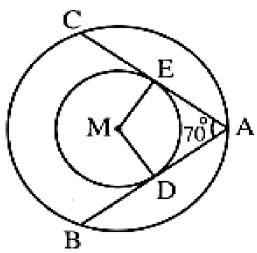
Two concentric circles at M,

$\overline{AB}$ and $\overline{AC}$ are two tangent segments to

the smaller circles, m ($\angle$ A) = 70°

(1) **Find :** m ($\angle$ DME)

(2) **Prove that :** AB = AC

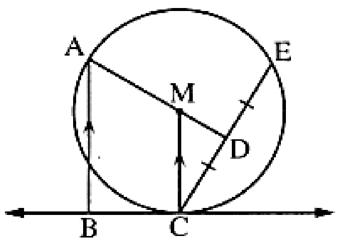


[b] In the opposite figure :

$\overleftrightarrow{BC}$ is a tangent to the circle M at C ,

D is the midpoint of $\overline{EC}$, $\overline{MC}$ // $\overline{AB}$

Prove that : ABCD is a cyclic quadrilateral.



2 Giza Governorate

Answer the following questions : *(Calculators are permitted)*

1 Choose the correct answer from the given ones :

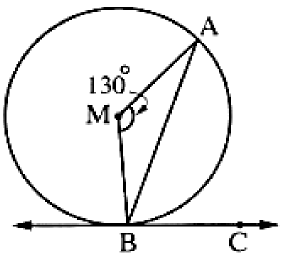
(1) **In the opposite figure :**

$\overrightarrow{BC}$ is a tangent to the circle M at B

if m (∠ AMB) = 130° , then m (∠ ABC) = ...

(a) 280° (b) 140°

(c) 70° (d) 65°



(2) The length of the arc which represents $\frac{1}{4}$ of circumference of a circle = ...

(a) 2 π r (b) π r (c) $\frac{1}{2}$ π r (d) $\frac{1}{4}$ π r

(3) In a cyclic quadrilateral , each two opposite angles are ....

(a) equal. (b) supplementary. (c) intersecting. (d) corresponding.

(4) If surface of circle M ∩ surface of circle N = ∅ , then the two circles are ....

(a) intersecting. (b) distant.

(c) touching internally. (d) touching externally.

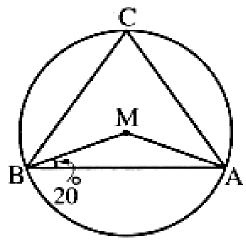
(5) **In the opposite figure :**

Circle M , if m (∠ MBA) = 20°

, then m (∠ C) = ...

(a) 120° (b) 70°

(c) 40° (d) 30°



(6) **In the opposite figure :**

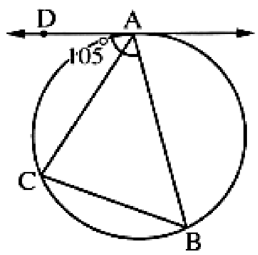
If $\overleftrightarrow{AD}$ is a tangent to the circle at A

, m (∠ DAB) = 105°

, then m (∠ ACB) = ...

(a) 75° (b) 60°

(c) 50° (d) 35°



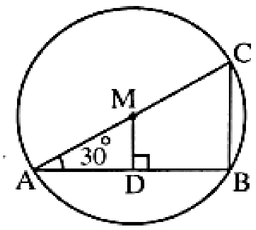
2 **[a] In the opposite figure :**

A circle of centre M , $\overline{MD} \perp \overline{AB}$,

If $m(\angle A) = 30°$

(1) **Prove that :** $\overline{MD} \parallel \overline{CB}$

(2) **Find :** $m(\angle C)$



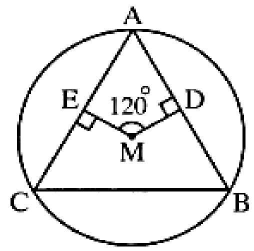
[b] In the opposite figure :

A circle M , $\overline{MD} \perp \overline{AB}$,

$\overline{ME} \perp \overline{AC}$, where MD = ME ,

$m(\angle DME) = 120°$

Prove that : the triangle ABC is equilateral.



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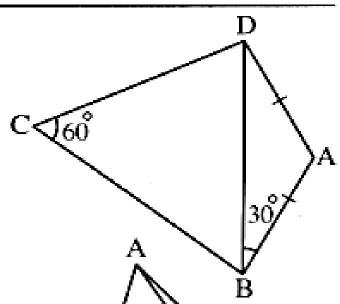
3 **[a] In the opposite figure :**

If : AB = AD , $m(\angle ABD) = 30°$, $m(\angle C) = 60°$

Prove that : ABCD is a cyclic quadrilateral.

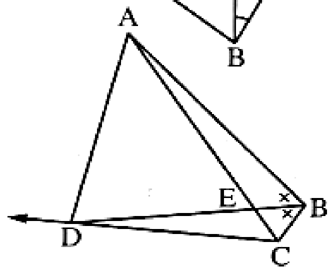


[b] In the opposite figure :

ABCD is a cyclic quadrilateral , $\overrightarrow{BD}$ bisects ∠ ABC ,

If $\overline{BD} \cap \overline{AC} = \{E\}$

Prove that : $\overrightarrow{CD}$ is a tangent to the circle passing through the vertices of Δ BEC



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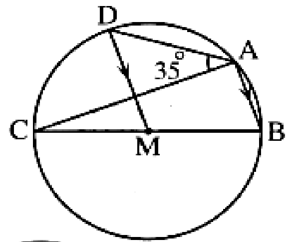
4 **[a] In the opposite figure :**

$\overline{BC}$ is a diameter in the circle M ,

$m(\angle CAD) = 35°$, $\overline{AB} \parallel \overline{DM}$,

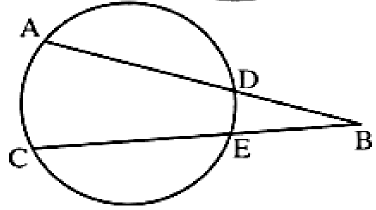
Find : $m(\angle ABC)$



[b] In the opposite figure :

If $m(\widehat{AC}) = 120°$, $m(\widehat{DE}) = 50°$

Find : $m(\angle ABC)$



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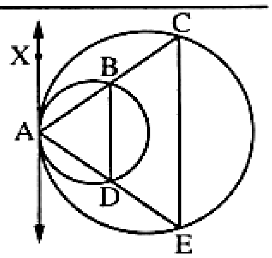
5 **[a] In the opposite figure :**

If $\overleftrightarrow{AX}$ is a common tangent to the two circles at A.

Prove that : $\overline{BD} \parallel \overline{CE}$



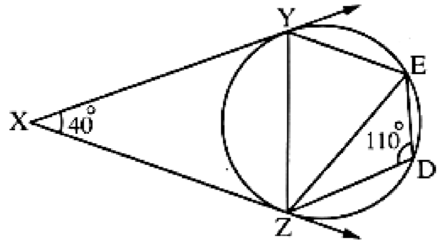
[b] In the opposite figure :

$\overrightarrow{XY}$ and $\overrightarrow{XZ}$ are two tangents

to the circle from the point X at Y , Z

, if m (∠ EDZ) = 110° , m (∠ YXZ) = 40°

Prove that : m $(\overset{\frown}{ZDE})$ = m $(\overset{\frown}{ZY})$



3 Alexandria Governorate

Answer the following questions : *(Calculators are permitted)*

1 Choose the correct answer from the given ones :

(1) The number of common tangents for the two tangent circles externally is ....

(a) 4 (b) 3 (c) 2 (d) infinite number.

(2) The figure which the circle doesn't passing through its vertices is ....

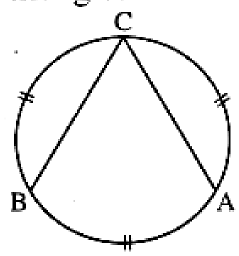
(a) square. (b) rectangle. (c) rhombus. (d) triangle.

(3) **In the opposite figure :**

m (∠ C) = ...

(a) 45° (b) 50°

(c) 30° (d) 60°



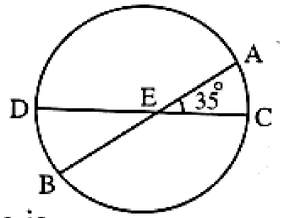
(4) **In the opposite figure :**

(∠ AEC) = 35°

, then m $(\overset{\frown}{AC})$ + m $(\overset{\frown}{DB})$ = ...

(a) 17.5° (b) 35°

(c) 70° (d) 140°



(5) The inscribed angle opposite to an arc greater than the semicircle is ....

(a) straight. (b) acute. (c) right. (d) obtuse.

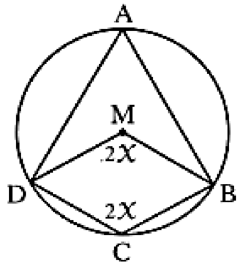
(6) **In the opposite figure :**

If m (∠ DMB) = m (∠ DCB) = 2 x

, then m (∠ A) = ...

(a) 60° (b) 70°

(c) 40° (d) 30°



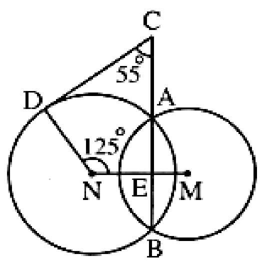
2 [a] In the opposite figure :

M and N are two intersecting circles at A and B

, C ∈ $\overrightarrow{BA}$, D ∈ the circle N

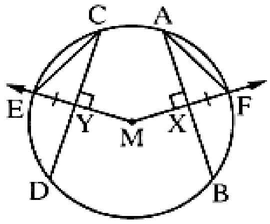
, m (∠ MND) = 125° and m (∠ BCD) = 55°

Prove that : $\overleftrightarrow{CD}$ is a tangent to circle N at D



[b] In the opposite figure : $\overline{AB}$ and $\overline{CD}$ are two chords in the circle M , $\overrightarrow{MX} \perp \overline{AB}$ and intersects the circle in F , $\overrightarrow{MY} \perp \overline{CD}$ and intersects the circle at E where FX = EY

Prove that : (1) AB = CD (2) AF = CE



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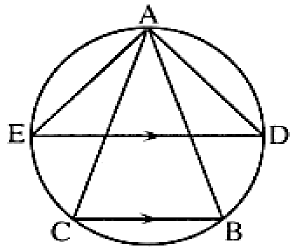
3 **[a] In the opposite figure :**

ABC is an inscribed triangle inside a circle , $\overline{DE}$ // $\overline{BC}$

Prove that : m (∠ DAC) = m (∠ BAE)

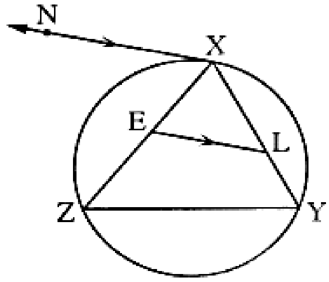


[b] In the opposite figure :

XYZ is an inscribed triangle in a circle , $\overline{LE}$ paralled tangent $\overrightarrow{XN}$

Prove that :

LYZE is cyclic quadrilateral.



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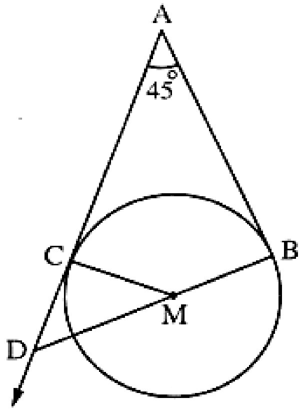
4 **[a] In the opposite figure :**

$\overline{AB}$, $\overline{AC}$ are two tangents to circle M at B , C , m (∠ A) = 45°

Prove that :

(1) ABMC is cyclic quadrilateral.

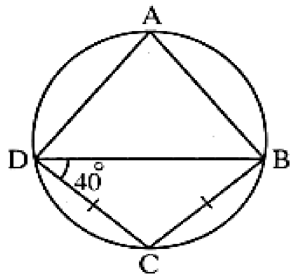
(2) AD = AB + MB



[b] In the opposite figure :

ABCD is a quadrilateral inscribed in circle , BC = CD , m (∠ BDC) = 40°

Find : m (∠ A)



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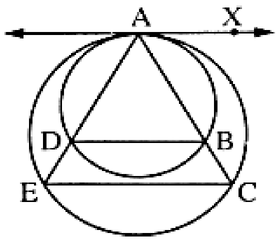
5 **[a] In the opposite figure :**

Prove that :

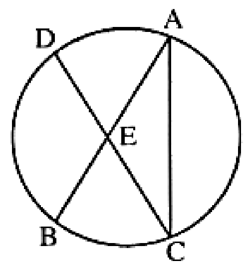
$\overline{BD}$ // $\overline{CE}$



[b] In the opposite figure :

$\overline{AB}$, $\overline{CD}$ are two equal chords in length

Prove that : the triangle ACE is an isosceles triangle.



4 El-Kalyoubia Governorate

Answer the following questions : *(Calculators are permitted)*

1 Choose the correct answer from the given ones :

(1) The diameter length of a circle is 8 cm. if the straight line L is at a distance 4 cm. from the centre , then the straight line L is ....

(a) a secant to the circle. (b) outside the circle.

(c) a tangent to the circle. (d) an axis of symmetry to the circle.

(2) The measure of the exterior angle at any vertex of a cyclic quadrilateral vertices .... the measure of the opposite interior of the adjacent angle.

(a) > (b) < (c) = (d) ≥

(3) The number of common tangents of two distant circles is ....

(a) 4 (b) 3 (c) 2 (d) infinite.

(4) The length of the arc opposite to the inscribed angle of measure 60° = ... circumference of the circle.

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) otherwise

(5) The inscribed angle drawn in a semicircle ....

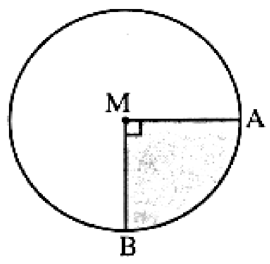
(a) acute. (b) obtuse. (c) reflex. (d) right.

(6) **In the opposite figure :**

$\overline{MA}$ and $\overline{MB}$ two radii in a circle M ,

$\overline{MA} \perp \overline{MB}$ and the radius length is 7 cm.

then the perimeter of the shaded part = ... cm.



(a) 14 (b) 21

(c) 38.5 (d) 25

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2 [a] In the opposite figure :

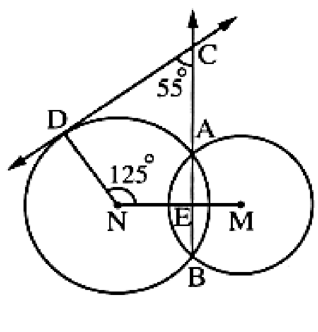
M and N are two intersecting circles at A and B ,

C ∈ $\overrightarrow{BA}$ and D ∈ the circle N , m (∠ MND) = 125° ,

m (∠ BCD) = 55°

Prove that :

$\overleftrightarrow{CD}$ is a tangent to the circle N at D



[b] $\overline{AB}$ and $\overline{CD}$ are two chords in the circle M , $\overrightarrow{MX}$ is drawn perpendicular to $\overline{AB}$ to intersect the circle in F and $\overrightarrow{MY}$ is drawn perpendicular to $\overline{CD}$ to intersect the circle at E , if FX = EY

Prove that :

(1) AB = CD (2) AF = CE

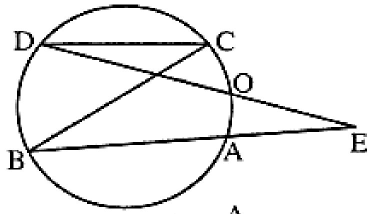
---

3 [a] **In the opposite figure :**

E is a point outside the circle

Prove that :

m (∠ DCB) > m (∠ E)



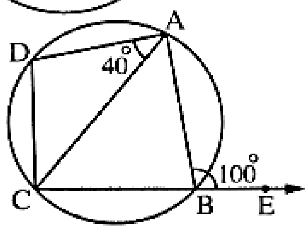
[b] **In the opposite figure :**

m (∠ ABE) = 100°

, m (∠ CAD) = 40°

Prove that :

m $(\overset{\frown}{CD})$ = m $(\overset{\frown}{AD})$



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4 [a] **Complete :**

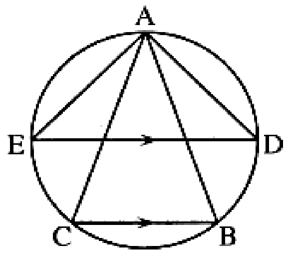
The straight line passing through the center of the circle and the intersection point of the two tangents are .... to the chord of tangency of those two tangents.

[b] **In the opposite figure :**

ABC is an inscribed triangle inside the circle

, $\overline{DE}$ // $\overline{BC}$

Prove that : m (∠ DAC) = m (∠ BAE)

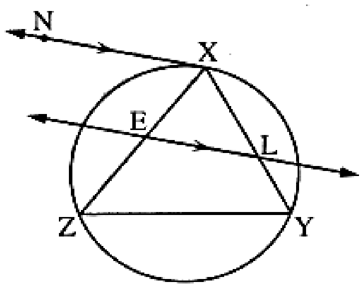


---

5 **In the opposite figure :**

XYZ is an inscribed triangle in a circle , if L ∈ $\overline{XY}$ and $\overleftrightarrow{LE}$ is drawn parallel to the tangent $\overrightarrow{XN}$ which touches the circle at X and intersects $\overline{XZ}$ at E

Prove that : LYZE is a cyclic quadrilateral.



5 El-Sharkia Governorate

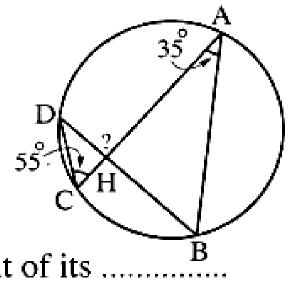
Answer the following questions : *(Calculators are permitted)*

1 **Choose the correct answer :**

(1) The measure of the circle with radius r is ....

(a) 2 π r (b) 180° (c) π r (d) 360°

(2) **In the opposite figure :**



m (∠ C) = 55° , m (∠ A) = 35°

, then m (∠ AHD) = ...

(a) 20° (b) 90°

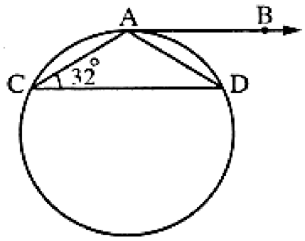
(c) 70° (d) 110°

(3) The centre of inscribed circle of a triangle is the intersection point of its ....

(a) altitudes. (b) medians.

(c) axes of symmetry of its sides. (d) bisectors of its interior angles.

(4) **In the opposite figure :**



$\overrightarrow{AB}$ is tangent to the circle ,

m (∠ C) = 32°

, then m (∠ BAD) = ...

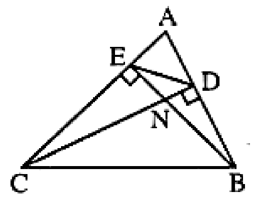
(a) 64° (b) 32°

(c) 148° (d) 58°

(5) If M and N are two touching externally circles with radii lengths 9 cm. and r cm. respectively , if MN = 14 cm. , then r = ... cm.

(a) 10 (b) 23 (c) 5 (d) 7

(6) **In the opposite figure :**



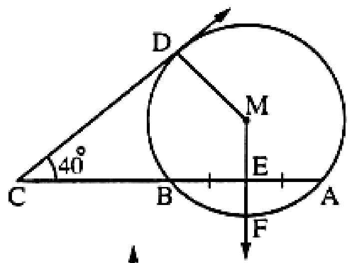
How many cyclic quadrilaterals ?

(a) 1 (b) 2

(c) 3 (d) 4

---

2 [a] **In the opposite figure :**



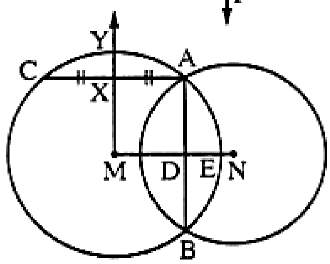
A circle M whose radius length is 10 cm. ,

m (∠ DCA) = 40° , AB = 16 cm.

, E is the midpoint of $\overline{AB}$, $\overrightarrow{CD}$ is a tangent to the circle

Find by proof : m (∠ DMF) , the length of $\overline{FE}$

[b] **In the opposite figure :**



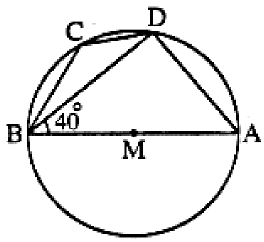
If M and N are two intersecting circles at A and B , AB = AC ,

X is the midpoint of $\overline{AC}$

Prove that : XY = DE

---

3 [a] **In the opposite figure :**



$\overline{AB}$ is a diameter of circle M ,

m (∠ ABD) = 40°

Find :

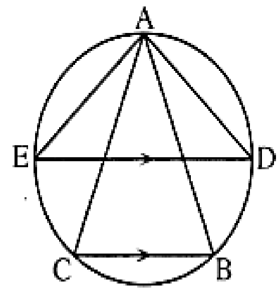
m (∠ A) , m (∠ C)

[b] In the opposite figure :

ABC is an inscribed triangle in the circle ,

$\overline{ED} \,//\, \overline{BC}$

Prove that : m (∠ DAC) = m (∠ BAE)



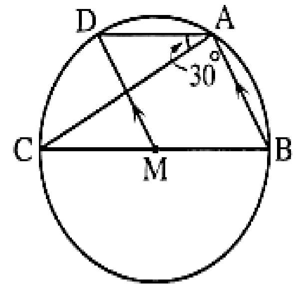
---

4 **[a] In the opposite figure :**

$\overline{CB}$ is a diameter of circle M ,

$\overline{AB} \,//\, \overline{DM}$, m (∠ DAC) = 30°

Find : m (∠ ACB)



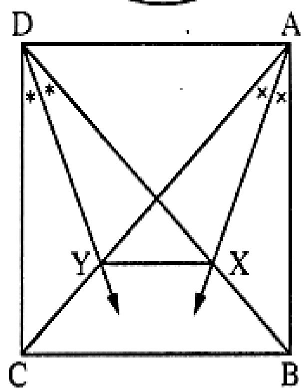
[b] In the opposite figure :

ABCD is a square , $\overrightarrow{AX}$ bisects ∠ BAC

and $\overrightarrow{DY}$ bisects ∠ CDB

(1) Prove that the figure AXYD is cyclic quadrilateral

(2) Find with proof m (∠ DXY)



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5 **In the opposite figure :**

$\overrightarrow{XZ}$ and $\overrightarrow{XY}$ are two tangents at Z and Y ,

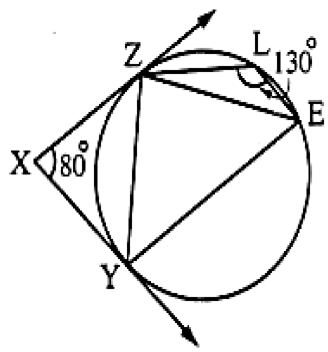
m (∠ YXZ) = 80° , m (∠ ELZ) = 130°

Prove that :

(1) ZE = ZY

(2) $\overline{XZ} \,//\, \overline{YE}$

(3) $\overline{ZE}$ is a tangent to the circle passing through the points X , Y and Z



Answers of governorates' examinations of geometry

Cairo 1

1

(1) (c) (2) (a) (3) (c)
(4) (c) (5) (b) (6) (c)

2

[a] ∵ X is the midpoint of $\overline{AB}$ ∴ $\overline{MX} \perp \overline{AB}$
∵ Y is the midpoint of $\overline{AC}$ ∴ $\overline{MY} \perp \overline{AC}$
∵ The sum of measures of the interior angles of the quadrilateral AXMY = 360°
∴ m (∠ XMY) = 360° – (70° + 90° + 90°) = 110°
∴ m (∠ DME) = 110° (Q.E.D. 1)
∵ AB = AC ∴ MX = MY
∵ MD = ME (lengths of two radii)
by subtracting
∴ XD = YE (Q.E.D. 2)

[b] ∵ $\overleftrightarrow{MN}$ is the line of centres , $\overline{AB}$ is the common chord
∴ $\overline{AB} \perp \overline{MN}$ ∴ m (∠ AEN) = 90°
∵ The sum of the measures of the interior angles of the quadrilateral CDNE = 360°
∴ m (∠ CDN) = 360° – (50° + 130° + 90°) = 90°
∴ $\overleftrightarrow{ND} \perp \overleftrightarrow{CD}$
∴ $\overleftrightarrow{CD}$ is a tangent to the circle N at D (Q.E.D.)

3

[a] ∵ $\overrightarrow{DC}$ // $\overline{AB}$
∴ m ($\overset{\frown}{AC}$) = m ($\overset{\frown}{BC}$)
, ∵ $\overline{AB}$ is a diameter in circle M
∴ m ($\overset{\frown}{ACB}$) = 180°
∴ m ($\overset{\frown}{AC}$) = 180° ÷ 2 = 90°
, ∵ m (∠ DCA) = $\frac{1}{2}$ m ($\overset{\frown}{AC}$)
∴ m (∠ DCA) = $\frac{1}{2}$ × 90° = 45° (The req.)

[b] ∵ $\overline{DE}$ // $\overline{BC}$ ∴ m ($\overset{\frown}{DB}$) = m ($\overset{\frown}{EC}$)
∴ m (∠ BAD) = m (∠ EAC)
adding m (∠ BAC) to both sides
∴ m (∠ DAC) = m (∠ BAE) (Q.E.D.)

4

[a] ∵ m (∠ EDC) = m (∠ EBC)
(two inscribed angles subtended by $\overset{\frown}{EC}$)
∴ m (∠ EBC) = 30°
, ∵ $\overline{AD}$ // $\overline{BE}$, $\overleftrightarrow{AB}$ is a transversal
∴ m (∠ A) + m (∠ ABE) = 180°
(two interior angles on the same side of the transversal)
∴ m (∠ ABE) = 180° – 100° = 80°
∴ m (∠ ABC) = 80° + 30° = 110°
, ∵ ABCD is a cyclic quadrilateral
∴ m (∠ ABC) + m (∠ ADC) = 180°
∴ m (∠ ADC) = 180° – 110° = 70° (The req.)

[b] ∵ m (∠ BCD) = $\frac{1}{2}$ m (BMD)
(inscribed and central angles subtended by $\overset{\frown}{BD}$)
∴ m (∠ BCD) = $\frac{1}{2}$ × 140° = 70°
, ∵ $\overline{AB}$ // $\overline{CD}$, $\overleftrightarrow{BC}$ is a transversal
∴ m (∠ ABC) = m (∠ BCD) = 70° (alternate angles)
, ∵ AB = AC
∴ m (∠ ABC) = m (∠ ACB) = 70°
∴ in Δ ABC :
m (∠ A) = 180° – (2 × 70°) = 40° (the req.)

5

[a] ∵ $\overline{AB}$ and $\overline{AC}$ are two tangents to the smaller circle
∴ $\overline{MD} \perp \overline{AB}$, $\overline{ME} \perp \overline{AC}$
∴ m (∠ MDA) = m (∠ MEA) = 90°
∴ From the quadrilateral ADME :
m (∠ DME) = 360° – (90° + 70° + 90°) = 110° (First req.)
∵ MD = ME (two radii in the smaller circle)
∴ AB = AC (Second req.)

[b] ∵ D is the midpoint of the chord $\overline{EC}$
∴ $\overline{MD} \perp \overline{EC}$ ∴ m (∠ MDC) = 90°
∵ $\overleftrightarrow{BC}$ is a tangent to the circle at C
∴ $\overline{MC} \perp \overline{BC}$ ∴ m (∠ MCB) = 90°
∵ $\overline{AB}$ // $\overline{MC}$, $\overleftrightarrow{BC}$ is a transversal to them
∴ m (∠ MCB) + m (∠ ABC) = 180°
(Two interior angles in the same side of the transversal)
∴ m (∠ ABC) = 180° – 90° = 90°
∴ m (∠ ADC) + m (∠ ABC) = 90° + 90° = 180°
∴ The figure ABCD is a cyclic quadrilateral.
(Q.E.D.)

Giza 2

1

(1) (d) (2) (c) (3) (b)

(4) (b) (5) (b) (6) (a)

2

[a] $\overline{MD} \perp \overline{AB}$ ∴ m (∠ ADM) = 90°

, ∵ $\overline{AC}$ is a diameter in the circle M

∴ m (∠ ABC) = 90°

, ∵ m (∠ ADM) = m (∠ ABC) = 90°

and they are corresponding angles.

∴ $\overline{MD}$ // $\overline{BC}$ (First req.)

In Δ ABC : ∵ m (∠ A) = 30° , m (∠ ABC) = 90°

∴ m (∠ C) = 180° – (30° + 90°) = 60° (Second req.)

[b] ∵ $\overline{MD} \perp \overline{AB}$ ∴ D is the midpoint of $\overline{AB}$

∵ $\overline{ME} \perp \overline{AC}$ ∴ E is the midpoint of $\overline{AC}$

∵ MD = ME ∴ AB = AC (1)

From the quadrilateral ADME

m (∠ A) = 360° – (120° + 90° + 90°) = 60° (2)

From (1) and (2) :

∴ Δ ABC is an equilateral triangle. (Q.E.D.)

3

[a] In Δ ABD : ∵ AB = AD

∴ m (∠ ABD) = m (∠ ADB) = 30°

∴ m (∠ A) = 180° – 2 × 30° = 120°

, ∵ m (∠ A) + m (∠ C) = 120° + 60° = 120°

∴ ABCD is a cyclic quadrilateral (Q.E.D.)

[b] ∵ ABCD is a cyclic quadrilateral

∴ m (∠ DCA) = m (∠ DBA) (1)

(drawn on $\overline{AD}$ and on the same side of it)

, ∵ $\overrightarrow{BD}$ bisects ∠ ABC

∴ m (∠ DBC) = m (∠ DBA) (2)

From (1) , (2) : ∴ m (∠ DBC) = m (∠ DCA)

∴ $\overrightarrow{CD}$ is a tangent to the circle passing through the vertices of Δ BEC (Q.E.D.)

4

[a] ∵ m (∠ CMD) = 2 m (∠ CAD)

(central and inscribed angles subtended by $\overset{\frown}{CD}$)

∴ m (∠ CMD) = 2 × 35° = 70°

, ∵ $\overline{AB}$ // $\overline{DM}$, $\overleftrightarrow{BM}$ is a transversal

∴ m (∠ ABC) = m (∠ CMD) (corresponding angles)

∴ m (∠ ABC) = 70° (The req.)

[b] ∵ m (∠ ABC) = $\frac{1}{2}$ [m ($\overset{\frown}{AC}$) – m ($\overset{\frown}{DE}$)]

∴ m (∠ ABC) = $\frac{1}{2}$ [120° – 50°]

= $\frac{1}{2}$ × 70° = 35° (The req.)

5

[a] In the small circle

∵ m (∠ XAB) (the tangency angle)

= m (∠ ADB) (the inscribed angle) (1)

In the great circle

∵ m (∠ XAC) (the tangency angle)

= m (∠ AEC) (the inscribed angle) (2)

From (1) and (2) :

∴ m (∠ ADB) = m (∠ AEC) but they are corresponding.

∴ $\overline{DB}$ // $\overline{EC}$ (Q.E.D.)

[b] ∵ $\overrightarrow{XY}$ and $\overrightarrow{XZ}$ are two tangents

∴ XY = XZ

∴ In Δ XYZ : m (∠ XZY) = m (∠ XYZ)

$= \frac{180° - 40°}{2} = 70°$

, ∵ m (∠ XZY) (tangency) = m (∠ YEZ) (inscribed)

∴ m (∠ YEZ) = 70°

, ∵ DEYZ is a cyclic quadrilateral

∴ m (∠ EYZ) + m (∠ D) = 180°

∴ m (∠ EYZ) = 180° – 110° = 70°

∴ m (∠ EYZ) = m (∠ YEZ) = 70°

∴ In Δ EZY : ZE = ZY

∴ m ($\overset{\frown}{ZDE}$) = m ($\overset{\frown}{ZY}$) (Q.E.D.)

Alexandria 3

1

(1) (b) (2) (c) (3) (d)

(4) (c) (5) (d) (6) (a)

2

[a] $\because$ $\overleftrightarrow{MN}$ is the line of centres , $\overline{AB}$ is the common chord

$\therefore$ $\overline{AB} \perp \overline{MN}$ $\quad$ $\therefore$ m ($\angle$ AEN) = 90°

$\because$ The sum of the measures of the interior angles of the quadrilateral CDNE = 360°

$\therefore$ m ($\angle$ CDN) = 360° – (55° + 125° + 90°) = 90°

$\therefore$ $\overleftrightarrow{ND} \perp \overleftrightarrow{CD}$

$\therefore$ $\overleftrightarrow{CD}$ is a tangent to the circle N at D (Q.E.D.)

[b] $\because$ MF = ME (lengths of two radii)

, XF = YE $\quad$ $\therefore$ MX = MY

$\because$ $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{CD}$

$\therefore$ AB = CD (Q.E.D. 1)

$\because$ $\overline{MX} \perp \overline{AB}$

$\therefore$ X is the midpoint of $\overline{AB}$ $\quad$ $\therefore$ AX = $\frac{1}{2}$ AB

$\because$ $\overline{MY} \perp \overline{CD}$

$\therefore$ Y is the midpoint of $\overline{CD}$ $\quad$ $\therefore$ CY = $\frac{1}{2}$ CD

$\because$ AB = CD $\quad$ $\therefore$ AX = CY

$\therefore$ Δ AXF , Δ CYE

In them $\begin{cases} AX = CY \\ XF = YE \\ m(\angle AXF) = m(\angle CYE) = 90° \end{cases}$

$\therefore$ Δ AXF $\equiv$ Δ CYE then we deduce that AF = CE (Q.E.D. 2)

3

[a] $\because$ $\overline{DE}$ // $\overline{BC}$ $\quad$ $\therefore$ m ($\widehat{BD}$) = m ($\widehat{CE}$)

$\therefore$ m ($\angle$ DAB) = m ($\angle$ CAE)

adding m ($\angle$ BAC) to both sides

$\therefore$ m ($\angle$ DAC) = m ($\angle$ BAE) (Q.E.D.)

[b] $\because$ $\overline{LE}$ // $\overrightarrow{XN}$, $\overrightarrow{XZ}$ is a transversal

$\therefore$ m ($\angle$ XEL) = m ($\angle$ NXZ) (alternate angles)

$\because$ m ($\angle$ Y) the inscribed = m ($\angle$ NXZ) of tangency

$\therefore$ m ($\angle$ Y) = m ($\angle$ XEL)

$\therefore$ The figure LYZE is a cyclic quadrilateral. (Q.E.D.)

4

[a] $\because$ $\overline{AB}$ touches the circle at B $\quad$ $\therefore$ $\overline{MB} \perp \overline{AB}$

$\because$ $\overline{AC}$ touches the circle at C $\quad$ $\therefore$ $\overline{MC} \perp \overline{AC}$

$\therefore$ m ($\angle$ ABM) + m ($\angle$ ACM) = 90° + 90° = 180°

$\therefore$ The figure ABMC is a cyclic quadrilateral (Q.E.D. 1)

$\because$ $\angle$ CMD is an exterior angle of it

$\therefore$ m ($\angle$ CMD) = m ($\angle$ A) = 45°

$\therefore$ In Δ MCD : m ($\angle$ D) = 180° – (90° + 45°) = 45°

$\therefore$ CD = MC (1)

$\because$ $\overline{AC}$, $\overline{AB}$ are two tangent segments to the circle

$\therefore$ AC = AB (2)

Adding (1) and (2) : $\therefore$ CD + AC = MC + AB

$\therefore$ AD = AB + MC ,

$\because$ MC = MB (the lengths of two radii)

$\therefore$ AD = AB + MB (Q.E.D. 2)

[b] In Δ CBD : $\because$ CB = CD

$\therefore$ m ($\angle$ CBD) = m ($\angle$ CDB) = 40°

$\therefore$ m ($\angle$ C) = 180° – 2 × 40° = 100°

, $\because$ ABCD is a cyclic quadrilateral

$\therefore$ m ($\angle$ A) + m ($\angle$ C) = 180°

$\therefore$ m ($\angle$ A) = 180° – 100° = 80° (The req.)

5

[a] In the small circle

$\because$ m ($\angle$ XAB) (the tangency angle)

= m ($\angle$ ADB) (the inscribed angle) (1)

In the great circle

$\because$ m ($\angle$ XAC) (the tangency angle)

= m ($\angle$ AEC) (the inscribed angle) (2)

From (1) and (2) :

$\therefore$ m ($\angle$ ADB) = m ($\angle$ AEC) but they are corresponding.

$\therefore$ $\overline{DB}$ // $\overline{EC}$ (Q.E.D.)

[b] $\because$ AB = CD $\quad$ $\therefore$ m ($\widehat{AB}$) = m ($\widehat{CD}$)

Subtracting m ($\widehat{BD}$) from both sides

$\therefore$ m ($\widehat{AD}$) = m ($\widehat{BC}$) $\quad$ $\therefore$ m ($\angle$ C) = m ($\angle$ A)

$\therefore$ Δ ACE is isosceles (Q.E.D.)

El-Kalyoubia 4

1

(1) (c)	(2) (c)	(3) (a)
(4) (b)	(5) (d)	(6) (d)

2

[a] ∵ $\overleftrightarrow{MN}$ is the line of centres , $\overline{AB}$ is the common chord

∴ $\overline{AB} \perp \overline{MN}$ ∴ m (∠ AEN) = 90°

∵ The sum of the measures of the interior angles of the quadrilateral CDNE = 360°

∴ m (∠ CDN) = 360° – (55° + 125° + 90°) = 90°

∴ $\overleftrightarrow{ND} \perp \overleftrightarrow{CD}$

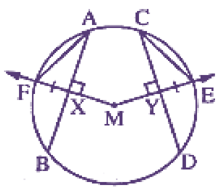
∴ $\overleftrightarrow{CD}$ is a tangent to the circle N at D (Q.E.D.)

[b] ∵ MF = ME

(lengths of two radii)

, XF = YE

∴ MX = MY



∵ $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{CD}$

∴ AB = CD (Q.E.D. 1)

∵ $\overline{MX} \perp \overline{AB}$

∴ X is the midpoint of $\overline{AB}$ ∴ AX = $\frac{1}{2}$ AB

∵ $\overline{MY} \perp \overline{CD}$

∴ Y is the midpoint of $\overline{CD}$ ∴ CY = $\frac{1}{2}$ CD

∵ AB = CD ∴ AX = CY

∴ Δ AXF , Δ CYE :

In them $\begin{cases} AX = CY \\ XF = YE \\ m(\angle AXF) = m(\angle CYE) = 90° \end{cases}$

∴ Δ AXF ≡ Δ CYE then we deduce that AF = CE (Q.E.D. 2)

3

[a] ∵ m (∠ E) = $\frac{1}{2}$ [m ($\overset{\frown}{BD}$) – m ($\overset{\frown}{AO}$)]

∴ m (∠ E) = $\frac{1}{2}$ m ($\overset{\frown}{BD}$) – $\frac{1}{2}$ m ($\overset{\frown}{AO}$)

, ∵ m (∠ DCB) = $\frac{1}{2}$ m ($\overset{\frown}{BD}$)

∴ m (∠ E) = m (∠ DCB) – $\frac{1}{2}$ m ($\overset{\frown}{AO}$)

∴ m (∠ DCB) = m (∠ E) + $\frac{1}{2}$ m ($\overset{\frown}{AO}$)

∴ m (∠ DCB) > m (∠ E) (Q.E.D.)

[b] ∵ ∠ ABE is an exterior angle of the cyclic quadrilateral ABCD

∴ m (∠ D) = m (∠ ABE) = 100°

In Δ ACD : m (∠ ACD) = 180° – (100° + 40°) = 40°

∴ m (∠ ACD) = m (∠ CAD)

∴ CD = AD ∴ m ($\overset{\frown}{CD}$) = m ($\overset{\frown}{AD}$) (Q.E.D.)

4

[a] an axis of symmetry.

[b] ∵ $\overline{DE}$ // $\overline{BC}$ ∴ m ($\overset{\frown}{BD}$) = m ($\overset{\frown}{CE}$)

∴ m (∠ DAB) = m (∠ CAE)

adding m (∠ BAC) to both sides

∴ m (∠ DAC) = m (∠ BAE) (Q.E.D.)

5

∵ $\overline{LE}$ // $\overleftrightarrow{XN}$, $\overleftrightarrow{XZ}$ is a transversal

∴ m (∠ XEL) = m (∠ NXZ) (alternate angles)

∵ m (∠ Y) the inscribed = m (∠ NXZ) of tangency

∴ m (∠ Y) = m (∠ XEL)

∴ The figure LYZE is a cyclic quadrilateral (Q.E.D.)

El-Sharkia 5

1

(1) (d)	(2) (b)	(3) (d)
(4) (b)	(5) (c)	(6) (b)

2

[a] ∵ $\overleftrightarrow{CD}$ is a tangent to the circle

∴ $\overline{MD} \perp \overleftrightarrow{CD}$ ∴ m (∠ MDC) = 90°

, ∵ E is the midpoint of $\overline{AB}$

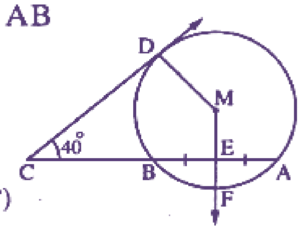
∴ $\overline{ME} \perp \overline{AB}$

∴ m (∠ MEC) = 90°

∴ m (∠ DMF)

= 360° – (40° + 90° + 90°)

= 360° – 220° = 140° (First req.)



, ∵ AE = $\frac{1}{2}$ AB = 8 cm.

, ∵ AM = r = 10 cm.

In Δ AEM : ∵ m (∠ AEM) = 90°

∴ $(ME)^2 = (AM)^2 - (AE)^2 = 100 - 64 = 36$

∴ ME = $\sqrt{36}$ = 6 cm.

∴ FE = MF – ME = 10 – 6 = 4 cm. (Second req.)

[b] ∵ $\overleftrightarrow{MN}$ is the line of centres

, $\overline{AB}$ is the common chord of the two circles

∴ $\overline{MN} \perp \overline{AB}$

, ∵ X is the midpoint of $\overline{AC}$

∴ $\overline{MX} \perp \overline{AC}$

, ∵ AB = AC ∴ MX = MD

, ∵ MY = ME (lengths of two radii)

∴ MY – MX = ME – MD

∴ XY = DE (Q.E.D.)

3

[a] ∵ $\overline{AB}$ is a diameter of the circle M

∴ m (∠ ADB) = 90°

In Δ ABD : ∵ m (∠ ABD) = 40°

∴ m (∠ A) = 180° – (90° + 40°) = 50°

, ∵ ABCD is a cyclic quadrilateral

∴ m (∠ C) + m (∠ A) = 180°

∴ m (∠ C) = 180° – 50° = 130° (The req.)

[b] ∵ $\overline{DE}$ // $\overline{BC}$

∴ m ($\widehat{BD}$) = m ($\widehat{CE}$)

∴ m (∠ DAB) = m (∠ CAE)

adding m (∠ BAC) to both sides

∴ m (∠ DAC) = m (∠ BAE) (Q.E.D.)

4

[a] ∵ m (∠ DMC) = 2 m (∠ CAD)

(central and inscribed angles subtended by $\widehat{CD}$)

∴ m (∠ DMC) = 2 × 30° = 60°

, ∵ $\overline{AB}$ // $\overline{DM}$, $\overline{BC}$ is a transversal

∴ m (∠ B) = m (∠ DMC) = 60° (corresponding angles)

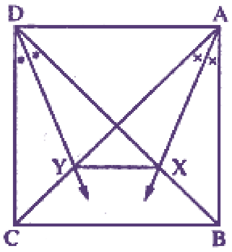
, ∵ $\overline{BC}$ is a diameter of circle M

∴ m (∠ BAC) = 90°

∴ In Δ ABC : m (∠ ACB) = 180° – (90° + 60°)

= 30° (The req.)

[b] ∵ ABCD is a square , $\overline{AC}$ and $\overline{BD}$ are two diagonals of the square



∴ m (∠ BAC) = m (∠ BDC)

∴ $\frac{1}{2}$ m (∠ BAC) = $\frac{1}{2}$ m (∠ BDC)

∴ m (∠ XAY) = m (∠ XDY) but they are drawn On $\overline{XY}$ and on one side of it

∴ The figure AXYD is a cyclic quadrilateral (Q.E.D. 1)

∴ m (∠ DXY) = m (∠ DAY) = 45°

(They are drawn on $\overline{DY}$ and on one side of it) (Q.E.D. 2)

5

∵ $\overline{XY}$, $\overline{XZ}$ are tangentsegments to the circle at Y and Z

∴ XY = XZ

∴ m (∠ XYZ) = m (∠ XZY) = $\frac{180° – 80°}{2}$ = 50°

∴ m (∠ ZEY) (the inscribed angle)

= m (∠ ZYX) (the tangency angle) = 50°

∵ The figure LEYZ is a cyclic quadrilateral

∴ m (∠ ZYE) = 180° – 130° = 50°

∴ m (∠ ZEY) = m (∠ ZYE) = 50°

∴ ZE = ZY (Q.E.D. 1)

∵ m (∠ XZY) = m (∠ ZYE) = 50°

but they are alternate angles.

∴ $\overline{XZ}$ // $\overline{YE}$ (Q.E.D. 2)

∵ In Δ ZYE : m (∠ EZY) = 180° – 2 × 50° = 80°

∴ m (∠ EZY) = m (∠ X) = 80°

∴ $\overline{ZE}$ is a tangent to the circle passing through The points X , Y and Z (Q.E.D. 3)